High Inflation:
Low Default Risk AND Low Equity Valuations*

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ABSTRACT

We develop an asset-pricing model with endogenous corporate policies that explains how inflation jointly impacts real asset prices and corporate default risk. Our model includes two empirically founded nominal rigidities: fixed nominal debt coupons (sticky leverage) and sticky cash flows. These two frictions result in lower real equity prices and credit spreads when expected inflation rises. A decrease in expected inflation has opposite effects, with even larger magnitudes. In the cross-section, the model predicts that the negative impact of higher expected inflation on real equity values is stronger for low leverage firms. We find empirical support for the model's predictions.

JEL Classification Numbers: E44, G12, G32, G33

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1 Introduction

Corporate default risk falls during times of higher expected inflation. But so do firms' equity values, despite lower bankruptcy risk. Figure 1 documents these two empirical facts for the U.S. over the period 1970Q2–2019Q4, after orthogonalizing expected inflation with respect to real economic conditions. Panel A illustrates the strong negative relationship between expected inflation and the number of quarterly defaults in the U.S., whereas Panel B shows a similar negative relationship between expected inflation and the price-dividend ratio. In this paper, we explain how shareholders can rationally value equity less favorably during periods of higher expected inflation, despite facing lower bankruptcy risk. We propose a theory that reconciles this apparent contradiction and provide novel empirical evidence that these relationships are robust features of the data.[1]

Existing theories have overlooked the connection between these two empirical relationships and only examined them separately. One branch of the literature focuses on the link between expected inflation and default risk, but yields counterfactual implications for equity valuation. In Bhamra, Fisher, and Kuehn (2011), Kang and Pflueger (2015), and Gomes, Jermann, and Schmid (2016), higher expected inflation increases both the nominal risk-free rate and the expected growth rate of a firm's nominal cash flows. Both effects reduce firms' indebtedness and default risk, but these models predict a counterfactual increase in equity prices. Another branch of the literature investigates the link between expected inflation and equity values, but this literature remains silent on implications for default risk (see, e.g., Modigliani and Cohn (1979), Feldstein et al. (1980), Ritter and Warr (2002), Sharpe (2002), and Campbell and Vuolteenaho (2004)).[2] A common explanation for the link between inflation and equity prices is money illusion: investors discount real cash flows with nominal discount rates.[3] In contrast to the existing literature, we propose a unified treatment of the empirical facts we illustrate in Figure 1.

We build on Bhamra, Kuehn, and Strebulaev (2010a,b) and Chen (2010) and construct an asset-pricing model with fluctuating levels of expected inflation to explain these apparently contradictory observations.[4] Our framework provides predictions on default risk and equity values from a corporate

[1]We show the relationships displayed in Figure 1 are not an artifact of firm aggregation: we find that the *same sets* of firms experience a decrease in default risk and equity valuation when expected inflation increases. Furthermore, firm-level regressions reveal that equity valuation and default risk jointly decrease with expected inflation after controlling for firm characteristics and variations in aggregate financial/economic conditions.

[2]Additional contributions to this literature are Lintner (1975), Bodie (1976), Fama and Schwert (1977), Miller, Jeffrey, and Mandelker (1976), Nelson (1976), Fama (1981), Schwert (1981), Geske and Roll (1983), Gultekin (1983), Solnik (1983), Pindyck et al. (1984), Kaul (1987), Pearce and Roley (1988), Kaul and Seyhun (1990), Boudoukh and Richardson (1993), and Bekaert and Wang (2010).

[3]Alternative explanations are the non-neutrality of inflation and the existence of an inflation risk premium. We describe the relevant literature in more detail below.

[4]Bhamra et al. (2010a,b) and Chen (2010) analyze firms' capital-structure and default decisions, as well as levered asset prices, in a consumption-based model with changing macroeconomic conditions. Our key modeling contribution relative to the above papers is the introduction of a new state variable (stochastic expected inflation) and two specific nominal rigidities (sticky leverage and sticky cash flows). Understanding how these two nominal rigidities impact the

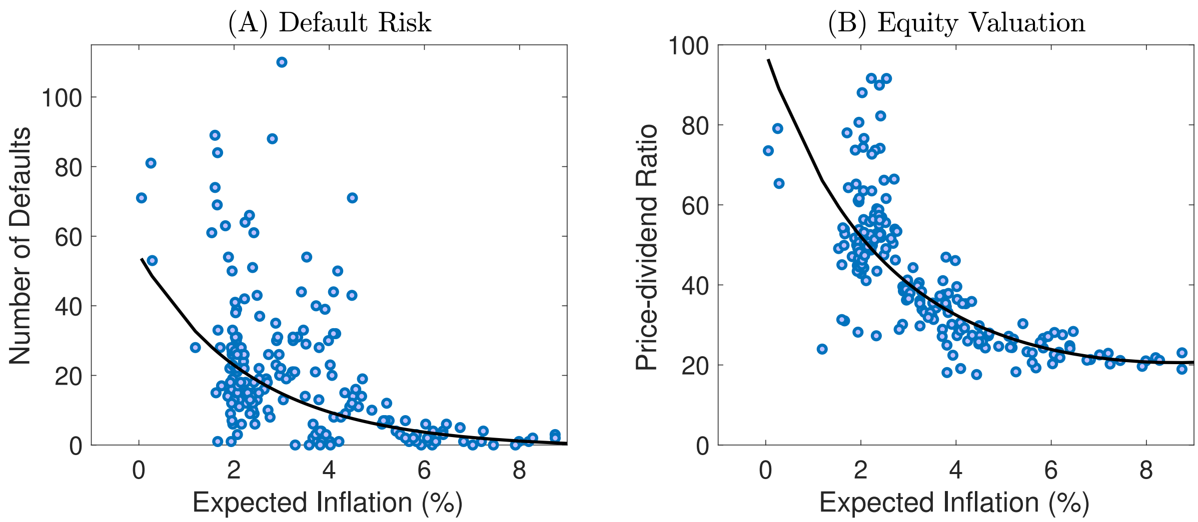


Figure 1: Default risk, equity valuation, and expected inflation in the U.S.

This figure illustrates the relationship between default risk, equity valuation, and expected inflation. Panel A reports the number of quarterly defaults of firms domiciled in the U.S. with debt rated by Moody's. Default data is from the Moody's Default and Recovery Database. Panel B displays the price-dividend ratio, computed as the value-weighted CRSP price index in the last month of the quarter divided by the sum of dividends paid in the last 12 months. Expected inflation is the one-year-ahead inflation forecast from the Survey of Professional Forecasters, which is orthogonalized with respect to real economic conditions, as measured by real consumption growth, NBER recessions, and a dummy for the Great Recession. The sample spans the period 1970Q2-2019Q4.

finance perspective, whereby firms' financing and default policies are endogenous. Firms issue nominal debt and equity, which are priced by a representative agent with Epstein-Zin-Weil preferences. The economy switches randomly between expansion and recession, creating intertemporal macroeconomic risk. A two-state Markov regime-switching model with parameter estimates based on quarterly U.S. consumption data over the period 1970Q2-2019Q4 determines the switches between real states. We introduce three expected inflation states (low, moderate, and high) via a second Markov regime-switching process that matches the one-year mean inflation forecast from the Survey of Professional Forecasters. We refer to fluctuations in the expected inflation rate as *inflation risk*, which is distinct from real macroeconomic risk. We consider both correlated and uncorrelated real macroeconomic risk and inflation risk regimes. Furthermore, we can allow for a time-varying correlation between shocks to consumption growth and inflation, consistent with the empirical evidence (see Bilal (2017), Boons, Duarte, de Roon, and Szymanowska (2020) and Campbell, Pflueger, and Viceira (2020)).

We impose two key frictions in our model, both of which act as nominal rigidities. First, firms keep their nominal debt coupons fixed. This *stickiness of leverage* means changes in expected inflation impact real asset prices via shifts in the real values of debt coupons. Second, price rigidity in the

way expected inflation affects endogenous corporate financial policies and real asset prices forms the novel theoretical core of this paper.

goods market implies sticky nominal cash flow growth in the short run, and so expected nominal cash flow growth changes less than one-for-one with changes in expected inflation. We denote this friction as *sticky cash flows* and find strong empirical support for this nominal rigidity in U.S. data. Our assumption is consistent with the evidence on the stickiness of output prices (see, e.g., Nakamura and Steinsson (2008), Gorodnichenko and Weber (2016)), Pasten, Schoenle, and Weber (2019) and D'Acunto, Liu, Pflueger, and Weber (2018) and nominal rigidities are also central to explain the real effects of large-scale asset purchase programs (Elenev (2019)).[5]

We show these two empirically motivated nominal frictions are sufficient to explain the stylized facts in Figure 1. Our model predicts an individual firm experiences both lower credit risk and lower equity valuation during periods of higher expected inflation. In addition, we extend our analysis to an economy of firms, whose distribution of leverage ratios is structurally similar to that in the data. We show the negative impact of expected inflation on equity valuation and default risk continues to hold–and thus does not cancel out–when aggregating over a cross-section of firms.

Each friction plays a distinct role in driving our theoretical results. The sticky leverage nominal rigidity is the key driver of the result that an increase in expected inflation reduces credit risk: an increase in expected inflation increases firm performance in nominal terms–or alternatively reduces the real debt coupon, which decreases the default probability. The sticky cash flow rigidity, however, determines how changes in expected inflation affect equity valuation, and does so via two distinct effects. First, a higher nominal cash flow growth rate caused by a rise in expected inflation increases the equity value. Second, firm cash flows are discounted at a higher nominal risk-free rate, which decreases the equity value. The latter discount rate effect dominates the former cash flow effect, because the nominal risk-free rate varies one-for-one with expected inflation, whereas the nominal cash flow growth rate varies less than one-for-one with expected inflation. Hence, equity value decreases with expected inflation, because of sticky cash flows.

The model also generates the convexity in the relations depicted in Figure 1. A decrease in expected inflation increases the value of equity, and it appears natural to assume an increase in expected inflation of the same size will result in an equal-sized decrease of equity values. But such an analysis is incomplete, because it ignores how the present value of firm cash flows depends non-linearly on the nominal risk-free rate, and thus on the level of expected inflation, via nominal discounting.[6] The relation between equity valuation and expected inflation is thus asymmetric, and low expected

[5]Nominal price rigidities are the leading explanation of the real effects of monetary policy. We show that sticky cash flows arise endogenously in a continuous-time New Keynesian model with aggregate risk, where firms have monopoly power and adjust their prices subject to quadratic costs as in Rotemberg (1982). It is also the case that menu-cost models with fixed costs generate a band of inaction, rationalizing price non-adjustment to shocks (see, e.g., Mankiw (1985) and Ball and Mankiw (1994)).

[6]This prediction arises although default probabilities are convex in the distance-to-default, which implies that an increase in default risk depresses the value of equity more than a decrease in default risk of the same size. But we show this effect is not sufficient to offset the asymmetry arising from nominal discounting.

inflation is not the mirror image of high expected inflation. Lower expected inflation increases equity prices more than higher expected inflation depresses them, which implies the presence of fluctuations in expected inflation increases equity valuation on average.[7] Hence, the presence of inflation risk has a positive–and not a negative–effect on real asset values. Our paper thus contributes to understanding the impact of inflation risk on asset pricing and in showing that the existence of inflation fluctuations can be economically beneficial to investors.

At first glance, it may appear the sticky leverage assumption drives the model's results for credit risk while the sticky cash flow assumption drives the equity valuation results. If the two assumptions operated independently, it could call into question the rationale behind studying how expected inflation affects both equity valuation and credit risk. However, the sticky leverage assumption does impact equity valuation: in the cross section, the impact of expected inflation on equity values varies with firm leverage, because changes in expected inflation affect asset prices through two opposing channels: discounting and default risk. First, higher expected inflation decreases the value of equity through sticky cash flows, i.e., nominal cash flow growth does not increase one-for-one with expected inflation, while the nominal risk-free rate does. This discounting effect is independent of leverage. Second, default risk decreases when expected inflation goes up, and this relationship strengthens with leverage. The reduction in default risk partially offsets the decrease in equity valuation, especially for more highly levered firms. The model thus predicts equity prices of more highly levered firms are less sensitive to changes in expected inflation than those of less levered firms, because of sticky leverage.

We provide a detailed empirical investigation of the impact of expected inflation on both equity valuation and default risk. Our empirical analysis has two aims. First, we test the cross-sectional prediction that financial leverage reduces the sensitivity of equity values to expected inflation. Second, we verify that the negative and asymmetric relations are robust at the firm level. We use CRSP-Compustat merged data from April 1972 to December 2019 and exploit two firm-level measures of equity valuation: the market-to-book (M/B) ratio and the price-dividend ratio.[8] We compute a firm's financial distress risk and its implied physical default probability following Campbell, Hilscher, and Szilagyi (2008). A portfolio analysis with firms sorted on their financial leverage ratios shows that default risk and equity valuation decrease with the level of expected inflation, for both low and high-leverage firms. The reduction in equity valuation is, however, stronger for less-levered firms, which means that higher default risk reduces, rather than exacerbates, the sensitivity of equity prices to changes in nominal conditions. The validation of this cross-sectional prediction provides support for our model.

[7]To reach this conclusion, we compare the model's prediction with that of an hypothetical economy with expected inflation set at its unconditional mean.

[8]The availability of forecasts for inflation determines the starting point of the sample.

A potential concern is that variations in expected inflation reflect changes in economic or financial conditions, and that the resulting changes in default risk and equity valuation at the portfolio level are due to grouping heterogeneous firms. We address these issues by running firm-level regressions with 798,288 firm-quarter observations and a rich set of financial, macroeconomic, and firm-level controls. We find the negative and asymmetric impact of expected inflation on a firm's equity valuation and default risk are highly statistically significant, and the results remain robust to different samples of firms and subperiods. The results continue to hold when we condition on firms that remain in our sample throughout the period, which ensures a firm-selection effect does not explain our findings. Furthermore, consistent with our model's predictions, the negative relation between expected inflation and equity values strengthens for firms with lower leverage or with more cash flow stickiness, using the frequency of price adjustments from Pasten, Schoenle, and Weber (2017). Our empirical analysis thus provides robust support that U.S. firms display lower equity values, despite lower default risk, when expected inflation increases and that the sensitivity of a firm's equity value to expected inflation is decreasing in leverage and increasing in price stickiness.

Our paper makes several contributions. First, we build a model of multiple firms issuing debt and equity with the option to default, in which expected inflation impacts firms' asset prices because of sticky leverage and sticky cash flows. We explain the negative relation between equity valuation and expected inflation with sticky cash flows. Our model generates the negative impact of expected inflation on default risk through sticky leverage, which induces variations in real leverage. Second, we find that equity prices and default risk are more sensitive to a change in expected inflation when expected inflation is currently low than when it is high, which suggests a fundamental asymmetry in the effects of inflation risk. This asymmetry is important in light of the extremely low inflation levels we have observed during and after the Great Recession. Third, in the cross section, we show equity prices vary more with expected inflation for firms with less leverage and stickier cash flows. Finally, we empirically validate all these predictions at the firm level, which provides new evidence regarding the joint response of equity valuation and default risk to variations in expected inflation.

Existing studies going back to Fama (1981) provide explanations for the negative relation between equity valuation and expected inflation, based on the idea that inflation is non-neutral because it has a negative effect on real growth.[9] Agents demand a positive inflation risk premium, which reduces equity prices (e.g., Eraker, Shaliastovich, and Wang (2015)). Our model shows that sticky leverage combined with sticky cash flows is sufficient to generate the relations between expected inflation, equity valuation, and default risk we observe in the data. We can thus obtain negative relations between equity valuation and expected inflation without any inflation risk premium.

[9]Other studies exploring theoretically the interaction between inflation and equity returns include Day (1984), Stulz (1986), Wachter (2006), Gabaix (2008), Hess and Lee (1999), Chen (2010), Bansal and Shaliastovich (2013), and Gomes et al. (2016).

We also explore the impact of a non-zero correlation between real and nominal conditions, based on the evidence of an unconditionally small but strongly time-varying inflation risk premium (Boons et al., 2020).[10] We uncover two new results. First, when the unconditional inflation risk premium becomes sufficiently large, our model no longer generates the relationship between equity values and expected inflation shown in Figure 1. Our model in tandem with the data therefore places an upper bound on the unconditional inflation risk premium. Second, with significant time-variation in the correlation between shocks to consumption growth and inflation, within our model, equity values and credit spreads still decrease with expected inflation. Therefore, an unconditionally small, but time-varying inflation risk premium is consistent with the data.

To further assess the theoretical underpinnings of our model, we confront it with the empirical data on the term structure of equity yields and credit spreads. Empirically, we find the level of equity yields is increasing with expected inflation, in particular at the short end of the term structure.[11] Furthermore, we find the slope of the equity yield term-structure is downward sloping when expected inflation is high and upward sloping when expected inflation is medium or low. Our model is able to generate both these features of the data, because of cash flow stickiness. We also find the level of finite-maturity credit spreads is decreasing with expected inflation,[12] thus complementing our empirical findings that financial distress risk and physical default probabilities decrease with expected inflation. The difference in spreads is particularly high in the medium and long end of the term structure. Our model-implied finite-maturity credit spreads is able to reproduce this feature of the data, because of sticky leverage.

This paper contributes to the literature exploring empirically the relation between inflation and equity returns. Chen, Roll, and Ross (1986) and Ang, Briere, and Signori (2012) find inflation risk is priced in the cross section of U.S. equity returns, whereas Boons et al. (2020) show the inflation risk premium varies over time, conditional on the relation between inflation and the real economy. We provide a complementary approach to understanding the impact of expected inflation on equity values, which hinges on cash flow stickiness. Whereas Weber (2015) shows how inflation risk impacts equity returns via a sticky-price channel, we combine the idea of *sticky cash flows* with *sticky leverage*. Finally, Kang and Pflueger (2015), which studies how inflation risk impacts corporate bond prices, is another closely related paper. Our paper complements this study by jointly studying expected inflation, default

[10]While Piazzesi and Schneider (2006) shows inflation predicts consumption growth negatively, Boons et al. (2020) suggest the relation is time-varying. This finding is consistent with the evidence inflation periods do not always reflect a bad state of the economy. See, for example, Bekaert and Wang (2010), Campbell, Sunderam, and Viceira (2017), and David and Veronesi (2013).

[11]We use model-implied data from Giglio, Kelly, and Kozak (2021), which matches the available empirical data from Bansal, Miller, and Yaron (2017) and Van Binsbergen and Koijen (2017), but goes further back in time and includes the 1970's high inflation period.

[12]Following Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017), we combine different bond data sets to construct the corporate credit spreads over the period 1974Q3-2019Q4: the Lehman Brothers Fixed Income Database, TRACE, Mergent FISD/NAIC, and Datastream.

risk, and equity prices in a unified framework. Furthermore, we provide novel empirical evidence and a theoretical explanation for the asymmetric relation between asset prices and expected inflation.

In Section 2, we present a simple model to clarify the qualitative relationships between equity valuation, credit risk, and expected inflation, whereas in Section 3, we describe the fully fledged consumption-based asset-pricing model with inflation risk. In Section 4, we derive asset prices together with optimal default and capital-structure decisions. We discuss the model's theoretical predictions in Section 5. In Section 6, we provide novel empirical evidence for the relationship between expected inflation, equity values and debt values. We also confront our model with the data. In Section 7 we conclude.

2 Intuition from a Simple Model

In this section, we consider a simple, static model with exogenous financing and default policies. We develop intuition for the negative impact of expected inflation on equity valuation and credit risk. We also discuss why equity prices and credit risk are more sensitive to a decrease in expected inflation than to an increase in expected inflation, that is, why the relations are asymmetric. The Online Appendix OA.A provides details on derivations and proofs.

2.1 Economy

To value nominal asset prices, we specify a price index P_t that satisfies

$$\frac{dP_t}{P_t} = \mu_P dt,$$

where μ_P is expected inflation, which is constant.[13] We assume the price index is locally risk free. Therefore, the nominal risk-free rate is equal to the real interest rate r plus expected inflation μ_P, i.e. $r^{\$} = r + \mu_P$.

Consider a firm with time t nominal cash flow X_t. Under the risk-neutral probability measure $\mathbb{Q}$, the dynamics of X_t are given by

$$\frac{dX_t}{X_t} = \widehat{\mu}_X dt + \sigma_X dW_t^{\mathbb{Q}},$$

where $W_t^{\mathbb{Q}}$ is a standard Brownian motion under $\mathbb{Q}$. The nominal cash-flow growth volatility σ_X equals real cash-flow growth volatility, σ_Y, as the price index is locally risk free. Expected nominal cash-flow

[13] In the simple model of this section, there are no shocks to inflation, so realized inflation and expected inflation are the same. In the full model of Section 3, we introduce shocks to inflation, and realized inflation and expected inflation are no longer equal.

growth is the sum of real expected cash-flow growth $\widehat{\mu}_Y$ and a multiple φ of expected inflation μ_P, that is, $\widehat{\mu}_X = \widehat{\mu}_Y + \varphi\mu_P$, where φ captures the sensitivity of nominal cash-flow growth to expected inflation. Cash flows are sticky when $\varphi < 1$. In Section OA.O of the Online Appendix, we provide a model to show that sticky cash flows can arise endogenously when firms with monopoly power optimally adjust prices subject to menu costs.

Strong empirical evidence exists supporting the notion of sticky cash flows (see, e.g., Nakamura and Steinsson (2008) and Gorodnichenko and Weber (2016)), which we confirm by estimating the parameter φ with U.S. data. We regress the consensus forecast for the growth rate of corporate profits over the next 12 months on the consensus forecast for inflation over the same period, using data from the Survey of Professional Forecasters. The estimate of φ is 0.407 in column (4) of Table 1, which controls for variations in real macroeconomic conditions. This estimate is significantly lower than 1 (t-stat of 2.48), which indicates that the relation between expected nominal cash flow growth and expected inflation is less than one-for-one. Hence, cash flows are sticky with respect to changes in nominal conditions.

Table 1 [about here]

The firm issues equity and a bond. The corporate bond pays out a fixed nominal coupon of c dollars per unit of time until default which occurs at the first passage time $\tau_D = \inf_{t>0}\{X_t \leq X_D\}$, for some fixed default threshold X_D. The debt coupon is constant in nominal terms, that is, leverage is sticky. The firm has no residual value when default occurs and there are no taxes.[14]

Consider first the case of a bond with no default risk. The nominal price of this bond is given by

$$B^{\$}_{f,t} = \frac{c}{r + \mu_P},$$

from which we can immediately obtain the real bond price $B_{f,t} = B^{\$}_{f,t}/P_t$. Sticky leverage implies that both the real and nominal prices of a bond without default risk are decreasing in expected inflation, simply because the real value of the nominal coupon decreases with expected inflation. We report this relation in Panel A of Figure 2, using three levels of expected inflation: low ($\mu_P = 1\%$), moderate ($\mu_P = 3\%$), and high ($\mu_P = 5\%$).

Figure 2 [about here]

A corporate bond subject to default risk has a different exposure to expected inflation. The nominal price of such a bond is equal to

$$B^{\$}_t = B^{\$}_{f,t}\left[1 - q^{\$}_{D,t}(\mu_P)\right],$$

[14] We relax these assumptions in the full model (Section 3).

where the term $0 < q^{\$}_{D,t}(\mu_P) < 1$ is the Arrow-Debreu default claim, that is, the date t price of the security that pays out one dollar at the time of default. An increase in expected inflation improves firm performance in nominal terms, which decreases the Arrow-Debreu default claim (for any value of $\varphi > 0$). This effect originates from the nominal debt coupons that are constant and, thus, not adjusted with expected inflation.[15] This is a direct consequence of the stickiness of leverage, as in Bhamra et al. (2011), Kang and Pflueger (2015), and Gomes et al. (2016).

A change in expected inflation now has two opposing effects on corporate debt valuation (both present because of sticky leverage): both the present value of the nominal coupons, $B^{\$}_{f,t}$, and the Arrow-Debreu default claim, $q^{\$}_{D,t}(\mu_P)$, decreases with expected inflation. The pricing of default risk dampens the price sensitivity of a risky corporate bond to expected inflation, relative to that of a risk-free bond (Panel A). The yield of a risky corporate bond ($y^{\$} = c/B^{\$}_t$) then moves less than one-for-one with expected inflation, whereas the nominal risk-free rate ($r^{\$} = c/B^{\$}_{f,t}$) moves one-for-one with expected inflation (Panel B). The credit spread, which is defined as the nominal yield of the corporate bond minus the nominal risk-free rate

$$s_t = y^{\$} - r^{\$} = \frac{c}{B^{\$}_t} - \frac{c}{B^{\$}_{f,t}}, \tag{1}$$

thus decreases with expected inflation, because of sticky leverage (Panel C).[16,17] Without sticky leverage, the corporate bond subject to default risk displays a similar sensitivity to expected inflation as the risk-free counterpart, thereby turning off the negative exposure of credit risk to expected inflation.

With sticky leverage, the relation between the credit spread and expected inflation is not only negative but also convex, as expected inflation impacts default risk non-linearly via the Arrow-Debreu default claim (Panel D).[18] Intuitively, an increase in expected inflation decreases default risk more when default risk is currently high, which is when expected inflation is low.

We turn now to equity valuation. The nominal value of equity is given by the nominal firm value less the nominal bond value

$$S^{\$}_t = V^{\$}_t - B^{\$}_t,$$

[15]Without leverage stickiness, the firm would optimally increase the level of the nominal debt coupon to offset the decrease in default risk when expected inflation increases.

[16]See page OA-3 of Section OA.A in the Online Appendix for a proof.

[17]We can directly interpret the predictions on the credit spread as predictions on default risk, because the bond has no value in default.

[18]The Arrow-Debreu default claim satisfies $q^{\$}_{D,t}(\mu_P) = e^{-a(\mu_P)(x_t - x_D)}$, with $x_t = \ln X_t$ and $x_D = \ln X_D$. Note that $x_t - x_D$ is the distance-to-default in logarithms and so $a(\mu_P)$ acts as a discount rate, which is increasing in expected inflation. For a given distance-to-default, the relation between the Arrow-Debreu default claim $q^{\$}_{D,t}(\mu_P)$ and expected inflation μ_P is negative and convex, as verified in Appendix OA.A.

with

$$V_t^{\$} = \frac{X_t - X_D q_{D,t}^{\$}}{r^{\$} - \widehat{\mu}_X} = \frac{X_t - X_D q_{D,t}^{\$}}{r - \widehat{\mu}_Y + (1-\varphi)\mu_P},$$

where $V_t^{\$}$ is the present value of the firm's cash flows up until default. Observe that $V_t^{\$}$ is decreasing with expected inflation when $\varphi < 1$ (Panel E), and so the equity value, $S_t^{\$}$, is also decreasing with expected inflation when cash flows are sticky (Panel F).

The sticky cash flows assumption implies that expected inflation affects equity valuation through two distinct channels. First, higher expected inflation increases the nominal cash flow growth rate, which increases equity valuation. Second, firm cash flows are discounted at a higher nominal risk-free rate, which decreases equity valuation. The latter effect dominates the former because the nominal risk-free rate varies one-for-one with expected inflation, whereas the nominal cash flow growth rate varies less than one-for-one with expected inflation when $\varphi < 1$. Hence, equity valuation decreases with expected inflation, and this relation arises from sticky cash flows. A change in the nominal risk-free rate impacts equity values non-linearly via nominal discounting, which implies that the impact of changes in expected inflation on equity value is stronger when expected inflation is lower. Hence, the relation between equity valuation and expected inflation is negative and asymmetric.

To sum up, Figure 2 shows that equity values and credit risk are both negatively related to expected inflation. Hence, a firm displays lower equity prices and, at the same time, faces lower credit spreads (or default risk) when expected inflation increases. Furthermore, a change from moderate to low expected inflation has a greater impact than a change from moderate to high expected inflation, although we consider symmetric variations in expected inflation. Hence, low expected inflation is not the mirror image of high expected inflation.

This analysis demonstrates that firms can have lower levered equity valuations *and* lower default risk when expected inflation increases. This simple model assumes no arbitrage and does not make specific assumptions about preferences. The two critical features driving both relations are sticky cash flows, for which we find strong support in the data, and sticky leverage, which is an empirically grounded friction in the corporate debt market. Based on these insights, we now consider a dynamic version of the model with endogenous corporate policies to study how fluctuations in expected inflation jointly impact equity valuation and credit risk in a richer environment.

3 Model

This section presents a dynamic asset-pricing model with firms facing real and nominal risk. We first define aggregate consumption and inflation and derive the real and nominal stochastic discount factors, using an Epstein-Zin-Weil representative agent. We then derive the asset values of firms, which issue nominal debt and equity, and describe their optimal financing and default decisions.

3.1 Aggregate economic variables

We now specify the joint dynamics of aggregate consumption and inflation. Aggregate consumption at time t is denoted by C_t and the time t level of the price index by P_t, where

$$\frac{dC_t}{C_t} = \mu_{C,t}dt + \sigma_{C,t}dZ_t,$$
$$\frac{dP_t}{P_t} = \mu_{P,t}dt + \sigma_{P,t}dZ_{P,t},$$

and Z_t and $Z_{P,t}$ are standard Brownian motions under the physical probability measure $\mathbb{P}$ such that $E_t[dZ_{P,t}dZ_t] = \rho_{PC,t}$.

The conditional first and second moments of aggregate consumption growth, $\mu_{C,t}$, $\sigma_{C,t}$, conditional expected inflation, $\mu_{P,t}$ together with the volatility $\sigma_{P,t}$ and the correlation between shocks to consumption growth and the price index, $\rho_{PC,t}$ are all stochastic.[19]

We use a six state Markov chain to describe the joint real-nominal state of the economy. The current state of the Markov chain is denoted by s_t, which switches randomly between the six states described in the table below.

	s_t	g_t	$\sigma_{C,t}$	$\mu_{P,t}$	$\rho_{PC,t}$
Recession & Low Expected Inflation (RL)	1	$\mu_{C,R}$	$\sigma_{C,R}$	$\mu_{P,L}$	$\rho_{PC,RL}$
Recession & Moderate Expected Inflation (RM)	2	$\mu_{C,R}$	$\sigma_{C,R}$	$\mu_{P,M}$	$\rho_{PC,RM}$
Recession & High Expected Inflation (RH)	3	$\mu_{C,R}$	$\sigma_{C,R}$	$\mu_{P,H}$	$\rho_{PC,RH}$
Expansion & Low Expected Inflation (EL)	4	$\mu_{C,E}$	$\sigma_{C,E}$	$\mu_{P,L}$	$\rho_{PC,EL}$
Expansion & Moderate Expected Inflation (EM)	5	$\mu_{C,E}$	$\sigma_{C,E}$	$\mu_{P,M}$	$\rho_{PC,EM}$
Expansion & High Expected Inflation (EH)	6	$\mu_{C,E}$	$\sigma_{C,E}$	$\mu_{P,H}$	$\rho_{PC,EH}$

We have $\mu_{C,R} < \mu_{C,E}$ and $\sigma_{C,R} > \sigma_{C,E}$ to ensure the mean and volatility of consumption growth are procyclical and countercyclical, respectively. Also, $\mu_{P,L} < \mu_{P,M} < \mu_{P,H}$, to be consistent with the labelling described in the above table.

We allow for correlation between real and nominal states, because expected inflation can change at the same time as the moments of real consumption growth. The physical probability of the joint real-nominal state switching from s_{t-} to s_t, where $s_t \neq s_{t-}$ within a time interval of length dt is

[19] See, e.g., Boons et al. (2020), who document a small negative unconditional correlation between expected consumption growth and expected inflation in addition to a time-varying correlation, the latter being the focus of their analysis.

given by $\lambda_{s_{t-},s_t} dt$, where λ_{s_{t-},s_t} is the physical intensity of switching from state s_{t-} to s_t. Since the Markov chain has $N = 6$ states, there are $N(N-1) = 30$ such physical intensities λ_{ij}, where $i \neq j$ and $i, j \in \{1, \ldots, 6\}$. In the special case of independent real and nominal regimes, the 6×6 intensity matrix arises from two nested intensity matrices, associated with a 2-regime real Markov chain and a 3-regime nominal chain. As such, the probability of switching from a nominal regime to another becomes independent of the current real state.

3.2 Representative agent and stochastic discount factors

The representative agent has the continuous-time analog of Epstein-Zin-Weil preferences.[20] The real stochastic discount factor (SDF) at time t, π_t, depends on the state of the real economy and is given by (see Online Appendix OA.B for the derivation)

$$\pi_t = \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t} e^{\int_0^t p_{C,u}^{-1} du}\right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}},$$

where β is the rate of time preference, γ is the coefficient of relative risk aversion (RRA), and ψ is the elasticity of intertemporal substitution under certainty (EIS). The date t value of the claim to aggregate consumption per unit of time is denoted by $p_{C,t}$.[21] This price-consumption ratio depends on the state of the economy, denoted by s_t:

$$p_{C,t} = p_{C,i}, \text{ if } s_t = i.$$

The real stochastic discount factor at date-t, π_t, evolves as follows

$$\left.\frac{d\pi_t}{\pi_t}\right|_{s_{t-}=i, s_t=j} = -r_i dt - \gamma \sigma_{C,i} dZ_t + \sum_{j \neq i} (\omega_{ij} - 1) dN_{ij,t}^P, \; i, j \in \{1, \ldots, N\}, \; j \neq i, \tag{3}$$

where r_i is the equilibrium real risk-free interest rate in state $i \in \{1, \ldots, N\}$ and $N_{s_{t-},s_t,t}^P$ is a compensated Poisson process given by

$$dN_{s_{t-},s_t,t}^P = dN_{s_{t-},s_t,t} - \sum_{k \neq s_{t-}} \lambda_{s_{t-},k} dt, \; s_{t-}, s_t \in \{1, \ldots, N\},$$

[20] The continuous-time version of the recursive preferences introduced by Epstein and Zin (1989) and Weil (1989) is known as stochastic differential utility, and is derived in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness for the finite-horizon case.

[21] The price-consumption ratios for each real state are derived from a coupled system of nonlinear algebraic equations given in equation (OA.4) of the Online Appendix.

where $N_{s_{t-},s_t,t}$ is a Poisson process that jumps up by 1 when the state of the economy switches; that is, $N_{s_{t-},s_t,t} = 1$ if $s_t \neq s_{t-}$. The real interest rates are identical to those of Bhamra et al. (2010a) and Bhamra et al. (2010b), and given in Online Appendix OA.B.

Two distinct types of risk are priced. First, the increment in the standard Brownian motion, i.e. dZ_t, represents small but frequent changes in unexpected consumption growth, and $\gamma\sigma_{C,i}$ is the associated price of risk. Second, the increment in the compensated Poisson process, i.e. $dN^P_{s_{t-},s_t,t}$, is a martingale (under the physical measure $\mathbb{P}$) that represents the risk that the state of the economy changes, and the associated price of risk is $\omega_{s_{t-},s_t} - 1$. If the state of the economy moves from i to j, that is, if $s_{t-} = i$ and $s_t = j$, then $\omega_{ij} = \omega_j/\omega_i = (p_{C,j}/p_{C,i})^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}$, where the $N-1$ constants, $\omega_1, \ldots, \omega_{N-1}$ are determined by a system of $N-1$ nonlinear algebraic equations (see equation (OA.18) in Section OA.D of the Online Appendix). Observe that if $p_{C,j} < p_{C,i}$, then $\omega_{ij} > 1$, and so the real SDF increases because the price-consumption ratio falls – we interpret the change in state from i to j as a negative shock to the economy.

In general, a change in expected inflation triggers a jump in the real SDF. Changes in nominal conditions are priced positively (negatively) when a rise in expected inflation is associated with better (worse) real economic conditions. Our framework also nests the special case, where changes in real and nominal regimes are independent: transitions between nominal states are then unpriced and the price of risk $\omega_{ij} - 1$ associated with strictly nominal transitions is zero.

The pricing of securities is based on the risk-neutral switching probabilities per unit of time (that is, transition intensities), $\hat{\lambda}_{s_{t-},s_t}$, which are related to the physical switching probabilities, λ_{s_{t-},s_t}, via

$$\hat{\lambda}_{s_{t-},s_t} = \omega_{s_{t-},s_t}\lambda_{s_{t-},s_t},\ s_{t-} \neq s_t,$$

where $\omega_{s_{t-},s_t} = 1$ if $s_{t-} = s_t$.

Hence, under Epstein-Zin-Weil preferences, ω_{s_{t-},s_t} acts as a distortion factor, distorting physical transition intensities. The representative agent cares about future consumption growth and prefers early resolution of intertemporal risk ($\gamma > 1/\psi$), and so $\omega_{s_t,s_{t-}} > 1$, when s_t is a worse state than s_{t-}, which implies the risk-neutral probability per unit of time of the economy worsening is higher than the physical probability.

Financial securities have nominal prices, which requires us to consider a nominal stochastic discount factor for asset pricing. The date-t nominal SDF, denoted by $\pi^{\$}_t$, is defined as

$$\pi^{\$}_t = \frac{\pi_t}{P_t},$$

whose dynamics satisfy

$$\left.\frac{d\pi_t^{\$}}{\pi_t^{\$}}\right|_{s_{t-}=i,s_t=j} = -r_i^{\$}dt - \gamma\sigma_{C,i}dZ_t + \sum_{j\neq i}(\omega_{ij}-1)dN_{ij,t}^P.$$

and $r_i^{\$}$ is the nominal interest rate in state i, given by

$$r_i^{\$} = r_i + \mu_{P,i} - \gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i} - \sigma_{P,i}^2.$$

The nominal interest rate depends on both real and nominal states and can thus takes six different values; it changes when the conditional moments of consumption growth change and also when expected inflation changes. The nominal risk-free rate is lowest during the recession/low-inflation state and highest during the expansion/high-inflation state.

3.3 Firm cash flows

The date-t level of the real cash flow of an individual firm is denoted by Y_t and evolves under the physical probability measure $\mathbb{P}$ according to the process

$$\frac{dY_t}{Y_t} = \mu_{Y,t}dt + \sigma_{Y,t}dW_t.$$

Real cash flows have a conditional expected growth rate $\mu_{Y,t}$ and a conditional volatility $\sigma_{Y,t}$. Both moments are identical across firms. Increments in the standard Brownian motion W (under $\mathbb{P}$) represent frequent but small shocks to the firm's cash flow growth. We assume cash flow shocks are independent across firms and from shocks to consumption growth.[22] Consequently, systematic risk in real cash flows is exclusively originating from low-frequency but severe changes in economic conditions. The expected growth rate is higher in expansions than in recessions, whereas the conditional volatility is lower in expansions than in recessions.

Because firms issue nominal securities and pay nominal taxes, investors care about the dynamics of nominal cash flows. The firm's nominal date t cash flow level is then given by X_t, where

$$X_t \equiv Y_t P_t^{\varphi}, \tag{4}$$

[22]We ignore a non-zero correlation between real cash flows and consumption, because the asset-pricing and corporate financing implications are negligible. See, for example, Bhamra et al. (2010a, 2010b).

which thus satisfies

$$\frac{dX_t}{X_t} = \mu_{Y,t} + \varphi \left(\mu_{P,t} + \rho_{PY,t}\sigma_{Y,t}\sigma_{P,t} \right) dt + \sigma_{Y,t} dW_t + \varphi \sigma_{P,t} dZ_{P,t}.$$

If we assume that shocks to real cash flow growth and inflation are uncorrelated, i.e. $\rho_{PY,t} = 0$, then the above expression reduces to

$$\frac{dX_t}{X_t} = \mu_{X,t} dt + \sigma_{Y,t} dW_t + \varphi \sigma_{P,t} dZ_{P,t},$$

where

$$\mu_{X,t} = \mu_{Y,t} + \varphi \mu_{P,t}$$

and the volatility of nominal cash flow growth is given by

$$\sigma_{X,t} = \sqrt{\sigma^2_{Y,t} + \varphi^2 \sigma^2_{P,t}}.$$

The sticky cash flow parameter, φ, captures the extent to which changes in inflation expectations affect the firm's cash flow growth rate.

Overall, firms exhibit heterogeneity in their cash flows due to firm-specific shocks but, at the same time, all firms have identical conditional moments for the cash flow growth rate.

4 Asset Prices and Corporate Financing Decisions

In this section, we derive asset prices together with optimal default and capital-structure decisions.

4.1 Nominal debt and leverage stickiness

Firms pay taxes on nominal cash flows X_t and issue debt to shield profits from taxes. Each firm has a debt contract that is characterized by a constant and perpetual nominal debt coupon c. Leverage is sticky because the coupon is fixed in nominal terms. Hence, when the nominal state changes, the real coupon changes, which affects asset valuations. Consequently, sticky leverage acts as a nominal rigidity. In other words, firms cannot adjust the nominal quantity of debt to news about the inflation state.

4.2 Liquidation value

A firm is liquidated when its nominal cash flows reach a state-dependent boundary $X_{D,i}$, which equityholders select to maximize equity value.

The nominal asset value at the time of liquidation, denoted by $A^{\$}_{i,t}$ in state $i \in \{1, \ldots, N\}$, corresponds to the present value of the after-tax nominal unlevered cash flows:

$$A^{\$}_{i,t} = (1-\eta) X_t \frac{1}{r_{A,i}},$$

where η is the corporate tax rate and $\frac{1}{r_{A,i}}$ is defined by

$$\frac{1}{r_{A,i}} = E_t \left[\int_t^{\infty} \frac{\pi^{\$}_u}{\pi^{\$}_t} \frac{X_u}{X_t} du \middle| s_t = i \right].$$

The value of $r_{A,i} = v^{-1}_{A,i}$ is given by the reciprocal of the i'th element of the vector $V_A = [v_{A,1}, \ldots, v_{A,N}]^{\top}$ where

$$V_A = (R_A - \widehat{\Lambda})^{-1} 1_{N \times 1}. \tag{5}$$

$1_{N\times 1}$ is a $N \times 1$ vector of ones, R_A is the following $N \times N$ diagonal matrix

$$R_A = \text{diag}(r^{\$}_1 - \mu_{X,1}, \ldots, r^{\$}_N - \mu_{X,N}),$$

and $\widehat{\Lambda}$ is the $N \times N$ risk-neutral generator matrix of the Markov chain characterizing the real and nominal states of the economy, defined by

$$[\widehat{\Lambda}]_{ij} = \widehat{\lambda}_{ij},\ i, j \in \{1, \ldots, N\},\ j \neq i,$$
$$[\widehat{\Lambda}]_{ii} = -\sum_{j \neq i} \widehat{\lambda}_{ij},\ i \in \{1, \ldots, N\}.$$

We can interpret $r_{A,i}$ as the discount rate for a perpetuity with stochastic expected growth rate $\mu_{X,t}$, which is currently equal to $\mu_{X,i}$. If the economy stays in state i forever, the discount rate reduces to the standard expression $r_{A,i} = r^{\$}_i - \mu_{X,i}$. In general, however, the economy can change state, and so the discount rate depends on the risk-neutral generator matrix of the Markov chain governing the economy's transitions. The presence of the risk-neutral generator matrix as opposed to the physical generator matrix incorporates the pricing of risk.

4.3 Arrow-Debreu default claims

Default risk is central to firm valuation. We now express the value of a firm's assets as a function of a set of Arrow-Debreu default claims. We define an Arrow-Debreu default claim as an asset that pays out \$1 if default occurs in state j and the current state is i. We denote the nominal price of such a security by $q^{\$}_{D,ij,t}$, which satisfies (see Online Appendix OA.H)

$$q^{\$}_{D,ij,t} = E_t \left[\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t} I_{\{s_{\tau_D}=j\}} \middle| s_t = i \right], \tag{6}$$

where τ_D is the time at which default occurs and $I_{\{s_{\tau_D}=j\}}$ is an the indicator function that equals 1, if default occurs in state j, and zero otherwise.

When valuing assets that depend on the level of cash flows at the time of default, X_{τ_D}, we have to consider additional Arrow-Debreu securities, because our economy features "deep defaults." These defaults can occur when the state of the economy jumps from its current state to a worse state. Default boundaries are countercyclical and can suddenly move upward when the economy deteriorates. In such a situation, a fraction of firms may immediately default upon a change in state. Consider a firm that has a nominal cash flow level of \$10 while the default boundary is \$8. If the economy suddenly deteriorates by moving into a new state where the default boundary is \$11, the firm will immediately default. In fact, all firms with a nominal cash flow level below \$11 would default, thereby creating a default cluster. More formally, we can consider a firm with a nominal cash flow level X_{τ_D-}, at time τ_D-, which is the time just before default, where X_{τ_D-} is below the new state's default boundary, $X_{D,j}$. This firm will default as soon as the economy enters the new state, and so $X_{\tau_D-} = X_{\tau_D} < X_{D,j}$ ($X_{\tau_D-} = X_{\tau_D}$ because X is a continuous process). Hence, it is not necessarily the case that at default, a firm's cash flow level is at the default boundary. Consequently, to value securities that depend on a firm's cash flows, we need a modified set of Arrow-Debreu default claims. We derive them in Online Appendix OA.I.

This second type of Arrow-Debreu default claims pay out $\frac{X_{\tau_D}}{X_{D,j}}$ at default if default occurs in state j and the current state is i. The date-t nominal price of this security is denoted by $\tilde{q}^{\$}_{D,ij,t}$, where

$$\tilde{q}^{\$}_{D,ij,t} = E_t \left[\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t} \frac{X_{\tau_D}}{X_{D,j}} I_{\{s_{\tau_D}=j\}} \middle| s_t = i \right].$$

Overall, $N^2 = 36$ Arrow-Debreu default prices exist for each type, because $N = 6$ states characterize the aggregate economy.

4.4 Corporate bond value

A firm that issues debt promises to pay the nominal coupon c per unit time. If the firm defaults, debtholders recover a fraction of the after-tax unlevered asset value of the firm, whereas the remaining fraction is lost due to liquidation costs. We denote the state-dependent recovery rate by α_j if default occurs in state j. Hence, the time t nominal value of corporate debt, conditional on the current state being i, is given by

$$B_{i,t}^{\$} = cE_t\left[\int_t^{\tau_D} \frac{\pi_u^{\$}}{\pi_t^{\$}} du\right] + E_t\left[\frac{\pi_{\tau_D}^{\$}}{\pi_t^{\$}} \alpha_{s_{\tau_D}} A_{s_{\tau_D}}^{\$}(X_{\tau_D}) du\right].$$

The above expression is simply the present value of future coupon flows up until some random default time, τ_D, plus the present value of the unlevered firm assets net of liquidation costs. We can rewrite the above expression as

$$B_{i,t}^{\$} = c\left(\frac{1}{r_{P,i}^{\$}} - \sum_{j=1}^{N} q_{D,ij,t}^{\$} \frac{1}{r_{P,j}^{\$}}\right) + \sum_{j=1}^{N} \alpha_j A_j^{\$}(X_{D,j}) \tilde{q}_{D,ij,t}^{\$}, \tag{7}$$

where $r_{P,i}^{\$}$ is the nominal discount rate for a perpetuity paying a flow of \$1, conditional on the current state being i. Observe that

$$\frac{1}{r_{P,i}^{\$}} = E_t\left[\int_t^{\infty} \frac{\pi_u^{\$}}{\pi_t^{\$}} du | s_t = i\right].$$

To gain intuition for corporate bond price (7), note that $c\frac{1}{r_{P,i}^{\$}}$ is the present value in nominal terms of a default-free bond paying a coupon flow of c dollars in perpetuity. The expression $c\sum_{j=1}^{N} q_{D,1,ij}^{\$} \frac{1}{r_{P,j}^{\$}}$ is the present value of coupons lost because of the possibility of default, and $\sum_{j=1}^{N} \alpha_j A_j^{\$}(X_{D,j})\tilde{q}_{D,ij,t}^{\$}$ is the present value of the assets recovered.

The nominal discount rate for a constant nominal perpetuity, $r_{P,i}^{\$}$, is given by $r_{P,i}^{\$} = v_{B,i}^{-1}$, where $v_{B,i}$ is the i-th element of the vector $V_B = [v_{B,1}, ..., v_{B,6}]'$

$$V_B = (R^{\$} - \widehat{\Lambda})^{-1} 1_{N\times 1},$$

and $R^{\$}$ represents the $N \times N$ diagonal matrix such that $R_{ii}^{\$} = r_i^{\$}$. Therefore, $r_{P,i}^{\$}$ accounts for the possibility that the nominal risk-free rate takes different future values as macroeconomic fundamentals and expected inflation fluctuate over time.

4.5 Equity value

Shareholders are entitled to the firm's cash flows net of taxes and debt servicing as long as the firm does not default. When the firm is in default, which occurs at some random time τ_D, shareholders recover nothing and lose their rights to any future cash flows. The nominal value of equity at date t, conditional on the current state i, is then given by

$$S^{\$}_{i,t} = (1-\eta)E_t\left[\int_t^{\tau_D} \frac{\pi^{\$}_u}{\pi^{\$}_t}(X_u - c)du \middle| s_t = i\right]$$
$$= A^{\$}_i(X_t) - (1-\eta)\frac{c}{r^{\$}_{P,i}} - \sum_{j=1}^{N}\left(A^{\$}_j(X_{D,j})\tilde{q}^{\$}_{D,ij,t} - (1-\eta)q^{\$}_{D,ij,t}\frac{c}{r^{\$}_{P,j}}\right). \quad (8)$$

The first two terms of (8) represent the present value of cash flows net of coupon payments in the absence of default, whereas the summation term captures the present value of the net cash flows that shareholders lose in the case of default.

The equity risk premium in state-i is given by

$$\mu^{\$}_{R,i} - r^{\$}_i = \sum_{j\neq i}(1-\omega_{ij})\frac{S^{\$}_{j,t} - S^{\$}_{i,t}}{S^{\$}_{i,t}}\lambda_{ij} + \varphi\frac{X_t}{S^{\$}_{i,t}}\frac{\partial S^{\$}_{i,t}}{\partial X_t}\left(\gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i} + \sigma^2_{P,i}\right),$$
$$i, j \in \{1, \ldots, N\}. \quad (9)$$

Changes in nominal conditions affect the equity risk premium through two distinct channels: i) the correlation between real and nominal regimes generates a risk premium shown in the first component of (9) via a jump in the SDF; ii) the correlation between shocks to consumption and inflation, $\rho_{PC,i}$, generates an additional risk premium (second component of (9)). The portion of the equity risk premium induced by the pricing of random changes in nominal conditions is the inflation risk premium.

4.6 Default and capital structure decisions

Shareholders maximize the value of their default option by choosing when to default. The state-contingent endogenous default boundary X_{D,s_t} depends on the current real and nominal states of the economy, that is, $s_t \in \{1, ..., N\}$. Expected inflation matters for default decisions because a change in the nominal cash flow growth is not offset by a change in the nominal coupon rate; that is, leverage is sticky. Hence, equityholders are entitled to smaller expected future cash flows when expected inflation is low than when expected inflation is high.

The default boundaries satisfy the following $N = 6$ standard smooth-pasting conditions

$$\left.\frac{\partial S^{\$}_{s_t}(X)}{\partial X}\right|_{X=X_{D,s_t}} = 0,\ s_t \in \{1, ..., N\}. \quad (10)$$

Shareholders also choose the optimal nominal coupon to maximize firm value at time 0 by balancing marginal tax benefits from debt against marginal expected distress costs. Two features are noteworthy.

First, as is standard in the capital-structure literature (Leland, 1994), by maximizing firm value, shareholders internalize debtholders' value at time 0. However, in choosing default times, they ignore the considerations of debtholders. This feature creates the usual conflict of interest between equity- and debtholders. Second, the optimal coupon depends on the state of the economy at date 0. We denote the time 0 coupon by c_{s_0}, where, to emphasize this dependence, s_0 is the date 0 state of the economy. Shareholders choose the coupon to maximize date-0 firm value $F^{\$}_{s_0,0} = B^{\$}_{s_0,0} + S^{\$}_{s_0,0}$

$$c_{s_0} = \arg\max_{c} \ F^{\$}_{s_0,0}(c). \tag{11}$$

We obtain the optimal default and capital-structure decisions numerically by maximizing equation (11) subject to the conditions in equation (10). As a result, the optimal default boundaries depend on the debt policy, which the initial financing state determines. Hence, if the economy is in state i, the default boundary for nominal cash flows is given by $X_{D,i}(c_{s_0})$, where i denotes the dependence on the current state and c_{s_0} the dependence on the optimal coupon chosen in the initial state.

5 Theoretical Predictions

This section discusses how changes in expected inflation affect corporate asset prices and default risk.

5.1 Calibration

We calibrate the model to the U.S. economy over the period 1970Q2-2019Q4. The real states (R & E) are characterized by the conditional moments of quarterly real per capita consumption expenditures and real earnings growth. For the nominal regimes (L, M, & H), we use the quarterly mean of the one-year-ahead inflation forecasts from the Survey of Professional Forecasters, as reported by the Federal Reserve Bank of Philadelphia. We set the unconditional probabilities of being in the low (L) or high (H) expected-inflation regimes to be 25%. The sensitivity of nominal cash-flow growth to expected inflation is set to $\varphi = 0.407$, using the empirical estimate reported in Table 1.

In the core of our analysis, we intentionally consider a restricted version of the model in which inflation risk is absent from the stochastic discount factor, such that the inflation risk premium does not drive any of our predictions. In this benchmark case, no inflation risk premium exists, because (i) expected consumption growth and expected inflation change independently (due to the way the Markov chain governing the state of the economy s_t is specified) and (ii) shocks to consumption growth and expected inflation are uncorrelated. We thus analyze how higher expected inflation can negatively affect equity prices although inflation risk remains unpriced. We relax the assumption of uncorrelated real and nominal conditions in Section 5.7. Appendix A provides additional details on the calibration, while Table 2 summarizes the parameter values.

Table 2 [about here]

Table 3 reports the firm-level predictions in the case of independent real and nominal regimes and zero correlation between shocks to consumption growth and inflation. Unconditionally, the firm-level risk premium is 4.57%, while the credit spread is 154 bps with a leverage ratio of 37.8%. These model-implied moments are consistent with their empirical counterparts for an average Baa firm, which displays an average leverage ratio of 43.28% and a bond spread of 158 bps (Huang and Huang, 2012). Similarly, Kang and Pflueger (2015) report a leverage ratio of 41% and a credit spread of 153 bps.

Table 3 [about here]

5.2 Expected inflation, equity valuation, and default risk

Equity valuation decreases with the level of expected inflation (see solid-blue line in Panel A of Figure 3). Two opposing effects of an increase in expected inflation on equity valuation exist: a discounting channel, which reduces equity values via an increase in the nominal risk-free rate; and a cash flow channel, which increases equity values via an increase in nominal cash flow growth. The discounting channel dominates the cash flow channel, because of the sticky cash flow assumption. The reason is that the nominal risk-free rate changes one-for-one with expected inflation ($r_t^{\$} = r_t + \mu_{P,t} - \sigma_{P,t}^2$, when $\rho_{PC,t} = 0$), whereas the expected nominal cash flow growth rate changes less than one-for-one with expected inflation ($\mu_{X,t} = \mu_{Y,t} + \varphi\mu_{P,t}$) when cash flows are sticky ($\varphi < 1$).

Figure 3 [about here]

We now explain why the sticky leverage assumption implies that an increase in expected inflation reduces credit spreads (see solid-blue line in Panel B of Figure 3). A rise in expected inflation decreases the real value of debt coupons, because coupons are set in nominal terms, i.e. leverage is sticky. The fall in real coupon value creates a reduction in default risk and, thus, in credit spreads. Stickiness in leverage is the central driver of the negative relation between expected inflation and credit spreads, following the work of Bhamra et al. (2011), Kang and Pflueger (2015), and Gomes et al. (2016).[23]

Importantly, when it comes to equity valuations the default risk channel is not strong enough to fully counteract the discounting channel: equity valuations will still fall as expected inflation rises. Naturally, this effect will be muted for higher leverage firms, leading to the cross-sectional implications described in Section 5.6.

We find that the equity risk premium remains similar across nominal states and, as a result, is not driving our main finding regarding the negative relation between expected inflation and equity

[23]Shareholders' option value of defaulting, as captured by the level of the optimal default boundaries, also varies with expected inflation. An increase in expected inflation translates into a higher default boundary, but this effect on default risk remains modest.

valuation. The lack of correlation between real and nominal conditions means that the way leverage changes with respect to nominal conditions drives any link between changes in nominal conditions and the equity risk premium. Corporate bond prices actually fall with expected inflation (see solid-blue line in Panel C) together with equity values, so leverage is stable across nominal states (see solid-blue line in Panel D). The equity risk premium therefore does not vary materially with expected inflation (see Panel B of Table 3) in the baseline calibration. We investigate the effect of a non-zero correlation in Section 5.7.

5.3 Varying cash flow stickiness

The degree of cash flow stickiness shapes the relations between equity valuation and expected inflation, and between credit risk and expected inflation. We find that the negative relation between expected inflation and equity values strengthens for firms with more cash flow stickiness (see Panel A of Figure 3). As a counterfactual, turning off cash flow stickiness completely in the model ($\varphi = 1$) inverts the relation between equity prices and expected inflation: equity prices rise with expected inflation, at odds with our motivating empirical evidence (Figure 1). The intuition is straightforward: the discount rate is canceled out by the cash flow effect, and so, the only way expected inflation affects equity valuation is through the default risk channel. Without cash flow stickiness, higher expected inflation lowers default risk, which then increases equity values and reduces the equity risk premium (see Panel A of Table 4).

Table 4 [about here]

In contrast with equity valuation, the negative relation between credit spreads and expected inflation weakens for firms with more cash flow stickiness. This illustrates the tension inherent in using sticky cash flows to generate a joint decrease in equity valuation and default risk when expected inflation increases. Our model shows that, with reasonable degrees of cash flow and leverage stickiness, equity valuation and default risk jointly decrease with expected inflation. Thus far, these relations have only been studied separately in the existing literature.

5.4 Low versus high expected inflation: Asymmetry

Our model uncovers another new prediction: the relation between equity valuation and expected inflation is asymmetric, as illustrated by Figure 3, which implies that higher expected inflation is not the mirror image of lower expected inflation. Specifically, equity valuation increases by 16.1% (from 13.94 to 16.18) when the economy switches from moderate to low expected inflation, whereas it decreases by only 5.1% (from 13.94 to 13.23) when expected inflation switches from moderate to high expected inflation, as reported in Table 3.[24] The impact of a decrease in expected inflation on equity prices is therefore stronger than the impact of an increase in expected inflation, although both states are equally likely.

The reason underlying this result is the convexity of equity values with respect to expected inflation, which exists because of sticky cash flows. Importantly, a change in expected inflation has stronger effects on equity valuation when the denominator in the traditional Gordon growth formula is small, that is, when either expected inflation or the real risk-free rate, or both, are small. This prediction arises although default probabilities are convex in the distance-to-default, which implies that an increase in default risk depresses the value of equity more than a decrease in default risk of the same size. We find this latter effect is not sufficient to offset the asymmetry arising from nominal discounting.

Two direct implications of this asymmetry exists. First, when moving out of a low inflation environment, the initial increase in expected inflation has a more negative impact on asset prices than subsequent increases. Second, the presence of inflation risk increases unconditional asset valuation. Given the convex relation between equity value and expected inflation, the average equity value across the low and high expected inflation states is higher than the equity value during an average expected inflation state. Following the same reasoning, inflation risk increases debt and firm valuation, on average.

To quantify the role of inflation risk, we compare the results of the full model (Table 3) with the case in which we switch off variations in the nominal state (Table 5). In this latter specification, the expected inflation rate is set at its unconditional mean, which corresponds to the "moderate inflation" regime. Table 6 indicates that inflation risk increases asset valuations, on average, adding up to 0.47% of equity value and 1.2% of total firm value. This prediction translates, using a simple back-of-the-envelope calculation, into an increase in aggregate firm valuation of approximately US$1.13 trillion, given a total market capitalization of public U.S. companies of US$37.7 trillion (as of December 2019) and a leverage ratio of 40%. The existence of inflation fluctuations therefore has economically important asset pricing implications for investors.

Tables 5 and 6 [about here]

5.5 Representative firm vs. aggregation of firms

The results discussed so far are for a single firm with optimal capital structure. In the real world, firms' leverage ratios frequently deviate from their optimal levels. These deviations are not symmetric and do not cancel each other in the cross-section. We now verify that our predictions continue to

[24] We consider a firm being in the expansion state, but the message is qualitatively similar when considering the recession state.

hold for a distribution of firms. For this exercise, we consider an economy of 1000 firms, with optimal policies reported in Table 3. We specify a cross-section of firms that differ in their distance-to-default, such that the distribution of leverage ratios is structurally similar to that in the data.

Figure 4 shows that the impact of expected inflation on equity valuation and credit spreads does not depend on whether we consider an individual firm or an economy of firms. The results are moreover similar if we aggregate firms using an equally-weighted (Panels A and B) or a value-weighted (Panels C and D) approach, which indicates that small, risky firms are not driving the relations. The joint relations between equity valuations, default risk, and expected inflation are thus robust, as these relations do not vanish when aggregating a large cross-section of firms.

Figure 4 [about here]

5.6 Cross-sectional predictions

Leverage plays a central role in the model, so we can expect equity valuations and credit spreads of low and high-leverage firms to be differentially exposed to variations in expected inflation. In our analysis, firms with higher leverage are those with lower cash flow levels (and thus lower distance-to-default) than firms with lower leverage.

Insightful cross-sectional predictions arise because changes in expected inflation affect asset prices through two opposing channels: discounting and default risk. First, higher expected inflation decreases the value of equity through sticky cash flows, i.e., nominal cash flow growth does not vary one-for-one with expected inflation, while the nominal risk-free rate does. This discounting effect is independent of leverage. Second, default risk decreases when expected inflation goes up, and this relationship strengthens with leverage. The reduction in default risk partially offsets the decrease in equity valuation, especially for more highly levered firms. The model thus predicts that equity prices of firms with higher leverage are less sensitive to changes in expected inflation than firms with lower leverage.

We illustrate the impact of leverage on the sensitivity of equity values with respect to changes in expected inflation in Table 7. We report equity prices by nominal conditions for firms with low versus high financial leverage, with ratios of 35% and 55%, respectively. An increase in expected inflation (from L to H) generates a greater fall in equity valuation (3.86 vs 1.78) for less-levered firms than for more-levered firms. Hence, higher leverage reduces–rather than exacerbates–the sensitivity of equity valuation to changes in nominal conditions. Consistent with this mechanism, Table 7 shows that credit risk is more sensitive to expected inflation for firms with higher leverage: an increase in expected inflation (from L to H) generates a stronger fall in credit spreads (24.44 vs. 15.60 bps) for more-levered firms than for less-levered firms.

Table 7 [about here]

5.7 The inflation risk premium

We now relax the assumption of independent real-nominal conditions, based on findings in Boons et al. (2020), who document an unconditionally small but strongly time-varying inflation risk premium. First, we consider a time-varying correlation between shocks to consumption growth and shocks to inflation, denoted by $\rho_{PC,t}$ (*shock correlation*). Second, we allow for a non-zero correlation between expected consumption growth, $\mu_{C,t}$, and expected inflation, $\mu_{P,t}$ (*regime correlation*), by allowing both to switch at the same time.

We uncover two new results. First, accounting for significant time-variation in the consumption-inflation correlation has no impact on our main model predictions: equity values and credit spreads still decrease with respect to expected inflation. Second, a large unconditional inflation risk premium does not generate the relationship between equity values and expected inflation we document in Figure 1, suggesting that the unconditional inflation risk premium cannot be too large.

Table 4 reports the results regarding the equity risk premium, which we discuss in detail within Appendix B. Panel B presents the predictions when we introduce a time-varying *shock correlation*, $\rho_{PC,t}$. Panel C considers different levels of the unconditional inflation risk premium (IRP), arising for different calibrations of the *regime correlation*. We summarize the main findings below.

With a *shock correlation*, equity values and credit spreads continue to decrease when expected inflation increases (see Figure B.1 in the Appendix), as was the case for zero correlation (see Figure 3). However, the equity risk premium now decreases with expected inflation, because the shock correlation decreases with expected inflation: the correlation between consumption and inflation shocks is 51.6% when expected inflation is low, -3.7% when expected inflation is moderate, and -24.2% when expected inflation is high.[25] A negative consumption-inflation correlation in times of higher expected inflation implies nominal cash flows become less correlated with consumption, thereby reducing the equity risk premium. The covariance between shocks to consumption growth and inflation is, however, small, because consumption growth and inflation are not very volatile, and so the inflation risk premium remains modest. Introducing a correlation between shocks to consumption growth and inflation therefore does not change our model's predictions.

When we allow for *regime correlation*, the equity risk premium is higher in the low inflation regime than in the high inflation regime. In addition, the relationship between equity valuation and expected inflation is no longer convex and loses its monotonicity for an unconditional inflation risk premium of 0.5% or more (see Figure B.2 in the Appendix). This result implies that (i) there is an upper bound on the *unconditional* inflation risk premium of around 0.25% per annum, and (ii) any significant inflation risk premium beyond this magnitude must be time varying. This finding is consistent with

[25] Our calibrated consumption-inflation correlation is unconditionally small, although highly time varying, consistent with the findings of Bilal (2017), Boons et al. (2020) and Campbell et al. (2020), among others.

the empirical evidence suggesting that, over the last 50 years, the inflation risk premium has switched sign and is unconditionally close to zero (Boons et al., 2020).

In sum, our analysis suggests the inflation risk premium can be time varying, but cannot be too high unconditionally. These findings provide further support for our baseline model. In addition, we find that the equity risk premium is highest when expected inflation is low in both of these alternative cases. Hence, introducing an inflation risk premium cannot rationalize the finding that equity valuation *decreases* with expected inflation, as suggested by Figure 1.

5.8 Equity and credit spread term structures

We now explore the model's implications for the term structures of equity yields and credit spreads.[26] We construct the term structure of earnings yields, where payoffs are impacted by default risk. The time-t nominal value of the unlevered equity strip paying off X_T at time T, conditional on being in state i is denoted by $S^{\$}_{i,T-t}$. Therefore, we have

$$\begin{aligned} S^{\$}_{i,T-t} &= (1-\eta)E_t\left[\frac{\pi^{\$}_T}{\pi^{\$}_t} X_T I_{\{\tau_D \geq T\}}\right] \\ &= (1-\eta)X_t e^{-y^{\$}_{i,T-t}(T-t)}, \end{aligned}$$

where $I_{\{\tau_D \geq T\}}$ is an indicator function that equals one if the firm does not default before time T, and zero otherwise. The implied earnings yield, denoted by $y^{\$}_{i,T-t}$ with time horizon $T-t$, is then equal to

$$y^{\$}_{i,T-t} = -\frac{1}{T-t}\ln\frac{E_t\left[\frac{\pi^{\$}_T}{\pi^{\$}_t} X_T I_{\{\tau_D \geq T\}}\right]}{X_t}. \tag{12}$$

To the best of our knowledge, no closed-form solutions can be obtained for the finite-maturity expectations in (12). We thus rely on Monte Carlo simulations to compute these expectations (see Appendix C).

We now describe the model-implied term structure of earnings yields. Figure 5 shows the term structure of earnings yields conditional on real (Panel A) and nominal (Panel B) states. In the baseline calibration with sticky cash flows, we uncover three predictions: First, the term structure is upward sloping in expansions and downward sloping in recessions, as documented in Bansal, Miller, Song, and Yaron (2021) and Giglio et al. (2021). Second, earnings yields increase with expected inflation, and the effect is strongest for short maturities. Third, except in the very short run, we obtain a downward sloping term structure in the high inflation state (H) and an upward sloping term structure in the

[26] We use the terms equity yields and earnings yields interchangeably.

lower inflation states (M and L). These results imply a convergence in earnings yields across nominal states as the horizon increases. In a counterfactual exercise, we compare the above results with the case of zero cash flow stickiness ($\varphi = 1$) in Figure 6. In the model without cash flow stickiness, we observe much less variation in the conditional term structure of earnings yields with respect to expected inflation and the ordering of states changes. Indeed, when expected inflation goes up, earnings yields become (modestly) lower.

Figures 5 and 6 [about here]

We then explore predictions on the term structure of corporate credit spreads in Panels C and D of Figure 5. We focus on the credit spreads of finite-maturity consol bonds, which we derive in Online Appendix OA.L. We find credit spreads are higher in recessions and in the low inflation state for any maturity. The credit spreads display a hump shape, that is, an upward-sloping term structure in the short term but downward-sloping term structure for longer horizons. Furthermore, the difference between credit spreads in high vs. low expected inflation states is stable over time due to sticky leverage.[27] In the model, firms do not adjust their capital structure as nominal conditions vary, which increases their default risk in times of lower expected inflation. This finding is consistent with the empirical evidence that firms tend to adjust their capital structure conservatively (Graham, 2000), which suggests that sticky leverage is a reasonable friction in our model.

5.9 Summary of theoretical predictions

We show that a rational model can explain why shareholders value stocks less favorably when default risk decreases, that is, in times of higher expected inflation. The asset-pricing implications of expected inflation do not vanish when shareholders optimally adjust the firm's capital structure and the timing of default to the presence of inflation risk. In addition, we find these relations hold in the case of endogenous corporate policies, both for a representative firm and a cross-section of firms, over different horizons, and when accounting for macroeconomic risk or the correlation between real and nominal conditions.

Our theory highlights the minimum set of frictions that are necessary to explain the seemingly conflicting relations between default risk, equity valuation, and expected inflation in an asset pricing model with optimal corporate financing decisions. The key channel for the relation between default risk and expected inflation is the presence of sticky leverage, whereas sticky cash flows drive the negative relation between equity valuation and expected inflation.[28] Therefore, we find that both sticky cash

[27] The patterns of the credit spread term structure are similar with (Figure 5) and without (Figure 6) cash-flow stickiness, given that the primary channel behind the relation between credit spreads and expected inflation is not cash-flow stickiness but sticky leverage.

[28] Alternatively, investors may discount real cash flows with nominal discount rates, which induces real equity valuations to decrease with expected inflation. In this paper, we assume that the agent is fully rational and thus does not suffer from

flows and sticky leverage, which are plausible channels, help us understand how expected inflation jointly affects equity valuation and default risk.

6 Empirical Analysis

This section has four aims. First, we provide robust evidence for the empirical relations that arise in our theoretical model: equity valuation and default risk jointly decrease with expected inflation. Second, we verify that these relations are asymmetric. Third, we test our theoretical cross-sectional predictions that the relation between equity valuation and expected inflation is stronger for firms with less leverage and with more sticky cash flows. Fourth, we show that the term structures of equity yields and credit spreads are consistent with our model's predictions.

6.1 Data

Our empirical analysis is based on the following data. Expected inflation is the year-on-year expected GDP-deflator inflation from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters. We consider two measures of equity valuation: the firm's market-to-book (M/B) equity ratio and the price-dividend ratio. Default risk is measured by a firm's financial-distress risk, following Campbell, Hilscher, and Szilagyi (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Appendix D provides details on the computation of these measures. Accounting variables are from Compustat Fundamental Quarterly data, whereas stock price variables are from CRSP. The dataset spans from April 1972 to December 2019. Table 8 displays the summary statistics.

Table 8 [about here]

6.2 Relations between equity valuation, default risk, and expected inflation

We first compute the average price-dividend ratios, market-to-book ratios, and default risk for each of the six states in the model. Table 9 reports the results. Expansions (E) and recessions (R) are determined by the median real GDP growth. Low (L) and high (H) expected inflation states are determined by the bottom and top quartile of expected inflation; the moderate (M) state spans the interquartile range. The results show that both price-dividend ratios and market-book ratios decrease as expected inflation goes up, while distress risk and implied default probabilities fall. Also, price-dividend ratios and market-book ratios are lower in economic downturns, while distress risk and default

any type of money illusion. Accounting for this behavioral channel would merely reinforce the quantitative predictions of this paper.

probabilities are higher. Equity valuation thus decrease with expected inflation even though default risk falls, in line with the model predictions.

Table 9 [about here]

We then analyze the relation between equity valuations and default risk with expected inflation. Figure 7 displays the results for the price-dividend ratio (top panels), the market-to-book ratio (middle panels), and the implied bankruptcy probabilities (bottom panels). The panels plot the (value-weighted) averages of the firm characteristics against the level of expected inflation observed in the corresponding quarter. We disentangle the relations by level of financial leverage, which we define as long-term debt and debt in current liabilities over the sum of the numerator and stockholders' equity. The left panels report portfolios of firms with below-median leverage, whereas the right panels report firms with above-median leverage. Each panel uses a quadratic regression to fit the data.

Figure 7 [about here]

This graphical analysis suggests the price-dividend ratio, the market-to-book ratio, and the bankruptcy probability are all negatively related to the level of expected inflation. Importantly, each portfolio contains the same set of firms, thereby indicating a decrease in expected inflation simultaneously increases *both* a firm's equity valuation and its default risk. Furthermore, as our model predicts, the relations based on equity valuation appear to be stronger for low-leverage firms, whereas the relation based on default risk appears to be stronger for high-leverage firms.

6.3 Portfolio sorts

As a formal test of these cross-sectional relations, we now exploit portfolio double sorts. We first sort all firms into two portfolios based on their financial leverage. We then create three equal-sized portfolios depending on the level of expected inflation.

Table 10 reports the results. Panel A shows for conditional double sorts that both equity valuation and default risk decrease in expected inflation. The high expected inflation-minus-low expected inflation estimates are all negative and statistically significant within each leverage sort. In terms of magnitude, firms with low (high) leverage display an average price-dividend ratio of 97.1 (58.9) when expected inflation is low and 46.2 (24.8) when expected inflation is high. The market-to-book ratios are 3.78 (2.08) and 1.87 (0.95), respectively. These differences are economically large. Furthermore, the double differences by leverage ratios (that is, the difference between estimates of the high expected inflation minus low expected inflation estimates across high- and low-leverage firms) are also highly statistically significant.[29] These tests show that the relation between equity valuation and expected

[29]We bootstrap the double difference to calculate standard errors.

inflation is negative and stronger for firms with lower levels of financial leverage, consistent with our theory.

The conditional double sorts also indicate that the negative relation between distress risk and expected inflation is weakly stronger for high-leverage firms than for low-leverage firms. Both the sign and the (low) magnitude of this difference are consistent with the cross-sectional prediction of our model. Panel B of Table 10 confirms these results when we perform unconditional double sorts.

Table 10 [about here]

6.4 Firm-level regressions

We now show that the negative relations are robust features of the data and, in particular, hold at the individual firm level. To this end, we examine how valuation ratios and default risk at the firm level vary with expected inflation, while keeping constant other firm characteristics and aggregate economic conditions.

Our main regression specification is as follows:

$$E_{i,j,t} = \delta_P \mu_{P,t} + \mathbf{X}'_{i,j,t}\delta_{C_1} + \mathbf{Y}'_t \delta_{C_2} + \gamma_j + \epsilon_{i,j,t},$$

where $E_{i,j,t}$ denotes the equity valuation for firm i in industry j at quarter t, measured as the price-dividend ratio or the market-to-book ratio. In the analysis of default risk, $E_{i,j,t}$ captures firm i's default probability computed in quarter t. Keeping the same notation as in the model, $\mu_{P,t}$ reflects expected inflation in quarter t. We denote by $\mathbf{X}_{i,j,t}$ and $\mathbf{Y}_t$ the vectors of firm and global characteristics that we use as control variables. We include industry fixed effects (γ_j) to control for time-invariant differences across industry groups and cluster standard errors $\epsilon_{i,j,t}$ at the quarter level to allow for correlations in error terms of unknown form across firms in a given quarter.

Equity valuations and default probabilities vary with firm characteristics; therefore, accounting for such drivers is critical. Following Fama and French (2015), we consider the level of investment, profitability, and firm size as firm-level controls (see Appendix D for details of the variable definitions). We also include the year-on-year growth rate of U.S. industrial production, a recession indicator based on the NBER business-cycle dates, the trailing one-year return of the S&P 500 index, and the slope of the yield curve measured by the yield spread between the 10-year Treasury note and the three-month Treasury bill, because these factors predict U.S. defaults.[30] We also control for the recent period of unconventional monetary policies by including a dummy variable that is equal to 1 over the

[30] See, for example, Das, Duffie, Kapadia, and Saita (2007), Duffie, Saita, and Wang (2007), Campbell et al. (2008), Duffie, Eckner, Horel, and Saita (2009), Giesecke, Longstaff, Schaefer, and Strebulaev (2011), and Azizpour, Giesecke, and Schwenkler (2018).

2008Q1–2019Q4 period, and zero otherwise. These data are from the Federal Reserve Bank of St. Louis.

Table 11 reports the regression results. We see in columns (1)–(2) that expected inflation is a strong driver of the price-dividend ratio and the market-to-book ratio, beyond the information contained in firm fundamentals and economic/financial conditions. A one-standard-deviation decrease in the expected inflation rate (1.865) increases the price-dividend ratio by 17.29, which is economically sizable. Column (3) reports similar results for the level of distress risk. Results are similar if we end the sample in 2007 in columns (4)–(6) ensuring that the decade of low inflation after the Global Financial Crisis does not drive our results.

Table 11 [about here]

We now turn to another central prediction of the model: a decrease in expected inflation has a stronger impact on equity valuation and default risk than an increase in expected inflation. The following analysis tests for such asymmetry in the data. To investigate a potential non-linearity in the relation between the valuation ratios (or default risk) and expected inflation, we interact expected inflation with a dummy variable, $\mathcal{D}_{L,M}$, that takes the value of 1 when expected inflation is below the 75^{th} percentile. This choice follows from our calibration, in which high expected inflation corresponds to the top quartile. Table 12 show the relation between equity valuations and expected inflation is stronger when expected inflation is lower. The difference in the sensitivity to expected inflation is economically and statistically significant. The total effect of expected inflation on the price-dividend ratio is -15.15 when expected inflation is below the 75^{th} percentile, whereas it is only -2.65 for expected inflation in the top quartile. The same result holds for distress risk. The empirical support for such asymmetry confirms that an increase in expected inflation is not the mirror image of a decrease in expected inflation.

Table 12 [about here]

6.5 Cross-sectional analysis

We now use our regression-based analysis to test two key cross-sectional predictions of the model. First, we verify that the effect of expected inflation on the price-dividend ratio is weaker for high leverage firms. Columns (1) to (3) of Table 13 introduce an interaction term between expected inflation and firm leverage that confirms our theoretical prediction. In particular, the impact of expected inflation is statistically weaker for firms with higher leverage, controlling for expected drivers of financial leverage such as firm performance and aggregate economic/financial conditions.

Second, we verify the prediction that the relation between equity valuation and expected inflation becomes weaker when prices are less sticky. Columns (4) to (7) of Table 13 introduce an interaction between expected inflation with the frequency of price adjustments from Pasten et al. (2017). A higher frequency of price adjustments implies less sticky output prices, which we interpret as a lower degree of cash flow stickiness through the lens of our model. The interaction term is positive and statistically significant. The relation between the price-dividend ratio and expected inflation thus becomes weaker with a higher frequency of price adjustments, that is when prices are less sticky. The same result obtains for the relation between market-to-book ratio and expected inflation.[31] We can conclude that the strong negative relation between equity valuation and expected inflation observed in the data reflects a high degree of cash flow stickiness. This finding provides further support to our model.

Table 13 [about here]

6.6 Robustness analysis

In this section, we report several alternative tests to probe the robustness of our empirical findings. We first address the potential concern that variations in expected inflation reflect changes in economic or financial conditions, in particular given the low inflation levels observed during and after the Great Recession. It is therefore critical to exploit a measure of expected inflation that is independent of the business cycle. To this end, we first orthogonalize the level of expected inflation with respect to the NBER recession indicator and reproduce our portfolio analysis of Table 10 with this orthogonalized measure. Table 14 displays the results. Alternatively, Table 15 focuses on the 1972Q2–2007Q4 period to ensure that observations during and subsequent to the Great Recession do not drive the relation between expected inflation with equity valuation and default risk. In both analyses, the results continue to hold with the same economic magnitude and statistical significance.

Tables 14 and 15 [about here]

We go through the same exercise for the firm-level regressions, which all control for indicators of NBER recession and post-2007 years. Columns (4)–(6) of Tables 11 and 12 repeat the baseline analysis of columns (1)–(3) but for the 1972Q2–2007Q4 period, thereby excluding observations during which equity valuation and default risk are most sensitive to expected inflation. The relations continue to be negative and asymmetric. Furthermore, our rich set of financial, macroeconomic, and firm-level controls allows us to disentangle the impact of nominal and real conditions. Hence, we can rule out the concern that a high or a low inflation environment reflects a bad state of the economy, thereby driving equity valuation and default risk.

[31] By contrast, the relation between default risk and expected inflation becomes only marginally weaker with a higher frequency of price adjustments. To see that, a one-standard-deviation increase in the frequency of price adjustments reduces the impact of expected inflation on default risk from -0.184 ($= -0.19 + 0.03 \times 0.32$) to -0.178 ($= -0.19 + 0.03 \times (0.32 + 0.19)$). This change is economically negligible.

Our robustness analysis also considers alternative samples. First, we compare the findings with and without financial firms and utilities in Table 16, because they operate in regulated markets or have special capital structures. Second, columns (1)–(3) of Table 17 exclude all tech firms, which tend to display relatively high equity valuations. The results remain similar in all of these cases.

Tables 16 and 17 [about here]

Furthermore, we address the concern that the high levels of expected inflation in the 1970s may be a driver of our results. We thus exclude the pre-1980 period and report the results in columns (4)–(6) of Table 17. This analysis shows that our findings are not driven by the changes in equity valuation and default risk as a consequence of the large variations in expected inflation during that period. Expected inflation decreased substantially over time and at the same time valuation ratios have, on average, increased. To ensure that such a secular trend does not drive our baseline results, Table 18 directly controls for a linear time trend. The results remain qualitatively similar, that is, equity valuations decrease in expected inflation even after controlling for a time trend.

Table 18 [about here]

Finally, we verify that the results are not driven by the time-varying comovement between stocks and bonds. Over the last few decades, the correlation between stocks and bonds has switched signs (Bilal, 2017; Campbell et al., 2020; Boons et al., 2020), which might impact the relation between expected inflation, equity valuations, and default risk. Table 19 shows that the empirical effect of expected inflation on equity valuation and default risk is negative and statistically significant in periods of negative and positive stock-bond correlation, which we compute using the unsmoothed correlation of Campbell et al. (2020). Interestingly, the relations become steeper in times of positive stock-bond correlation, i.e., when an increase in expected inflation reduces consumption growth and is thus viewed as bad news.

Table 19 [about here]

6.7 Equity and credit spread term structures

Nominal price stickiness in output markets can temporarily generate sticky cash flows but this effect is likely of moderate persistence. To study these implications, we explore the equity and credit spread term structures in the data and compare them to the model predictions.[32] We empirically analyze the conditional term structure of forward equity yields, using the model-implied data from Giglio et al. (2021). The term structures of forward equity yields conditional on real and nominal states are

[32]We thank the editor and two referees for motivating this analysis.

displayed in Panels A and B of Figure 8. As in Giglio et al. (2021), the term structure is upward sloping in expansions and downward sloping in recessions. We provide evidence that the slope also varies strongly with expected inflation. Panel B of Figure 8 shows the slope of the term structure of equity yields is positive for states of low and moderate inflation, but turns negative for states of high inflation. The difference in the slope slowly compresses as the time horizon increases. Furthermore, the equity yields always increase with the level of expected inflation for any maturity, thereby providing further support for one of the key empirical results we address with our model: equity valuation decreases with expected inflation. These findings are all consistent with the predictions of our model with sticky cash flows.

We then estimate the average credit spreads by maturity using an extensive dataset of 20,068 corporate bonds issued by 2,123 firms, spanning the period matching the equity yield sample.[33] Panels C and D of Figure 8 show the value-weighted results conditional on real and nominal states, respectively. As illustrated in Panel C, credit spreads display a hump shape, that is, an upward-sloping term structure in the short term but a downward-sloping term structure for longer horizons, that is similar to the model-implied patterns reported in Figure 5. Credit spreads are also higher during periods of lower inflation for any maturity, confirming the main results of our paper that default risk increases with lower expected inflation. This result provides further evidence on the negative relation between expected inflation and default risk, as measured by the implied default probability in our main empirical analysis. The difference between the credit spreads in high vs. low expected inflation is persistent over time, which implies moderate convergence across nominal states in the long run. Again, the predictions of our model are consistent with this new set of observations.

Overall, one may expect that price rigidity in the goods market implies sticky nominal cash flow growth in the short run. Whether the price rigidity is transitory or not should be reflected in the response of the term structure of equity yields. Similarly, any expected adjustment in firms' capital structure should be reflected in the term structure of credit spreads. Our analysis suggests the difference in term structures across nominal conditions remains of the expected sign and stays large for a relatively long horizon, consistent with the size of the frictions (the degree of cash flow and leverage stickiness) we assume theoretically.

Figure 8 [about here]

6.8 Summary

Our empirical investigation of the impact of expected inflation on equity valuation and default risk highlights several main findings. First, we document that the relations are robust feature of the

[33] Appendix D describes the construction of the corporate credit spreads by maturity and provides a description of the bond data.

data and, in particular, hold for individual firms. Firm-level regressions reveal equity valuation and default risk jointly decrease with expected inflation, even after controlling for firm characteristics or for variations in aggregate financial, economic, and monetary conditions. Second, the relations are asymmetric, that is, a decrease in expected inflation has a stronger impact on a firm's default risk and equity valuation when expected inflation is low than when it is high. Third, we validate the cross-sectional prediction of our theory that the relation between equity valuation and expected inflation is stronger for less levered firms. Leverage thus reduces, rather than exacerbates, the sensitivity of equity valuations to changes in nominal conditions. Hence, our analysis provides novel empirical evidence that the relations are negative and asymmetric at the firm-level, and vary across financial leverage ratios. Fourth, we show firms with less sticky output prices are less sensitive to movements in expected inflation. Finally, we find our model implies term structures of equity yields and credit spreads that closely match those observed in the data, thereby lending further support for the model.

7 Conclusion

Default risk decreases in times of higher expected inflation, despite a fall in equity valuations. Our empirical contribution is to provide new evidence that these relations are robust features of the data, not only at the market level but also for individual firms. In particular, we show these relations are asymmetric and vary with firm leverage. Our theoretical contribution is to develop a model which jointly rationalizes these stylized patterns in the data. In the model, inflation risk impacts real asset prices via two empirically grounded nominal frictions: sticky leverage and sticky cash flows. Two key mechanisms are at play. First, long-term nominal debt coupons are fixed, but expected inflation varies. This stickiness in leverage makes expected future real debt coupons dependent on future expected inflation, ensuring that inflation risk impacts real corporate bond values and hence default risk. Second, the expected cash flow growth rate is less sensitive to variations in expected inflation than the nominal risk-free rate. This stickiness in cash flows makes equity prices decreasing in the nominal risk-free rate and hence in expected inflation.

Our model thus implies that higher expected inflation simultaneously decreases default risk and real asset values. Importantly, the relations are asymmetric, as a decrease in expected inflation increases real equity values by more than an increase in expected inflation of equal size. The effect on equity prices is also stronger for firms with less leverage. Hence, leverage dampens rather than exacerbates the sensitivity of equity valuation to inflation expectations. We find support for the model predictions in the data, lending credence to the idea that sticky leverage and sticky cash flows are important channels for understanding the impact of inflation risk on real asset values and corporate default risk.

References

Ang, A. A., M. Briere, and O. Signori (2012). Inflation and individual equities. *Financial Analysts Journal 68*(4), 36–55.

Azizpour, S., K. Giesecke, and G. Schwenkler (2018). Exploring the sources of default clustering. *Journal of Financial Economics 129*(1), 154–183.

Bai, J., T. G. Bali, and Q. Wen (2019). Common risk factors in the cross-section of corporate bond returns. *Journal of Financial Economics 131*(3), 619–642.

Ball, L. and N. G. Mankiw (1994). Asymmetric price adjustment and economic fluctuations. *The Economic Journal 104*, 247–261.

Bansal, R., S. Miller, D. Song, and A. Yaron (2021). The term structure of equity risk premia. *Journal of Financial Economics*.

Bansal, R., S. Miller, and A. Yaron (2017). Is the term structure of equity risk premia upward sloping. *Duke University and University of Pennsylvania Mimeo*.

Bansal, R. and I. Shaliastovich (2013). A long-run risks explanation of predictability puzzles in bond and currency markets. *Review of Financial Studies 26*(1), 1–33.

Beeler, J. and J. Y. Campbell (2012). The long-run risks model and aggregate asset prices: An empirical assessment. *Critical Finance Review 1*(1), 141–182.

Bekaert, G. and X. Wang (2010). Inflation risk and the inflation risk premium. *Economic Policy 25*(64), 755–806.

Bhamra, H. S., A. J. Fisher, and L.-A. Kuehn (2011). Monetary policy and corporate default. *Journal of Monetary Economics 58*(5), 480–494.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev (2010a). The aggregate dynamics of capital structure and macroeconomic risk. *Review of Financial Studies 23*(12), 4187–4241.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev (2010b). The levered equity risk premium and credit spreads: A unified framework. *Review of Financial Studies 23*(2), 645–703.

Bilal, M. (2017). Zeroing in: Asset pricing near the zero lower bound. *Available at SSRN 2897926*.

Bodie, Z. (1976). Common stocks as a hedge against inflation. *The Journal of Finance 31*(2), 459–470.

Boons, M., F. Duarte, F. de Roon, and M. Szymanowska (2020). Time-varying inflation risk and stock returns. *Journal of Financial Economics 136*, 444–470.

Boudoukh, J. and M. Richardson (1993). Stock returns and inflation: A long-horizon perspective. *The American Economic Review 83*(5), 1346–1355.

Campbell, J. Y., J. Hilscher, and J. Szilagyi (2008). In search of distress risk. *The Journal of Finance 63*(6), 2899–2939.

Campbell, J. Y., C. Pflueger, and L. M. Viceira (2020). Macroeconomic drivers of bond and equity risks. *Journal of Political Economy 128*(8), 3148–3185.

Campbell, J. Y., A. Sunderam, and L. M. Viceira (2017). Inflation bets or deflation hedges? the changing risks of nominal bonds. *Critical Finance Review 6*(2), 263–301.

Campbell, J. Y. and T. Vuolteenaho (2004). Inflation illusion and stock prices. *American Economic Review 94*(2), 19–23.

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *The Journal of Finance 65*(6), 2171–2212.

Chen, N.-F., R. Roll, and S. A. Ross (1986). Economic forces and the stock market. *Journal of Business*, 383–403.

Chordia, T., A. Goyal, Y. Nozawa, A. Subrahmanyam, and Q. Tong (2017). Are capital market anomalies common to equity and corporate bond markets? An empirical investigation. *Journal of Financial and Quantitative Analysis 52*(4), 1301–1342.

Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin (2001). The determinants of credit spread changes. *The Journal of Finance 56*(6), 2177–2207.

D'Acunto, F., R. Liu, C. Pflueger, and M. Weber (2018). Flexible prices and leverage. *Journal of Financial Economics 129*(1), 46–68.

Das, S. R., D. Duffie, N. Kapadia, and L. Saita (2007). Common failings: How corporate defaults are correlated. *The Journal of Finance 62*(1), 93–117.

David, A. and P. Veronesi (2013). What ties return volatilities to price valuations and fundamentals? *Journal of Political Economy 121*(4), 682–746.

Day, T. E. (1984). Real stock returns and inflation. *The Journal of Finance 39*(2), 493–502.

Duffee, G. R. (1998). The relation between treasury yields and corporate bond yield spreads. *The Journal of Finance 53*(6), 2225–2241.

Duffie, D., A. Eckner, G. Horel, and L. Saita (2009). Frailty correlated default. *The Journal of Finance 64*(5), 2089–2123.

Duffie, D. and L. G. Epstein (1992). Asset pricing with stochastic differential utility. *Review of Financial Studies 5*(3), 411–436.

Duffie, D., L. Saita, and K. Wang (2007). Multi-period corporate default prediction with stochastic covariates. *Journal of Financial Economics 83*(3), 635–665.

Duffie, D. and C. Skiadas (1994). Continuous-time security pricing: A utility gradient approach. *Journal of Mathematical Economics 23*(2), 107–131.

Elenev, V. (2019). Mortgage credit, aggregate demand, and unconventional monetary policy. *Working Paper*.

Elliott, R. J. (1982). *Stochastic calculus and applications*, Volume 2. Springer.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*(4), 937–969.

Eraker, B., I. Shaliastovich, and W. Wang (2015). Durable goods, inflation risk, and equilibrium asset prices. *The Review of Financial Studies 29*(1), 193–231.

Fama, E. F. (1981). Stock returns, real activity, inflation, and money. *American Economic Review 71*(4), 545–565.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of Financial Economics 116*(1), 1–22.

Fama, E. F. and G. W. Schwert (1977). Asset returns and inflation. *Journal of Financial Economics 5*(2), 115–146.

Feldstein, M. et al. (1980). Inflation and the stock market. *American Economic Review 70*(5), 839–47.

Freyberger, J., A. Neuhierl, and M. Weber (2017). Dissecting characteristics nonparametrically. Technical report, National Bureau of Economic Research.

Gabaix, X. (2008). Variable rare disasters: A tractable theory of ten puzzles in macro-finance. *American Economic Review 98*(2), 64–67.

Geske, R. and R. Roll (1983). The fiscal and monetary linkage between stock returns and inflation. *The Journal of Finance 38*(1), 1–33.

Giesecke, K., F. A. Longstaff, S. Schaefer, and I. Strebulaev (2011). Corporate bond default risk: A 150-year perspective. *Journal of Financial Economics 102*(2), 233–250.

Giglio, S., B. T. Kelly, and S. Kozak (2021). Equity term structures without dividend strips data. *Working paper*.

Gomes, J. F., U. J. Jermann, and L. Schmid (2016). Sticky leverage. *The American Economic Review 106*(12), 3800–3828.

Gorodnichenko, Y. and M. Weber (2016). Are sticky prices costly? Evidence from the stock market. *American Economic Review 106*(1), 165–99.

Graham, J. R. (2000). How big are the tax benefits of debt? *The Journal of Finance 55*(5), 1901–1941.

Gultekin, N. B. (1983). Stock market returns and inflation forecasts. *The Journal of Finance 38*(3), 663–673.

Hess, P. J. and B.-S. Lee (1999). Stock returns and inflation with supply and demand disturbances. *Review of Financial Studies 12*(5), 1203–1218.

Huang, J.-Z. and M. Huang (2012). How much of the corporate-treasury yield spread is due to credit risk? *The Review of Asset Pricing Studies 2*(2), 153–202.

Jobert, A. and L. C. Rogers (2006). Option pricing with markov-modulated dynamics. *SIAM Journal on Control and Optimization 44*(6), 2063–2078.

Kang, J. and C. E. Pflueger (2015). Inflation risk in corporate bonds. *The Journal of Finance 70*(1), 115–162.

Kaul, G. (1987). Stock returns and inflation: The role of the monetary sector. *Journal of Financial Economics 18*(2), 253–276.

Kaul, G. and H. N. Seyhun (1990). Relative price variability, real shocks, and the stock market. *The Journal of Finance 45*(2), 479–496.

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance 49*(4), 1213–1252.

Lintner, J. (1975). Inflation and security returns. *The Journal of Finance 30*(2), 259–280.

Mankiw, N. G. (1985). Small menu costs and large business cycles: A macroeconomic model of monopoly. *Quarterly Journal of Economics 100*(2), 529–538.

Miller, K. D., F. J. Jeffrey, and G. Mandelker (1976). The "fisher effect" for risky assets: An empirical investigation. *The Journal of Finance 31*(2), 447–458.

Modigliani, F. and R. A. Cohn (1979). Inflation, rational valuation and the market. *Financial Analysts Journal 35*(2), 24–44.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *Quarterly Journal of Economics 123*(4), 1415–1464.

Nelson, C. R. (1976). Inflation and rates of return on common stocks. *The Journal of Finance 31*(2), 471–483.

Pasten, E., R. Schoenle, and M. Weber (2017). Price rigidity and the origins of aggregate fluctuations. Technical report, National Bureau of Economic Research.

Pasten, E., R. Schoenle, and M. Weber (2019). The propagation of monetary policy shocks in a heterogeneous production economy. *Journal of Monetary Economics*.

Pearce, D. K. and V. V. Roley (1988). Firm characteristics, unanticipated inflation, and stock returns. *The Journal of Finance 43*(4), 965–981.

Piazzesi, M. and M. Schneider (2006). Equilibrium yield curves. *NBER macroeconomics Annual 21*, 389–472.

Pindyck, R. S. et al. (1984). Risk, inflation, and the stock market. *American Economic Review 74*(3), 335–351.

Ritter, J. R. and R. S. Warr (2002). The decline of inflation and the bull market of 1982–1999. *Journal of Financial and Quantitative Analysis 37*(1), 29–61.

Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy 90*(6), 1187–1211.

Schroder, M. and C. Skiadas (1999). Optimal consumption and portfolio selection with stochastic differential utility. *Journal of Economic Theory 89*(1), 68–126.

Schwert, G. W. (1981). The adjustment of stock prices to information about inflation. *The Journal of Finance 36*(1), 15–29.

Sharpe, S. A. (2002). Reexamining stock valuation and inflation: The implications of analysts' earnings forecasts. *The Review of Economics and Statistics 84*(4), 632–648.

Solnik, B. (1983). The relation between stock prices and inflationary expectations: The international evidence. *The Journal of Finance 38*(1), 35–48.

Stulz, R. M. (1986). Asset pricing and expected inflation. *The Journal of Finance 41*(1), 209–223.

Van Binsbergen, J. H. and R. S. Koijen (2017). The term structure of returns: Facts and theory. *Journal of Financial Economics 124*(1), 1–21.

Wachter, J. A. (2006). A consumption-based model of the term structure of interest rates. *Journal of Financial Economics 79*(2), 365–399.

Weber, M. (2015). Nominal rigidities and asset pricing. *Chicago Booth, Working paper*.

Weber, M. (2018). Cash flow duration and the term structure of equity returns. *Journal of Financial Economics 128*(3), 486–503.

Weil, P. (1989). The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics 24*(3), 401–421.

Table 1: Estimation of cash flow stickiness

This table reports estimates of the degree of cash flow stickiness, as determined by the sensitivity of expected cash flow growth to expected inflation. Expected cash flow growth is measured as the mean forecast for the one-year-ahead corporate profit growth rate, while expected inflation is measured as the mean forecast for one-year-ahead inflation. All growth rates are annualized. We report standard errors corrected for heteroskedasticity and serial correlation in parentheses. Statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively. Forecast data are obtained from the Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia. The control variables are retrieved from the Federal Reserve of St-Louis. The sample period is 1970Q2–2019Q4.

Dependent Variable: Expected Profit Growth				
	(1)	(2)	(3)	(4)
Expected Inflation	0.373**	0.383**	0.409**	0.407**
	(0.176)	(0.173)	(0.171)	(0.164)
Expected GDP Growth	3.806***	4.057***	4.262***	4.264***
	(0.283)	(0.320)	(0.313)	(0.309)
Consumption Growth		-0.225	0.009	0.011
		(0.166)	(0.167)	(0.177)
Industrial Production Growth			-0.198***	-0.197***
			(0.058)	(0.062)
NBER Recession				0.076
				(1.222)
Constant	-4.408***	-4.447***	-5.363***	-5.382***
	(0.970)	(0.965)	(0.942)	(1.004)
Nobs	199	198	198	198
R^2	0.578	0.582	0.606	0.606

Table 2: Model calibration

This table presents the parameter values of the model. Panel A reports the conditional moments of the economic environment. Panel B reports the conditional firm characteristics. We calibrate the model to the aggregate U.S. economy using real consumption data (non-durable goods plus service consumption expenditures). Expected inflation is measured as the mean forecast for one-year-ahead inflation. The moments of cash flows are estimated using Robert J. Shiller's aggregate earnings data. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. Each column displays the predictions for a specific state of the economy: the expected inflation rate can be low (L), moderate (M), or high (H), whereas the real economy can be in recession (R) or in expansion (E). The table also reports the unconditional predictions for a weighted average of these states. We retrieve the consumption data from the Bureau of Economic Analysis, while the forecast data are obtained from the Survey of Professional Forecasters provided by the the Federal Reserve Bank of Philadelphia. All estimates are in percentage points and annualized when applicable. The sample period is 1970Q2-2019Q4. The calibration is detailed in Section 5.1.

	Unconditional	Conditional					
		State 1 R & L	State 2 R & M	State 3 R & H	State 4 E & L	State 5 E & M	State 6 E & H
Panel A: Economic Environment							
Stationary Probability		3.27	6.54	3.27	21.74	43.45	21.73
Consumption Growth Rate	1.66	-1.85	-1.85	-1.85	2.19	2.19	2.19
Consumption Growth Volatility	1.06	1.52	1.52	1.52	0.99	0.99	0.99
Expected Inflation	3.46	1.78	3.46	5.15	1.78	3.46	5.15
Inflation Volatility	0.87	0.81	0.69	1.29	0.81	0.69	1.29
Real Interest Rate	4.06	3.77	3.77	3.77	4.10	4.10	4.10
Nominal Interest Rate	7.52	5.54	7.23	8.91	5.87	7.56	9.24
Risk-Free Discount Rate	7.44	6.33	7.57	8.19	6.35	7.60	8.22
Cash Flow Discount Rate	4.08	4.63	5.38	5.70	3.43	3.98	4.23
Panel B: Firm Characteristics							
Real Cash Flow Growth Rate	3.72	-24.33	-24.33	-24.33	7.94	7.94	7.94
Nominal Cash Flow Growth Rate	5.13	-23.61	-22.92	-22.23	8.66	9.35	10.04
Inflation Passthrough	0.407	0.407	0.407	0.407	0.407	0.407	0.407
Real Cash Flow Volatility	25.93	29.59	29.59	29.59	25.38	25.38	25.38
Recovery Rate	35.00	20.00	35.00	50.00	20.00	35.00	50.00
Tax Rate	15.00	15.00	15.00	15.00	15.00	15.00	15.00

Table 3: Firm policies and asset prices

This table presents the predictions of the model regarding endogenous firm policies and asset valuation. Panel A reports the coupon and the conditional default boundaries. The capital structure is chosen optimally in the state of expansion with moderate inflation. Panel B reports the conditional asset pricing quantities for the economy. Each column displays the predictions for a specific state of the economy: the expected inflation rate can be low (L), moderate (M), or high (H), whereas the real economy can be in recession (R) or in expansion (E). The table also reports the unconditional predictions for a weighted average of these states. Market leverage is the ratio of the market value of debt to the sum of the market values of debt and equity. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

	Unconditional	Conditional					
		State 1 R & L	State 2 R & M	State 3 R & H	State 4 E & L	State 5 E & M	State 6 E & H
Stationary Probability		0.0327	0.0654	0.0327	0.2174	0.4345	0.2173
Panel A: Corporate Policies							
Default Boundaries (Coupon: 0.7290)		0.2371	0.2475	0.2490	0.2018	0.2104	0.2115
Panel B: Asset Pricing Quantities							
Equity Value	13.63	10.17	8.76	8.33	16.18	13.94	13.23
Debt Value	8.16	8.33	7.42	7.02	9.24	8.10	7.60
Market Leverage (%)	37.76	45.02	45.86	45.73	36.34	36.74	36.48
Equity Risk Premium (%)	4.57	18.37	18.40	18.29	2.50	2.50	2.50
Credit Spreads (bps)	154.31	242.70	225.72	218.43	154.61	140.72	136.76

Table 4: Conditional equity risk premium under different model specifications

This table reports the conditional equity risk premium (in %) under alternative model specifications. Panel A reports the equity risk premium for our baseline model with and without sticky cash flows. Panel B reports the predictions of a model with correlated consumption and inflation shocks (shock correlation). Panel C reports the predictions of a model with correlated expected consumption growth and expected inflation (regime correlation). Each line of Panel C captures a different degree of regime correlation, which implies different levels of inflation risk premium (IRP). Each column reports model predictions for a different current state of the economy. The expected inflation rate can be low (L), moderate (M), or high (H), whereas the real economy can be in recession (R) or in expansion (E). The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

	Unc.	RL	RM	RH	EL	EM	EH
Panel A: No IRP							
Sticky cash flows	4.57	18.37	18.40	18.29	2.50	2.50	2.50
No stickiness	4.98	21.88	20.08	19.32	2.79	2.64	2.60
Panel B: Non-zero Shock Correlation							
Sticky cash flows	4.58	18.37	18.38	18.31	2.53	2.51	2.48
No stickiness	4.98	21.87	19.97	19.37	2.87	2.65	2.55
Panel C: Non-zero Regime Correlation							
IRP 25 bps	4.82	20.87	19.38	17.73	2.84	2.54	2.64
IRP 50 bps	5.07	23.67	20.40	17.27	3.21	2.58	2.77
IRP 75 bps	5.32	26.79	21.42	16.90	3.63	2.62	2.90
IRP 100 bps	5.57	29.94	22.31	16.63	4.05	2.66	3.02
IRP 125 bps	5.82	33.22	23.09	16.42	4.46	2.70	3.12

Table 5: Firm policies and asset prices – constant inflation

This table presents the predictions of the model without fluctuating nominal conditions. Expected inflation is constant and set to its unconditional mean over the sample period, which corresponds to the moderate expected inflation state (M). Panel A reports the coupon and the conditional default boundaries. The capital structure is chosen optimally in the state of expansion. Panel B reports the conditional asset pricing quantities for the economy. Each column displays the predictions for a specific state of the economy, which can be in recession (R) or in expansion (E). The table also reports the unconditional predictions for a weighted average of these states. Market leverage is the ratio of the market value of debt to the sum of the market values of debt and equity. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

	Unconditional	Conditional	
		State 2 R & M	State 5 E & M
Stationary Probability		0.1308	0.8692
Panel A: Corporate Policies			
Default Boundaries (Coupon: 0.7146)		0.2416	0.2054
Panel B: Asset Pricing Quantities			
Equity Value	13.56	8.99	14.25
Debt Value	7.97	7.39	8.06
Market Leverage (%)	37.29	45.11	36.12
Equity Risk Premium (%)	4.54	18.20	2.48
Credit Spreads (bps)	148.92	221.06	138.06

Table 6: Asset pricing implications of nominal risk

This table presents the impact of nominal risk on asset prices. It reports differences in asset pricing predictions between a model with fluctuating expected inflation and a model with constant expected inflation. In the latter case, the expected inflation rate is constant and set to its unconditional mean (i.e. moderate inflation state), and the model predictions are those of Table 5. The differences in asset values are in relative terms (%). The differences in leverage and equity risk premium are in percentage points, while the difference in credit spreads are in basis points. Each column reports model predictions for a different current state of the economy. The expected inflation rate can be low (L), moderate (M), or high (H), whereas the real economy can be in recession (R) or in expansion (E). The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

	Unconditional	Conditional					
		State 1 R & L	State 2 R & M	State 3 R & H	State 4 E & L	State 5 E & M	State 6 E & H
Change in Stationary Probability		0.0327	0.0654	0.0327	0.2174	0.4345	0.2173
Change in Equity Value	0.47	13.15	-2.61	-7.30	13.53	-2.21	-7.14
Change in Debt Value	2.44	12.73	0.40	-4.93	14.60	0.48	-5.66
Change in Firm Value	1.20	12.96	-1.25	-6.23	13.92	-1.24	-6.61
Change in Market Leverage (%)	1.25	-0.20	1.67	1.39	0.60	1.74	1.01
Change in Equity Risk Premium (%)	0.04	0.17	0.20	0.09	0.02	0.02	0.01
Change in Credit Spreads (bps)	5.39	21.64	4.66	-2.63	16.55	2.65	-1.30

Table 7: Cross-sectional predictions

This table presents the cross-sectional impact of nominal risk by market leverage. The table reports asset pricing predictions for firms that differ in their levels of cash flow, which generates cross-sectional differences in market leverage. Predictions are reported across nominal conditions for a firm with low (35%) and high (55%) leverage. The expected inflation rate can be low (L), moderate (M), or high (H), while the real economy is set at its unconditional state. The table also displays the difference in results between the high (H) and the low (L) expected inflation state, as well as the double difference. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

Expected Inflation	Equity		Credit Spread	
	Low Leverage	High Leverage	Low Leverage	High Leverage
L	21.18	9.74	130.33	231.76
M	18.26	8.37	117.96	214.36
H	17.32	7.96	114.73	207.32
H-L	−3.86	−1.78	−15.60	−24.44
Double Difference		2.08		−8.84

Table 8: Descriptive statistics

This table reports the summary statistics of the main variables. Financial variables at the firm level are value-weighted. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. The default probability is the marginal probability of bankruptcy or failure over the next quarter, which is computed as in Campbell et al. (2008). Section 6.1 provides details on the computation of the firm variables. N is the number of observations. The sample period is 1972Q2–2019Q4.

	Mean	SD	25% Perc	Median	75% Perc
Expected Inflation (%)	3.634	1.865	2.207	3.055	4.410
Price-Dividend Ratio	72.621	60.624	23.958	40.000	72.807
Market-Book Ratio	3.080	1.655	0.956	1.428	2.271
Distress Risk	-7.867	0.718	-8.266	-7.855	-7.383
Default Probability (bps)	4.561	0.376	2.570	3.875	6.211
Market Leverage	0.258	0.231	0.112	0.278	0.484
Net Income to Total Assets (%)	0.761	1.024	0.260	0.800	1.296
Excess Return (%)	3.721	35.122	-15.171	1.993	20.696
Return Volatility (%)	27.127	19.366	23.956	29.133	40.237
Size to Market	-8.605	2.636	-11.198	-9.102	-7.298
Short Term Assets to Total	0.054	0.071	0.015	0.037	0.083
Log Share Price	0.554	2.701	0.451	1.268	3.139
Change in Total Assets (%)	1.844	6.079	-0.702	1.542	4.341
Profitability	0.133	10.800	0.004	0.051	0.085
Size	7.047	1.965	4.704	6.198	7.637
IP Growth (%)	1.817	4.278	0.497	2.659	4.707
S&P500 (%)	9.732	15.787	1.388	10.926	19.345
Slope (%)	1.274	1.477	0.250	1.290	2.260
Frequency of Price Adjustment	0.323	0.191	0.163	0.253	0.486
N	798,288	798,288	798,288	798,288	798,288

Table 9: Conditional equity valuation and default risk

This table reports average price-dividend ratios, market-to-book ratios, and default risk by state. Reported estimates of distress risk are computed as in Campbell et al. (2008). The default probability is the marginal probability of bankruptcy or failure over the next quarter (reported in bps), whereas the distress risk measure corresponds to the logarithm of the default probability. Expansions (E) and recessions (R) are determined by the median real GDP growth. Low (L) and high (H) expected inflation states are determined by the bottom and top quartile of expected inflation; the moderate (M) state spans the interquartile range. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. The sample period is 1972Q2–2019Q4.

	State 1	State 2	State 3	State 4	State 5	State 6
	R & L	R & M	R & H	E & L	E & M	E & H
P/D Ratio	68.85	58.32	30.60	76.69	59.76	38.92
M/B Ratio	2.13	1.86	1.05	2.44	1.84	1.23
Distress Risk	−7.59	−7.80	−8.14	−7.61	−7.94	−8.29
Default Probability (bps)	5.07	4.10	2.92	4.95	3.55	2.52

Table 10: Equity valuation and default risk by expected inflation and leverage

This table reports double sorts of price-dividend ratios in columns (1)–(2), market-to-book ratios in columns (3)–(4) and default risk in columns (5)–(6) by firm market leverage and level of expected inflation. Panel A reports conditional double sorts, while Panel B reports unconditional double sorts. We value-weight variables at the portfolio level. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Reported estimates of default risk are computed as in Campbell et al. (2008). The default probability is the marginal probability of bankruptcy or failure over the next quarter (reported in bps), whereas the distress risk measure corresponds to the logarithm of the default probability. We bootstrap standard errors for the double differences. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4.

Panel A. Conditional Double Sorts									
						Distress Risk	Default Prob.	Distress Risk	Default Prob.
Expected Inflation		P/D Ratio		M/B Ratio					
		Low Leverage	High Leverage	Low Leverage	High Leverage	Low Leverage		High Leverage	
	Nobs	(1)	(2)	(3)	(4)	(5)		(6)	
L	190	97.08	58.89	3.78	2.08	−7.78	4.16	−7.53	5.39
		(1.72)	(0.90)	(0.05)	(0.02)	(0.02)		(0.02)	
M	189	92.23	53.36	3.61	1.88	−8.06	3.14	−7.68	4.62
		(1.66)	(1.03)	(0.05)	(0.09)	(0.01)		(0.02)	
H	189	46.18	24.81	1.87	0.95	−8.43	2.17	−7.99	3.40
		(0.98)	(0.56)	(0.04)	(0.02)	(0.01)		(0.02)	
H-L		−50.90	−34.08	−1.91	−1.13	−0.65		−0.46	
		(3.22)	(1.21)	(0.07)	(0.04)	(0.05)		(0.03)	
Double Difference			19.37		0.93			0.21	
			(0.12)		(0.04)			(0.01)	
Panel B. Unconditional Double Sorts									
L	190	90.94	58.29	3.76	1.81	−8.01	3.32	−7.58	5.10
		(1.76)	(0.86)	(0.05)	(0.02)	(0.01)		(0.02)	
M	189	90.48	51.12	3.72	1.76	−8.08	3.09	−7.61	4.96
		(1.66)	(0.99)	(0.08)	(0.02)	(0.02)		(0.02)	
H	189	49.22	26.43	1.99	1.00	−8.49	2.05	−8.02	3.28
		(0.82)	(0.47)	(0.03)	(0.01)	(0.02)		(0.02)	
H-L		−41.72	−31.87	−1.76	−0.81	−0.48		−0.44	
		(2.37)	(1.23)	(0.06)	(0.05)	(0.09)		(0.05)	
Double Difference			11.23		0.92			0.04	
			(0.14)		(0.05)			(0.01)	

Table 11: Regressions on expected inflation

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation, firm characteristics, and macro aggregates. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4 in columns (1)–(3) and 1972Q2–2007Q4 in columns (4)–(6). We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

	P/D Ratio	M/B Ratio	Default Risk	P/D Ratio	M/B Ratio	Default Risk
	(1)	(2)	(3)	(4)	(5)	(6)
Expected Inflation (μ_P)	−9.27	−0.14	−0.14	−9.90	−0.14	−0.18
	(0.52)	(0.01)	(0.01)	(0.51)	(0.00)	(0.01)
Investment	101.69	1.68	−0.77	107.26	1.69	−0.62
	(2.56)	(0.05)	(0.05)	(3.21)	(0.05)	(0.05)
Profitability	0.00	0.00	0.00	−0.01	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
log Size	3.44	0.25	−0.06	3.67	0.23	−0.04
	(0.17)	(0.00)	(0.00)	(0.22)	(0.01)	(0.00)
IP Growth	30.96	−0.11	−0.82	8.85	0.16	−0.51
	(8.69)	(0.15)	(0.22)	(10.53)	(0.16)	(0.24)
S&P Return	18.66	0.37	−0.10	12.18	0.42	0.07
	(2.81)	(0.05)	(0.06)	(3.18)	(0.05)	(0.06)
Yield Curve	−1.40	−0.06	−0.07	−2.03	−0.05	−0.07
	(0.29)	(0.00)	(0.01)	(0.30)	(0.00)	(0.01)
Leverage	−35.37	−1.91	1.56	−35.55	−1.80	1.50
	(0.76)	(0.03)	(0.02)	(0.83)	(0.02)	(0.01)
Recession	2.61	0.02	0.17	1.18	0.05	0.13
	(1.76)	(0.03)	(0.03)	(2.23)	(0.03)	(0.04)
$\text{Dummy}_{post\ 2008}$	−16.32	−0.23	0.05			
	(1.07)	(0.02)	(0.02)			
Industry FE	X	X	X	X	X	X
Nobs	798,288	798,288	798,288	592,819	592,819	592,819
R^2	16.71%	32.53%	49.34%	19.78%	35.95%	46.50%

Table 12: Regressions on expected inflation – Convexity

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation, including an interaction term capturing the asymmetry in the relations. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. $\mathcal{D}_{L,M}$ denotes a dummy variable that equals 1 when expected inflation is below the third quartile. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4 in columns (1)–(3) and 1972Q2–2007Q4 in columns (4)–(6). We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

	P/D Ratio	M/B Ratio	Default Risk	P/D Ratio	M/B Ratio	Default Risk
	(1)	(2)	(3)	(4)	(5)	(6)
Expected Inflation (μ_P)	−2.65	−0.08	−0.10	−4.18	−0.09	−0.01
	(0.55)	(0.01)	(0.01)	(0.38)	(0.01)	(0.02)
$\mu_P \times \mathcal{D}_{L,M}$	−12.50	−0.11	−0.08	−12.74	−0.13	−0.13
	(0.73)	(0.01)	(0.02)	(0.91)	(0.02)	(0.01)
Investment	101.02	1.68	−0.77	106.92	1.70	−0.62
	(2.55)	(0.05)	(0.05)	(3.08)	(0.04)	(0.05)
Profitability	0.00	0.00	0.00	0.19	0.01	0.00
	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)	(0.00)
log Size	3.21	0.25	−0.06	3.38	0.23	−0.04
	(0.16)	(0.00)	(0.00)	(0.22)	(0.01)	(0.00)
IP Growth	51.99	0.08	−0.68	26.45	0.41	−0.24
	(6.00)	(0.14)	(0.22)	(6.37)	(0.13)	(0.23)
S&P Return	17.34	0.36	−0.11	15.78	0.46	0.04
	(2.02)	(0.04)	(0.06)	(2.17)	(0.05)	(0.06)
Yield Curve	−0.05	−0.05	−0.06	−0.50	−0.03	−0.06
	(0.21)	(0.00)	(0.01)	(0.21)	(0.00)	(0.01)
Leverage	−35.19	−1.91	1.57	−35.44	−1.79	1.51
	(0.77)	(0.03)	(0.02)	(0.84)	(0.02)	(0.01)
Recession	1.94	0.02	0.17	0.03	0.05	0.14
	(1.23)	(0.02)	(0.03)	(0.02)	(0.02)	(0.04)
$\text{Dummy}_{post\ 2008}$	−22.70	−0.29	0.01			
	(0.91)	(0.02)	(0.03)			
Industry FE	X	X	X	X	X	X
Nobs	798,288	798,288	798,288	592,966	592,966	592,966
R^2	17.33%	32.60%	49.52%	20.62%	36.08%	47.04%

Table 13: Regressions on expected inflation – Interactions with leverage and price stickiness

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation by leverage and price stickiness. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Columns (1)–(3) add interaction terms between expected inflation and leverage, while columns (4)–(6) add interaction terms between expected inflation and the frequency of price adjustment (FPA) from Pasten et al. (2017). Higher frequencies imply lower price stickiness. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4. We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

	P/D Ratio (1)	M/B Ratio (2)	Default Risk (3)	P/D Ratio (4)	M/B Ratio (5)	Default Risk (6)
Expected Inflation (μ_P)	−11.65	−0.25	−0.16	−11.96	−0.30	−0.19
	(0.41)	(0.01)	(0.01)	(0.60)	(0.01)	(0.01)
$\mu_P \times$ Leverage	7.95	0.30	0.02			
	(0.47)	(0.01)	(0.00)			
$\mu_P \times$ FPA				4.57	0.41	0.04
				(0.57)	(0.02)	(0.01)
Investment	102.53	1.70	−0.66	112.36	1.79	−0.72
	(2.60)	(0.05)	(0.06)	(2.77)	(0.05)	(0.08)
Profitability	0.00	0.00	0.00	0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
log Size	3.36	0.25	−0.07	3.95	0.26	−0.07
	(0.17)	(0.00)	(0.00)	(0.15)	(0.00)	(0.00)
IP Growth	32.07	−0.10	−0.95	38.92	−0.23	0.42
	(8.45)	(0.14)	(0.19)	(9.70)	(0.19)	(0.22)
S&P Return	16.91	0.36	−0.22	17.12	0.36	0.13
	(2.73)	(0.05)	(0.05)	(2.54)	(0.05)	(0.05)
Yield Curve	−1.29	−0.06	−0.07	−0.25	−0.06	−0.06
	(0.28)	(0.00)	(0.01)	(0.29)	(0.00)	(0.01)
Leverage	−64.27	−3.01	2.29	−44.71	−1.97	3.95
	(2.00)	(0.06)	(0.01)	(0.92)	(0.02)	(0.02)
Recession	2.28	0.03	0.18	3.61	0.03	0.05
	(1.71)	(0.03)	(0.03)	(1.50)	(0.03)	(0.03)
$\text{Dummy}_{post\ 2008}$	−16.73	−0.26	0.00	0.92	−0.31	0.13
	(1.01)	(0.02)	(0.02)	(0.20)	(0.02)	(0.01)
FPA				−16.37	−1.82	0.91
				(2.57)	(0.09)	(0.04)
Industry FE	X	X	X	X	X	X
Nobs	798,288	798,288	798,288	445,728	445,728	445,728
R^2	17.04%	33.16%	50.56%	15.27%	32.44%	41.63%

Table 14: Equity valuation and default risk by orthogonolized expected inflation

This table reproduces Table 10 when the level of expected inflation is orthogonalized with respect to NBER recessions. We present double sorts of price-dividend ratios in columns (1)–(2), market-to-book ratios in columns (3)–(4) and default risk in columns (5)–(6) by firm market leverage and level of expected inflation. Panel A reports conditional double sorts, while Panel B reports unconditional double sorts. We value-weight variables at the portfolio level. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia and orthogonolized with respect to NBER recessions. Reported estimates of default risk are computed as in Campbell et al. (2008). The default probability is the marginal probability of bankruptcy or failure over the next quarter (reported in bps), whereas the distress risk measure corresponds to the logarithm of the default probability. We bootstrap standard errors for the double differences. We bootstrap standard errors for the double differences. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4.

Panel A. Conditional Double Sorts									
						Distress	Default	Distress	Default
Expected		P/D Ratio		M/B Ratio		Risk	Prob.	Risk	Prob.
Inflation		Low Leverage	High Leverage	Low Leverage	High Leverage	Low Leverage		High Leverage	
	Nobs	(1)	(2)	(3)	(4)	(5)		(6)	
L	190	93.67	59.22	3.87	1.91	−7.92	3.63	−7.58	5.11
		(1.72)	(0.91)	(0.06)	(0.02)	(0.02)		(0.02)	
M	189	89.93	53.19	3.53	1.86	−8.07	3.13	−7.67	4.64
		(1.63)	(0.99)	(0.05)	(0.09)	(0.01)		(0.03)	
H	189	45.99	24.66	1.86	0.95	−8.43	2.19	−7.98	3.43
		(0.96)	(0.55)	(0.04)	(0.02)	(0.01)		(0.02)	
H-L		−47.68	−34.56	−2.00	−0.97	−0.51		−0.40	
		(3.95)	(1.33)	(0.12)	(0.06)	(0.03)		(0.03)	
Double			16.93		1.12			0.12	
Difference			(0.09)		(0.03)			(0.01)	
Panel B. Unconditional Double Sorts									
L	190	89.59	58.21	3.78	1.83	−8.00	3.35	−7.59	5.08
		(1.73)	(0.87)	(0.05)	(0.02)	(0.01)		(0.02)	
M	189	81.99	51.28	3.69	1.75	−8.09	3.08	−7.61	4.96
		(1.49)	(0.98)	(0.08)	(0.02)	(0.02)		(0.02)	
H	189	49.08	26.37	2.00	1.00	−8.47	2.10	−8.01	3.31
		(0.80)	(0.46)	(0.03)	(0.01)	(0.02)		(0.02)	
H-L		−40.51	−31.85	−1.79	−0.82	−0.47		−0.43	
		(4.21)	(1.89)	(0.12)	(0.06)	(0.06)		(0.04)	
Double			10.21		0.96			0.05	
Difference			(0.12)		(0.04)			(0.01)	

Table 15: Equity valuation and default risk by expected inflation and leverage (1970–2007)

This table reports double sorts of price-dividend ratios in columns (1)–(2), market-to-book ratios in columns (3)–(4) and default risk in columns (5)–(6) by firm market leverage and level of expected inflation. Panel A reports conditional double sorts, while Panel B reports unconditional double sorts. We value-weight variables at the portfolio level. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Reported estimates of default risk are computed as in Campbell et al. (2008). The default probability is the marginal probability of bankruptcy or failure over the next quarter (reported in bps), whereas the distress risk measure corresponds to the logarithm of the default probability. We bootstrap standard errors for the double differences. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2007Q4.

Expected Inflation	Nobs	P/D Ratio Low Leverage (1)	P/D Ratio High Leverage (2)	M/B Ratio Low Leverage (3)	M/B Ratio High Leverage (4)	Low Leverage (5) Distress Risk	Low Leverage (5) Default Prob.	High Leverage (6) Distress Risk	High Leverage (6) Default Prob.
Panel A. Conditional Double Sorts									
L	138	130.15	72.69	4.66	2.35	−7.91	3.65	−7.56	5.22
		(1.58)	(0.96)	(0.08)	(0.04)	(0.02)		(0.03)	
M	135	72.62	41.25	3.21	1.59	−8.21	2.71	−7.73	4.39
		(1.14)	(0.93)	(0.05)	(0.03)	(0.02)		(0.02)	
H	138	44.08	23.15	1.79	0.90	−8.47	2.10	−7.96	3.49
		(0.86)	(0.55)	(0.03)	(0.02)	(0.02)		(0.02)	
H-L		−86.07	−49.54	−2.87	−1.45	−0.55		−0.40	
		(1.82)	(0.71)	(0.10)	(0.03)	(0.03)		(0.03)	
Double Difference			31.73		1.26			0.09	
			(0.03)		(0.00)			(0.00)	
Panel B. Unconditional Double Sorts									
L	138	121.99	68.83	4.40	2.21	−7.90	3.72	−7.50	5.55
		(1.43)	(1.02)	(0.08)	(0.03)	(0.02)		(0.03)	
M	135	72.38	41.02	3.19	1.58	−8.21	2.72	−7.73	4.40
		(1.13)	(0.91)	(0.04)	(0.03)	(0.02)		(0.02)	
H	138	48.68	26.77	2.02	1.03	−8.56	1.92	−8.07	3.13
		(0.93)	(0.64)	(0.04)	(0.02)	(0.02)		(0.02)	
H-L		−73.31	−42.06	−2.37	−1.18	−0.66		−0.57	
		(1.68)	(0.76)	(0.10)	(0.03)	(0.03)		(0.02)	
Double Differece			27.27		1.07			0.02	
			(0.03)		(0.00)			(0.00)	

Table 16: Regressions on expected inflation – Convexity: excl Finance and Utilities

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation excluding Finance and Utilities. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. $\mathcal{D}_{L,M}$ denotes a dummy variable that equals 1 when expected inflation is below the third quartile. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4 in columns (1)–(3) and 1972Q2–2007Q4 in columns (4)–(6). We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

	P/D Ratio (1)	M/B Ratio (2)	Default Risk (3)	P/D Ratio (4)	M/B Ratio (5)	Default Risk (6)
Expected Inflation (μ_P)	−3.75	−0.10	−0.11	−5.25	−0.10	−0.01
	(0.70)	(0.01)	(0.01)	(0.48)	(0.01)	(0.02)
$\mu_P \times \mathcal{D}_{L,M}$	−15.25	−0.05	−0.08	−18.62	−0.08	−0.14
	(0.96)	(0.02)	(0.02)	(0.87)	(0.02)	(0.01)
Investment	119.98	1.75	−0.83	122.42	1.81	−0.67
	(3.31)	(0.05)	(0.05)	(3.89)	(0.05)	(0.05)
Profitability	0.00	0.00	0.00	0.16	0.01	0.00
	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
log Size	3.30	0.33	−0.05	2.93	0.28	−0.03
	(0.19)	(0.01)	(0.00)	(0.25)	(0.01)	(0.00)
IP Growth	54.80	0.41	−0.69	27.31	0.64	−0.47
	(6.80)	(0.20)	(0.22)	(7.74)	(0.15)	(0.24)
S&P Return	19.96	0.49	−0.15	20.52	0.58	0.00
	(2.58)	(0.06)	(0.06)	(2.75)	(0.06)	(0.06)
Yield Curve	−0.37	−0.07	−0.06	−0.58	−0.04	−0.06
	(0.27)	(0.01)	(0.01)	(0.28)	(0.01)	(0.01)
Leverage	−33.21	−2.39	2.12	−34.90	−2.32	2.09
	(0.96)	(0.04)	(0.02)	(1.02)	(0.04)	(0.01)
Recession	2.69	−0.06	0.18	−0.34	0.08	0.17
	(1.59)	(0.03)	(0.03)	(1.43)	(0.02)	(0.03)
Dummy$_{post\ 2008}$	−31.19	0.01	−0.03			
	(1.16)	(0.04)	(0.02)			
Industry FE	X	X	X	X	X	X
Nobs	464,322	464,322	464,322	356,938	356,938	356,938
R^2	15.75%	34.70%	48.65%	19.01%	38.40%	48.31%

Table 17: Regressions on expected inflation – Convexity: no Tech or pre-1980

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation using different samples. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. $\mathcal{D}_{L,M}$ denotes a dummy variable that equals 1 when expected inflation is below the third quartile. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4 in columns (1)–(3) but excludes all tech firms, while it is 1980Q1–2019Q4 in columns (4)–(6). We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

	P/D Ratio	M/B Ratio	Default Risk	P/D Ratio	M/B Ratio	Default Risk
	(1)	(2)	(3)	(4)	(5)	(6)
Expected Inflation (μ_P)	−2.61	−0.09	−0.10	−2.20	−0.11	−0.13
	(0.55)	(0.01)	(0.01)	(0.53)	(0.01)	(0.02)
$\mu_P \times \mathcal{D}_{L,M}$	−12.58	−0.08	−0.08	−13.00	−0.05	−0.05
	(0.72)	(0.01)	(0.02)	(0.69)	(0.02)	(0.02)
Investment	100.56	1.67	−0.76	112.61	1.82	−0.84
	(2.57)	(0.05)	(0.05)	(2.29)	(0.05)	(0.04)
Profitability	0.00	0.00	0.00	0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
log Size	3.16	0.26	−0.06	3.38	0.27	−0.07
	(0.16)	(0.00)	(0.00)	(0.18)	(0.00)	(0.00)
IP Growth	51.13	0.25	−0.68	72.62	0.33	−0.16
	(5.98)	(0.16)	(0.22)	(7.70)	(0.20)	(0.26)
S&P Return	17.16	0.35	−0.11	15.79	0.27	−0.19
	(2.01)	(0.05)	(0.06)	(2.12)	(0.05)	(0.06)
Yield Curve	0.00	−0.06	−0.06	0.47	−0.07	−0.06
	(0.22)	(0.01)	(0.01)	(0.24)	(0.01)	(0.01)
Leverage	−34.79	−1.91	1.56	−38.90	−2.05	1.58
	(0.77)	(0.03)	(0.02)	(0.77)	(0.03)	(0.02)
Recession	1.98	−0.02	0.16	3.35	−0.06	0.17
	(1.24)	(0.02)	(0.03)	(1.58)	(0.03)	(0.04)
$\text{Dummy}_{post\ 2008}$	−22.70	−0.16	0.03	−23.13	−0.16	0.04
	(0.87)	(0.02)	(0.02)	(0.86)	(0.02)	(0.02)
Industry FE	X	X	X	X	X	X
Nobs	702,000	702,000	702,000	694,460	694,460	694,460
R^2	17.35%	32.34%	49.65%	14.82%	30.14%	47.40%

Table 18: Regressions on expected inflation – With time trend

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation, controlling for a linear time trend. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4 in columns (1)–(3) and 1972Q2–2007Q4 in columns (4)–(6). We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

	P/D Ratio	M/B Ratio	Default Risk	P/D Ratio	M/B Ratio	Default Risk
	(1)	(2)	(3)	(4)	(5)	(6)
Expected Inflation (μ_P)	−3.40	−0.07	−0.12	−2.48	−0.07	−0.07
	(0.41)	(0.01)	(0.02)	(0.43)	(0.01)	(0.01)
Investment	105.42	1.73	−0.75	113.80	1.77	−0.54
	(2.35)	(0.04)	(0.05)	(2.66)	(0.04)	(0.05)
Profitability	0.00	0.00	0.00	0.18	0.01	0.00
	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
log Size	2.91	0.25	−0.06	3.00	0.23	−0.05
	(0.17)	(0.00)	(0.00)	(0.23)	(0.01)	(0.00)
IP Growth	38.56	−0.01	−0.79	18.96	0.37	0.11
	(6.58)	(0.13)	(0.22)	(9.38)	(0.20)	(0.26)
S&P Return	23.71	0.44	−0.08	20.96	0.51	0.10
	(1.88)	(0.04)	(0.06)	(2.36)	(0.05)	(0.06)
Yield Curve	0.88	−0.03	−0.06	0.63	−0.02	−0.04
	(0.21)	(0.00)	(0.01)	(0.20)	(0.00)	(0.01)
Leverage	−33.85	−1.90	1.57	−33.50	−1.78	1.53
	(0.77)	(0.03)	(0.02)	(0.85)	(0.02)	(0.01)
Recession	4.42	0.05	0.18	0.77	0.05	0.08
	(1.51)	(0.02)	(0.03)	(1.57)	(0.02)	(0.03)
$\text{Dummy}_{post\ 2008}$	−32.00	−0.43	−0.02			
	(1.23)	(0.02)	(0.04)			
Industry FE	X	X	X	X	X	X
Time Trend	X	X	X	X	X	X
Nobs	798,288	798,288	798,288	592,819	592,819	592,819
R^2	17.36%	32.67%	49.43%	20.71%	36.08%	47.66%

Table 19: Regressions on expected inflation – Split by stock-bond correlation

This table reports regressions of price-dividend ratios, market-to-book ratios and default risk on expected inflation by stock-bond correlation. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Default risk is the level of distress risk computed as in Campbell et al. (2008), which corresponds to the logarithm of the marginal probability of bankruptcy or failure over the next quarter. Columns (1)–(3) focus on periods with negative stock-bond correlations, while columns (4)–(6) focus on periods with positive stock-bond correlations. We calculate the unsmoothed stock-bond correlation as in Campbell et al. (2020). Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4. We report standard errors in parentheses. Standard errors are clustered at the quarter level and all specifications include industry fixed effects at the Fama & French 17 industry classification.

	Negative Stock-Bond Correlation			Positive Stock-Bond Correlation		
	P/D Ratio	M/B Ratio	Default Risk	P/D Ratio	M/B Ratio	Default Risk
	(1)	(2)	(3)	(4)	(5)	(6)
Expected Inflation (μ_P)	−7.23	−0.06	−0.11	−8.29	−0.14	−0.19
	(2.60)	(0.02)	(0.04)	(0.58)	(0.01)	(0.01)
Investment	99.97	1.60	−0.66	103.13	1.80	−0.61
	(3.64)	(0.07)	(0.10)	(3.47)	(0.06)	(0.07)
Profitability	0.00	0.00	0.00	0.17	0.01	0.00
	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)	(0.00)
log Size	4.93	0.29	−0.10	1.84	0.21	−0.04
	(0.20)	(0.01)	(0.00)	(0.23)	(0.01)	(0.00)
IP Growth	52.54	−1.33	−0.24	41.20	0.62	1.39
	(10.94)	(0.31)	(0.33)	(8.58)	(0.22)	(0.38)
S&P Return	25.19	0.46	0.38	17.91	0.44	0.30
	(2.72)	(0.07)	(0.08)	(3.60)	(0.07)	(0.09)
Yield Curve	3.21	−0.06	−0.06	−2.94	−0.05	−0.05
	(0.44)	(0.01)	(0.01)	(0.33)	(0.01)	(0.01)
Leverage	−42.14	−2.17	4.94	−31.37	−1.75	4.68
	(1.18)	(0.05)	(0.02)	(0.84)	(0.03)	(0.02)
Recession	−24.36	−0.03	0.02	0.09	0.02	0.05
	(0.90)	(0.03)	(0.04)	(2.91)	(0.04)	(0.06)
$\text{Dummy}_{post\ 2008}$	6.97	−0.30	0.16	−10.85	0.00	0.02
	(1.66)	(0.02)	(0.02)	(1.55)	(0.05)	(0.04)
Industry FE	X	X	X	X	X	X
Nobs	330,374	330,374	330,374	467,914	467,914	467,914
R^2	13.28%	28.77%	41.55%	19.67%	35.67%	38.68%

Figure 2: Expected inflation and asset prices – Simple model

The figure illustrates the impact of inflation on debt value (Panel A), bond yield (Panel B), credit spread (Panel C), Arrow Debreu default price (Panel D), firm value (Panel E), and equity value (Panel F). The expected inflation rate is either low (1%), moderate (3%), or high (5%). Predictions are obtained with the static corporate finance model with exogenous capital structure and default policies discussed in Section 2. We set the parameter values to $\widehat{\mu}_Y = 2\%$, $\sigma_Y = 15\%$, $X_D = 0.5$, $X_0 = 1$, $c = 1$, $\varphi = 0.407$, and $r = 4\%$.

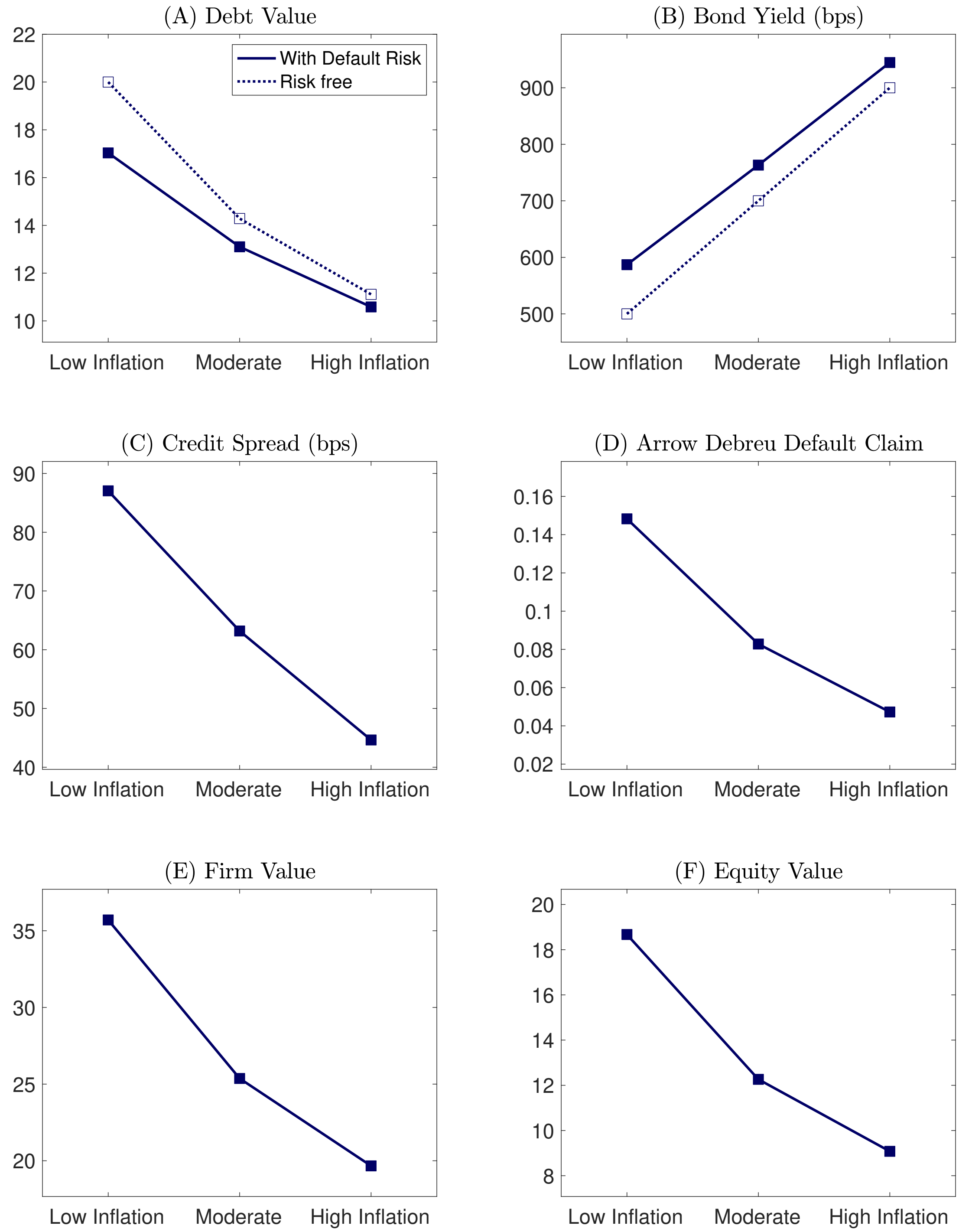


Figure 3: Expected inflation and asset prices – Full model

The figure illustrates the impact of expected inflation on equity value (Panel A), credit spread (Panel B), debt value (Panel C), and market leverage (Panel D). Each panel reports the predictions for different nominal conditions: low, moderate, and high expected inflation. Predictions for the full model (sticky cash flows, $\varphi = 0.407$) are compared to the predictions of a model without sticky cash flows ($\varphi = 1$). Light grey lines report the predictions when φ equals 0.6 and 0.8. All values are normalized to unity in the moderate expected inflation state. The baseline firm has the corporate policies presented in Table 3. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

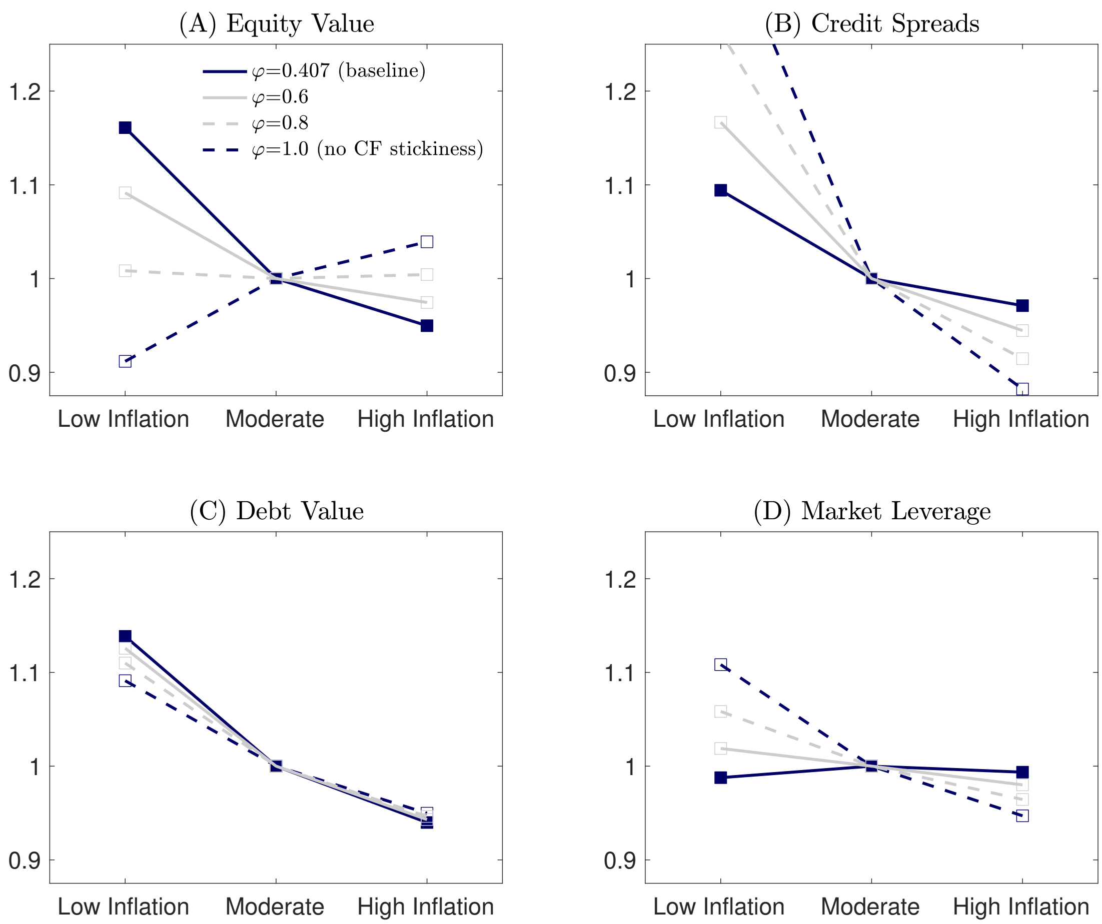


Figure 4: Expected inflation and asset prices – Economy of firms

The figure illustrates the impact of expected inflation on asset valuation for an economy of firms. Predictions are reported for equity value (Panel A), credit spread (Panel B), debt value (Panel C), and market leverage (Panel D). Each panel reports the predictions for different nominal conditions: low, moderate, and high expected inflation. Predictions for a representative firm are compared to the predictions for an economy of 1000 firms that differ in their leverage ratios. All values are normalized to unity in the moderate expected inflation state. All firms have initially the corporate policies presented in Table 3. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

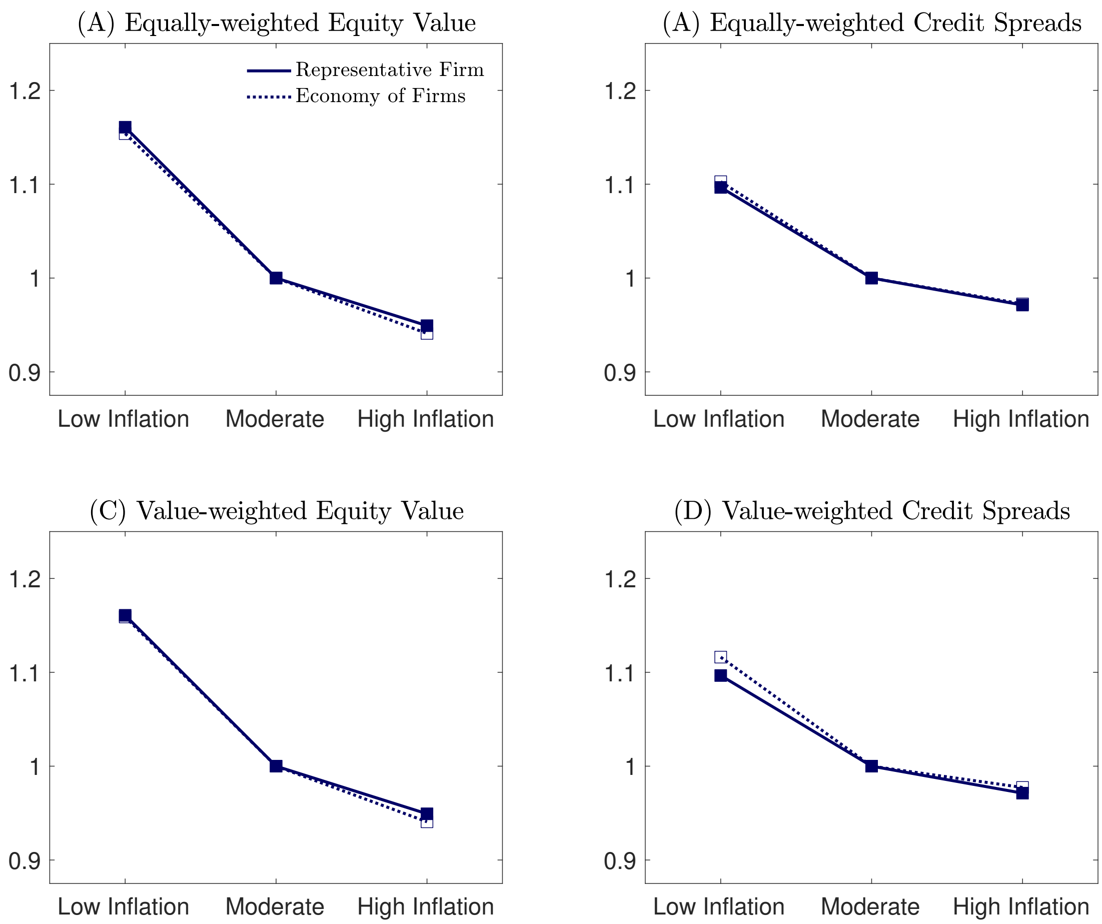


Figure 5: Term structure of equity yields and credit spreads – With sticky cash flows

The figure illustrates the term structure of equity yields (top panels) and credit spreads (bottom panels) in a model with sticky cash flows ($\varphi = 0.407$). The left panels report predictions by real conditions, while the right panels report predictions by nominal conditions. We construct the term structure of earnings yields from the nominal value of the unlevered equity strip for different maturities, while the term structure of credit spreads is for finite-maturity consol bonds. Results are based on Monte Carlo simulations; Appendix C provides the details. The firm initially has the corporate policies presented in Table 3. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

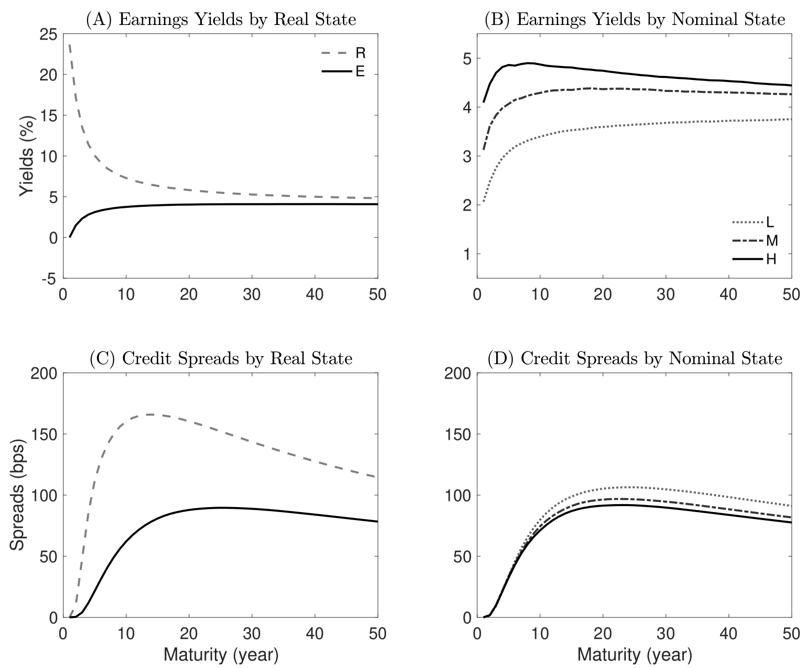


Figure 6: Term structure of equity yields and credit spreads – Without sticky cash flows

The figure illustrates the term structure of equity yields (top panels) and credit spreads (bottom panels) in a model without sticky cash flows ($\varphi = 1$). The left panels report predictions by real conditions, while the right panels report predictions by nominal conditions. We construct the term structure of earnings yields from the nominal value of the unlevered equity strip for different maturities, while the term structure of credit spreads is for finite-maturity consol bonds. Results are based on Monte Carlo simulations; Appendix C provides the details.

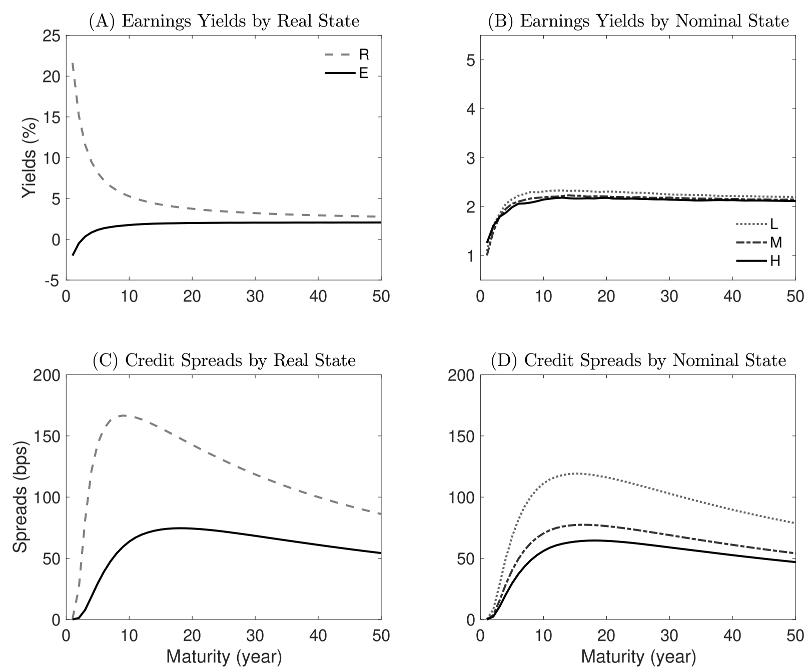


Figure 7: Equity valuation, default risk, and expected inflation – By leverage

This figure plots the relations between expected inflation and the price-dividend ratios (top panels), the market-to-book ratios (middle panels), and default risk (bottom panels). We report the relations by levels of market leverage. The left panels show portfolios of firms with below-median leverage, whereas the right panels report firms with above-median leverage. Each observation represents the value-weighted average of the valuation metric across firms for a given level of expected inflation. Expected inflation is the one-year-ahead inflation forecast from the Survey of Professional Forecasters, which is orthogonalized with respect to real consumption growth, NBER recessions, and a dummy for the Great Recession. Default risk is the marginal probability of bankruptcy or failure over the next quarter (reported in bps), which is computed as in Campbell et al. (2008). Section 6.1 provides additional details on the data. The sample period is 1972Q2–2019Q4.

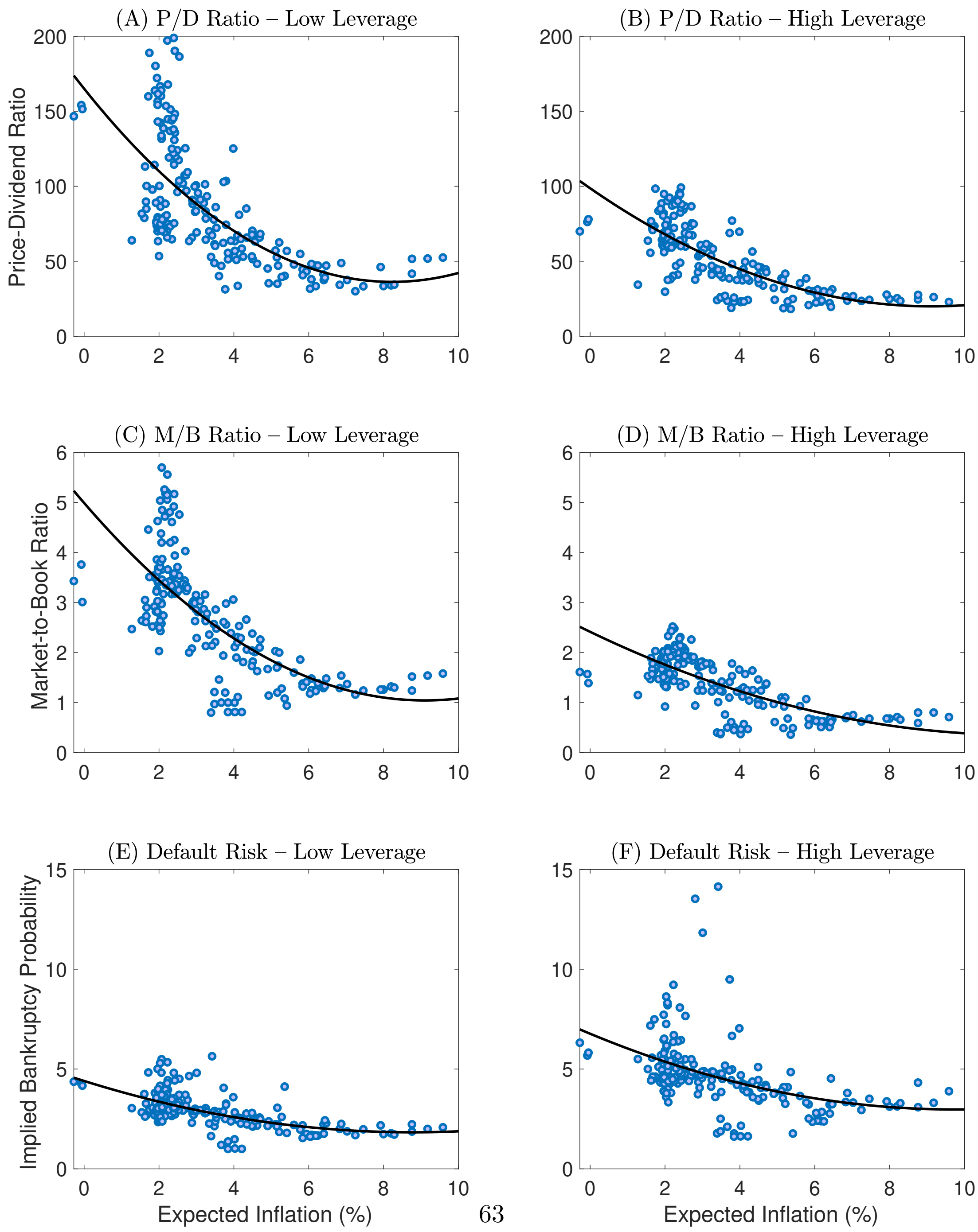
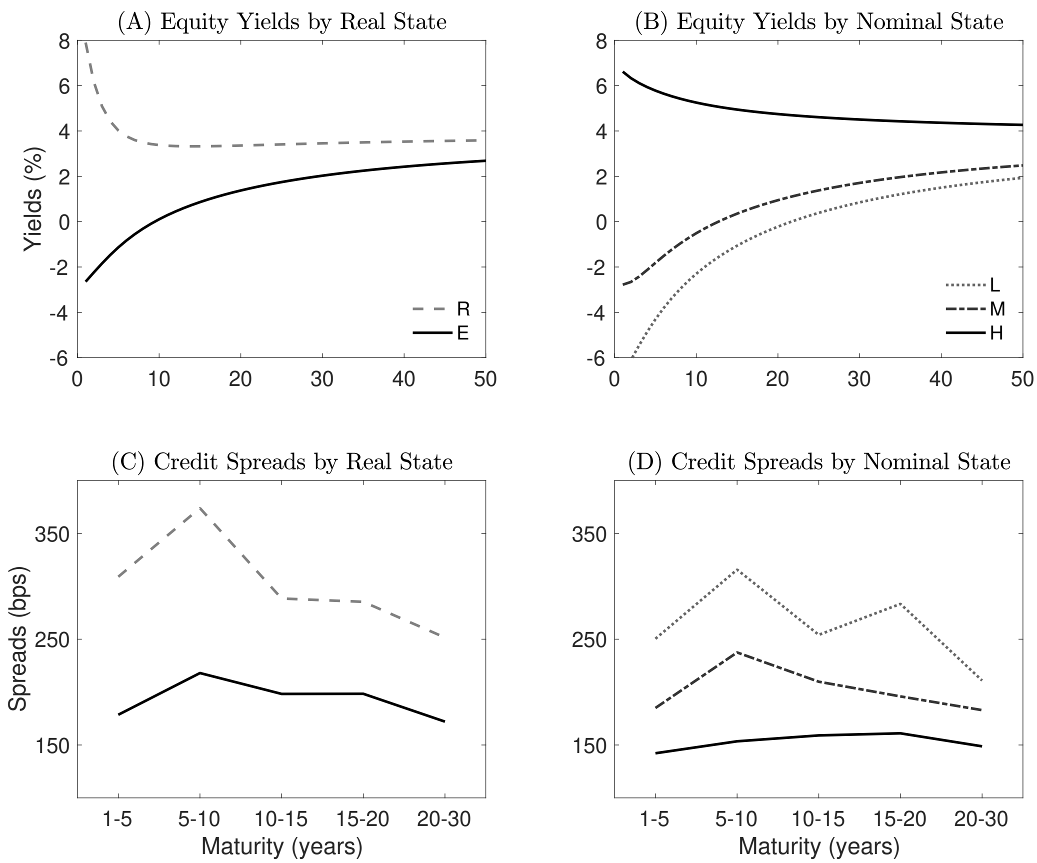


Figure 8: Term structure of equity yields and credit spreads – Data

The figure illustrates the term structure of equity yields (top panels) and credit spreads (bottom panels). The left panels report results by real conditions, while the right panels report results by nominal conditions. Expansions (E) and recessions (R) are determined by NBER-dated recessions. Low (L) and high (H) expected inflation states are determined by the bottom and top quartile of expected inflation; the moderate (M) state spans the interquartile range. Expected inflation is from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Equity yields data are from Giglio et al. (2021). Credit spreads data are described in Appendix D. The sample period is 1974Q3-2019Q4.



Appendix

A Calibration

This Appendix provides details on the calibration of the model. We distinguish between two distinct cases: with and without correlated real-nominal conditions.

We calibrate the model to the U.S. economy over the period 1970Q2-2019Q4.[34] The real states (R & E) are characterized by the conditional moments of quarterly real per capita consumption expenditures and real earnings growth.[35]

We obtain data on real per capita personal consumption expenditures and the corresponding price index from the Federal Reserve Bank of St. Louis (FRED). Nominal earnings for S&P 500 constituents, obtained from Robert J. Shiller's website, are deflated using the aforementioned price index.[36] NBER based recession indicators, also obtained from FRED, are used to determine (i) the long-run probability of being in recession / expansion and (ii) the moments of real consumption and earnings within those real regimes.

Regarding the nominal regimes (L, M & H), we use the quarterly mean of the one-year-ahead inflation forecasts from the Survey of Professional Forecasters, as reported by the Federal Reserve Bank of Philadelphia. Moments of inflation in the low (L) expected inflation regime are obtained from the lowest quartile in the data. Expected inflation in the moderate (M) regime is set to the unconditional mean. We then determine the high (H) expected inflation level such that the unconditional expected inflation in the calibration matches that in the data. Our calibration imposes a symmetry in the unconditional probabilities of being in the low (L) or high (H) expected-inflation regimes by setting them both to 25%. This choice ensures that any asymmetry in the response of asset prices to expected inflation is not driven by the calibration.

In the full model, expected inflation is non-neutral. With a correlation between real and nominal conditions, investors demand an inflation risk premium that impacts equity valuation. In the core of our analysis, we intentionally consider a restricted version of the model in which inflation risk is absent from the stochastic discount factor, such that the inflation risk premium does not drive any of our predictions. In this benchmark case, no inflation risk premium exists, because (i) expected consumption growth and expected inflation change independently (due to the way the Markov chain governing the state of the economy s_t is specified) and (ii) shocks to consumption growth and expected inflation are uncorrelated, i.e. $Cov_t\left(\frac{dC_t}{C_t}, \frac{dP_t}{dP_t}\right) = \rho_{CP,t}\sigma_{C,t}\sigma_{P,t}dt = 0$, because $\rho_{CP,t}dt = E_t[dZ_{P,t}dZ_t] =$

[34] The availability of the data on expected inflation determines our start date.

[35] Following Bhamra et al. (2010a,b), we account for an additional 22.58% of firm-specific volatility. The total cash-flow volatility is thus approximately 26% for our benchmark firm, which is the average volatility of firms with outstanding rated corporate debt.

[36] Real earning shocks are winsorized at the 1^{rst} and 99^{th} percentiles.

0. We thus analyze how higher expected inflation can negatively affect equity prices although inflation risk remains unpriced. In this baseline calibration, the 2×2 real transition matrix and the 3×3 nominal transition matrix are obtained separately, and are interacted to obtain the 6×6 chain for the model.

We relax the assumption of uncorrelated real and nominal conditions in Section 5.7 and adjust the calibration accordingly. There are two ways to account for a correlation between real and nominal variables within our model: (i) allowing for a non-zero *shock correlation*, whereby *shocks* to consumption growth and *shocks* to inflation exhibit an instantaneous correlation ρ_{CP,s_t} that varies with the current state of the economy, s_t; (ii) allowing for *regime correlation* by relaxing the assumption that the real and nominal Markov chains are independent, thereby introducing a correlation between *expected* consumption growth and *expected* inflation.

In the case of non-zero *shock correlation*, we obtain the state-dependent values of ρ_{CP,s_t} from the estimation of the Markov-regime switching model. We find that the correlation between shocks to inflation and consumption growth ρ_{CP,s_t} is -26.9% in RL, -61% in RM, 6.1% in RH, 63.4% in EL, 4.9% in EM, and -28.8% in EH.

In the case of *regime correlation*, we consider various calibrations that generate different unconditional levels of inflation risk premium, computed as the difference between the equity risk premium of the full model and the equity risk premium in the model with independent regimes. In each case, we directly estimate the 6×6 transition matrix, effectively adding 7 degrees of freedom to the estimation.[37] Importantly, none of these calibrations leads to a statistically significantly better fit to the macro data than the case of independent regimes. A log-likelihood ratio test of the nested setup – with independent regimes – against an estimation allowing for regime correlation yields a p-value of at least 98.4% across the different calibrations. While it is hard to distinguish econometrically between these alternative specifications based on macro data, the asset pricing implications across them are very different. We find that only a specification with low unconditional correlation between real and nominal regimes can generate the relations between equity valuation, credit risk, and expected inflation that we observe in the data.

The remaining parameters of the model are as follows. The sensitivity of nominal cash-flow growth to expected inflation is set to $\varphi = 0.407$, using the empirical estimate reported in Table 1. The corporate tax rate is set to $\eta = 15\%$. Following Chen (2010) and Bhamra et al. (2010a,b), we consider a state-dependent liquidation value in default, with $\alpha_L = 20\%$, $\alpha_M = 35\%$, and $\alpha_H = 50\%$. We normalize the initial value of the cash flow to $X_0 = 1$. Preferences involve a risk aversion of $\gamma = 10$,

[37] Technically, a free 6×6 intensity matrix has 25 parameters, one of which is constrained by the unconditional inflation risk premium target. In an unrestricted estimation, we find that 11 of these parameters are virtually 0. We thus rerun the estimation with 13 free parameters. Under the independence assumption, we interact (i) a nested real intensity matrix with 2 free parameters and (ii) a nested nominal matrix with 4 free parameters after setting the insignificant ones to 0. Thus, under the independence assumption, we have 6 free parameters.

an elasticity of intertemporal substitution (EIS) of $\psi = 2$, and a subjective time discount rate of $\beta = 0.035$ per annum. Table 2 summarizes the model calibration.

B Inflation Risk Premium

The core results of the paper are based on a calibration that intentionally abstracts from a correlation between real and nominal variables. Our aim is to provide an explanation of the negative relation between equity valuation and expected inflation without relying on an inflation risk premium, thereby complementing existing work (e.g., Eraker et al. (2015)).

In this Appendix, we account for a time-varying, non-zero correlation and investigate how the resulting inflation risk premium affects the results. Appendix A describes how we can do so by allowing for either a *shock correlation* or a *regime correlation*. We here discuss the results from both approaches.

B.1 Shock correlation

We first introduce a time-varying correlation between shocks to inflation and consumption growth. We obtain the state-dependent values of ρ_{CP,s_t} from the estimation of the Markov-regime switching model. We find that ρ_{CP,s_t} ranges between -0.61 in state RM and 0.63 in state EL, thereby generating substantial time-variation in the correlation between consumption growth and inflation. Specifically, the correlation is -24.2% when expected inflation is high (H), -3.7% when expected inflation is moderate (M), and 51.6% when expected inflation is low (L).[38]

Figure B.1 shows that the relations between expected inflation and the main output of the model remain very close to the baseline calibration (with $\rho_{CP,s_t} \equiv 0$). This analysis shows that accounting for a correlation between shocks to inflation and consumption growth has no impact on our model predictions, because the resulting inflation risk premium is small in all states, even when the correlation ρ_{CP,s_t} becomes sizable.

Figure B.1 [about here]

To see that, Panel B of Table 4 presents the predictions on the equity risk premium in the case of a *shock correlation*, ρ_{CP,s_t}. The equity risk premium becomes weakly decreasing in expected inflation. To understand how the inflation risk premium varies with expected inflation, recall that the correlation is positive in state L (51.6%) and negative in state H (-24.2%), consistent with the evidence that the sign switched from negative before 2000 to positive since the early 2000s (Boons et al., 2020).

[38] In line with Bilal (2017), Boons et al. (2020), or Campbell et al. (2020), we find that the correlation between consumption growth and inflation has turned from negative in the 1970-80s (mostly characterized by the H regime) to positive in recent years, which have been characterized by a period of low expected inflation (L regime).

While an inflation shock in state H is typically viewed as bad news (inflation correlates negatively with consumption), the negative correlation also reduces the systematic risk exposure of shareholders. Effectively, nominal cash flows become less correlated with consumption in state H than in state L, thereby reducing the levered equity risk premium. Observe that this effect is small, however, because the term $\varphi\gamma\sigma_{P,t}\sigma_{C,i}\rho_{PC,i}$ in (39) is quantitatively negligible, given that the product of inflation volatility ($\sigma_{P,t}$) and consumption growth volatility ($\sigma_{C,i}$) is economically small. As a result, the inflation risk premium and thus the equity risk premium decrease, but modestly, with expected inflation.

In summary, accounting for a *shock correlation* in the model does not materially impact the relationships between price-dividend ratios, credit spreads and expected inflation. The reason is that the covariance between shocks to consumption growth and inflation is economically modest, although strongly time-varying.

B.2 Regime correlation

We then allow for a *regime* correlation. The conditional moments of aggregate consumption growth inflation now evolve jointly, i.e., the real dynamics and inflation dynamics are not independent. Changes in expected inflation can then be correlated with shocks to the real SDF. The log likelihood function in our estimation does not change appreciably as the dependence between real and nominal regimes varies, so we can explore different calibrations. In each scenario, we compute the inflation risk premium as the difference between the equity risk premium of the full model and the equity risk premium of the nested model with independent regimes.

Figure B.2 shows how the inflation risk premium affects the relationship between the output variables and expected inflation. Our results regarding the negative impact of expected inflation on both equity valuation and credit risk continue to hold with regime correlation, as long as the *unconditional* inflation risk premium is small – around 0.25% per annum.

We can see, however, that for an unconditional inflation risk premium of 0.5% or above, the relationship between equity values and expected inflation is no longer convex and loses its monotonicity. The relationship between credit spreads and expected inflation retains its convexity, but loses its monotonicity. Based on our model, the negative relation between expected inflation and both equity valuation and credit risk implies that: (i) there is an upper bound on the *unconditional* inflation risk premium of around 0.25% per annum, and (ii) any significant inflation risk premium beyond this must be time varying, lending further support to the findings in Boons et al. (2020) and Campbell et al. (2020), among others. Their empirical evidence suggests that, over the last 50 years, the inflation risk premium has switched sign and is unconditionally close to zero.

Figure B.2 [about here]

Panel C of Table 4 considers different levels of the unconditional inflation risk premium, arising from different correlation structures between real and nominal states. The equity risk premium is now clearly higher in state L than in state H. The model can generate substantial *conditional* inflation risk premiums via correlated regimes; this is a direct consequence of the impact of a regime switch being persistent and, thus, having a much greater impact on the pricing kernel that transitory shocks.

Overall, we find that the equity risk premium tends to decrease with expected inflation in these two cases: (i) with correlated inflation-consumption shocks and (ii) with correlated real and nominal states. Hence, introducing an inflation risk premium cannot rationalize the negative relation between equity valuation and expected inflation observed in the data.

C Term Structure Analysis: Monte Carlo Simulation

This Appendix describes the Monte Carlo simulation we use to construct the term structures of equity yields, credit spreads, and nominal bond yields. Our model allows for closed-form solution for most perpetual claims (e.g., debt, equity), but several finite-maturity claims cannot be obtained in closed form, and so, we use the following simulation procedure.

For each of our 6 regimes, we simulate 20,000 paths, over 200 quarters, of the Markov chain and Brownian motions for the consumption C_t and price index P_t that characterize the evolution of the economy, starting in a given regime. For each of these paths, we consider a firm starting with the optimal corporate policies presented in Table 3, but experiencing 10,000 different streams of idiosyncratic cash flow shocks.[39]

The firm defaults when its cash flow X_t crosses the default boundary $X_{D,j}$ corresponding to the ongoing regime j in a given time period; a defaulted firm ceases to generate any cash flow.

In total, the simulation involves 240 billion $(6 \times 200 \times 20,000 \times 10,000)$ different realizations of cash flows and, thus, asset prices, which we use to generate the term structure of nominal equity yields and the term structure of nominal corporate credit spreads. We discuss our results in Section 5.8.

We also explore the term structure of nominal risk-free bond yields in Figure C.1. The nominal bond yield increases with expected inflation. The difference between states is especially strong in the short end of the term structure and decreases with the bond maturity. The resulting upward-sloping term structure in state L and downward-sloping term structure in state H reflects the transitory nature of expected inflation implied by our calibration.

Figure C.1 [about here]

[39] The expectations we compute depend both on trajectories of the pricing kernel and on the cash flow stream associated with realizations of the pricing kernel. It is thus important to simulate a large number of both paths. The chosen methodology allows for straightforward parallelization of the simulation procedure.

D Data Description

This Appendix describes the data used in our empirical analysis, presented in Section 6.

D.1 Valuation ratios

Market-to-book ratio (MB) is computed as ME/BE. Book equity (BE) is shareholders' equity ($SEQQ$ + $CEQQ$ + $PSTKQ$ or ATQ - LTQ), plus balance sheet deferred taxes and investment tax credit ($TXDITCQ$) if available, minus the book value of preferred stock ($PSTKQ$) as in Weber (2018). Market capitalization (ME) is the product of quarter-end price (PRC) and share outstanding ($Shrout$).

The price-dividend ratio is computed as the share price divided by the sum of dividend payments over the last 12 months. We construct dividend payments using cum-dividend return and ex-dividend returns, as in Beeler and Campbell (2012).

D.2 Default risk

We follow Campbell, Hilscher, and Szilagyi (2008) to calculate financial distress risk (FR) as the logit transformed bankruptcy probability, while excluding leverage in the measurement. FR is then calculated as

$$\begin{aligned} FR = & -9.16 - 20.26 * NIMTAVG - 7.13 * EXRETAVG + 1.41 * SIGMA \\ & - 0.045 * RSIZE - 2.13 * CASHMTA + 0.075 * MB - 0.058 * PRICE, \end{aligned}$$

where

$$NIMTAVG_t = \sum_{i=0}^{3} \frac{1-\phi^3}{1-\phi^{12}} (\phi^{3(i-1)} NITMA_{t-3i})$$

$$EXRETAVG_t = \sum_{i=0}^{11} \frac{1-\phi}{1-\phi^{12}} (\phi^{i-1} EXRET_{t-i})$$

$NIMTA$ and $EXRET$ are net income over total assets (NIQ/ATQ) and the log of gross excess returns over the value-weighted S&P500 returns, respectively. $SIGMA$ is the square root of the annualized sum of squared stock returns over a 3-month period. $RSIZE$ is the log of firm's market equity over the total valuation of all firms in the S&P500. $CASHMTA$ is cash and short-term investments over total assets ($CHEQ/ATQ$). MB is the market-to-book value of equity. $PRICE$ is the log of price per share. The associated 1-quarter bankruptcy probability for firm i at time t is then

$$P_{t-1}(Y_{i,t} = 1) = \frac{1}{1 + exp(-FR_{i,t-1})}.$$

D.3 Leverage, investment and profitability

Market leverage is the sum of long term debt and debt in current liabilities over the sum of of debt and market capitalization $((DLCQ + DLTTQ)/(DLCQ + DLTTQ + ME))$ as in Freyberger, Neuhierl, and Weber (2017).

Investment and profitability are calculated following Fama and French (2015) as revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity $(IBQ - COGSQ - XSGAQ - XINTQ)/BE$ and the percentage change in total asset.

D.4 Credit spreads

For our term structure analysis of credit spreads, we exploit a comprehensive dataset combining corporate bond data from four distinct sources: i) the Lehman Brothers Fixed Income Database, which covers the period 1973-1998; ii) the cleaned Enhanced TRACE data provided by WRDS, spanning the period 2002-2019; iii) the Mergent FISD/NAIC data, which comprises transaction level data for all trades in publicly traded bonds issued by life, property, and casualty insurance companies and health maintenance companies (HMOs) over the period 1994-2016; and finally iv) the Datastream database covering the period 1990-2019. We follow Chordia et al. (2017) to deal with overlapping observations and prioritize the different sources using the order above. We filter bonds using the following rules:

- Bond Type: We only include corporate bonds which are classified as US Corporate Debentures (‘CDEB’), US Corporate MTN (‘CMTN’) or US Corporate MTN Zero (‘CMTZ’).

- Public Firm: We exclude bonds that are not listed, or traded in the US public market, this includes bonds issued via private placement, bonds issued under the 144A rule and bond issuers not in the jurisdiction of the United States.

- Bond Coupon: We exclude bonds with a variable coupon (‘V’), i.e we only include bonds with a fixed (‘F’) or zero coupon (‘Z’).

- Convertible: We exclude all convertible bonds.

- Asset-Backed: We exclude all asset-backed bonds.

- Yankee Bonds: We exclude all Yankee bonds (a debt obligation issued by a foreign entity, such as a government or company, which is traded in the United States and denominated in U.S. dollars).
- Foreign Currency: We only include U.S. denominated bonds
- Embedded options: We exclude putable bonds but include callable bonds.
- Security Level: We exclude all junior bonds, this includes 'Junior', 'Junior Subordinate' and 'Subordinate' bonds.
- Rating: We exclude bonds which are 'Unrated'.

We then apply these additional filters: i) We remove observations if a corporate bonds monthly price is less than \$1 or above \$1000 and if the bonds time to maturity is less than 12 months, as in Bai, Bali, and Wen (2019); ii) To address the issue of stale prices, we follow Chordia et al. (2017) and exclude prices that do not change for more than 3 months. The final sample comprises 20,068 corporate bonds issued by 2,123 firms. We focus on the period 1974Q3-2019Q4 to match the equity yield sample.

We compute the average credit spread of each firm's outstanding bonds over a given quarter. We then construct a quarterly value-weighted average credit spread by maturity bucket. The credit spread of an individual bond is computed as the difference between the yield of the bond and the associated yield of the Treasury curve at the same maturity. We use the Benchmark Treasury rates from Datastream for maturities of 3, 5, 7, 10, and 30 years, and then use a linear interpolation scheme to estimate the entire yield curve, following Duffee (1998) and Collin-Dufresne, Goldstein, and Martin (2001) among others.

Figure B.1: Expected inflation and asset prices – Correlated consumption and inflation shocks

The figure illustrates the impact of expected inflation on asset valuation in the case of conditional correlations between consumption and inflation shocks (shock correlation). Predictions are reported for equity value (Panel A), credit spread (Panel B), debt value (Panel C), and market leverage (Panel D). Each panel reports the predictions for different nominal conditions: low, moderate, and high expected inflation. Predictions for the baseline model with sticky cash flows ($\varphi = 0.407$) are compared to the predictions of a model without sticky cash flows ($\varphi = 1$). All values are normalized to unity in the moderate expected inflation state. Panel B truncates extreme values for improved visibility. Firms have endogenous corporate policies. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.

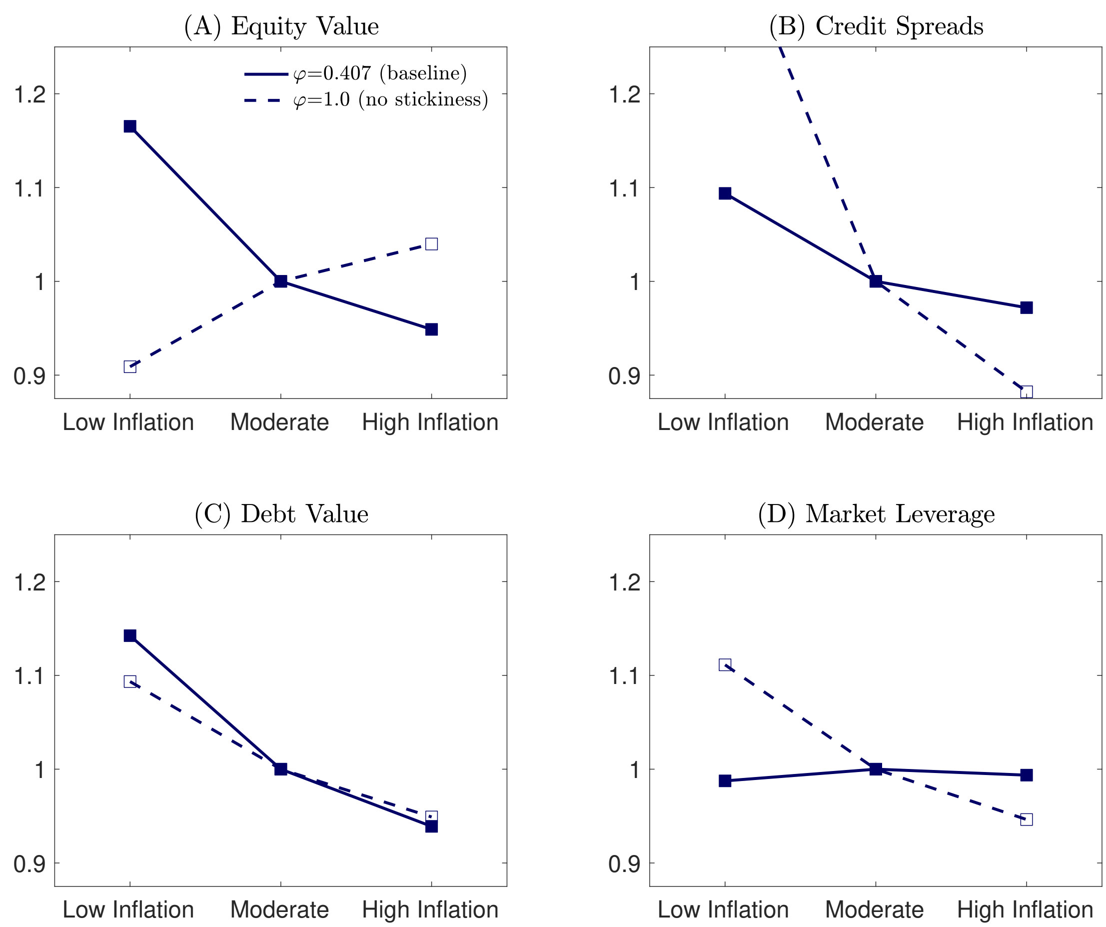


Figure B.2: Expected inflation and asset prices – Correlated real and nominal regimes

The figure illustrates the impact of expected inflation on asset valuation when real and nominal regimes are correlated (regime correlation). The results compare calibrations for different regime correlations generating different levels of inflation risk premium (IRP). The solid blue line captures the baseline case of independent real and nominal regimes (also reported in Figure 3), while the grey lines reflect unconditional levels of inflation risk premium ranging between 25 and 125 bps. Predictions are reported for equity value (Panel A), credit spread (Panel B), debt value (Panel C), and market leverage (Panel D). Each panel reports the predictions for different nominal conditions: low, moderate, and high expected inflation. All values are normalized to unity in the moderate expected inflation state. Panel B truncates extreme values for improved visibility. All firms have endogenous corporate policies. The parameter values of the model are reported in Table 2 and discussed in Section 6.1.

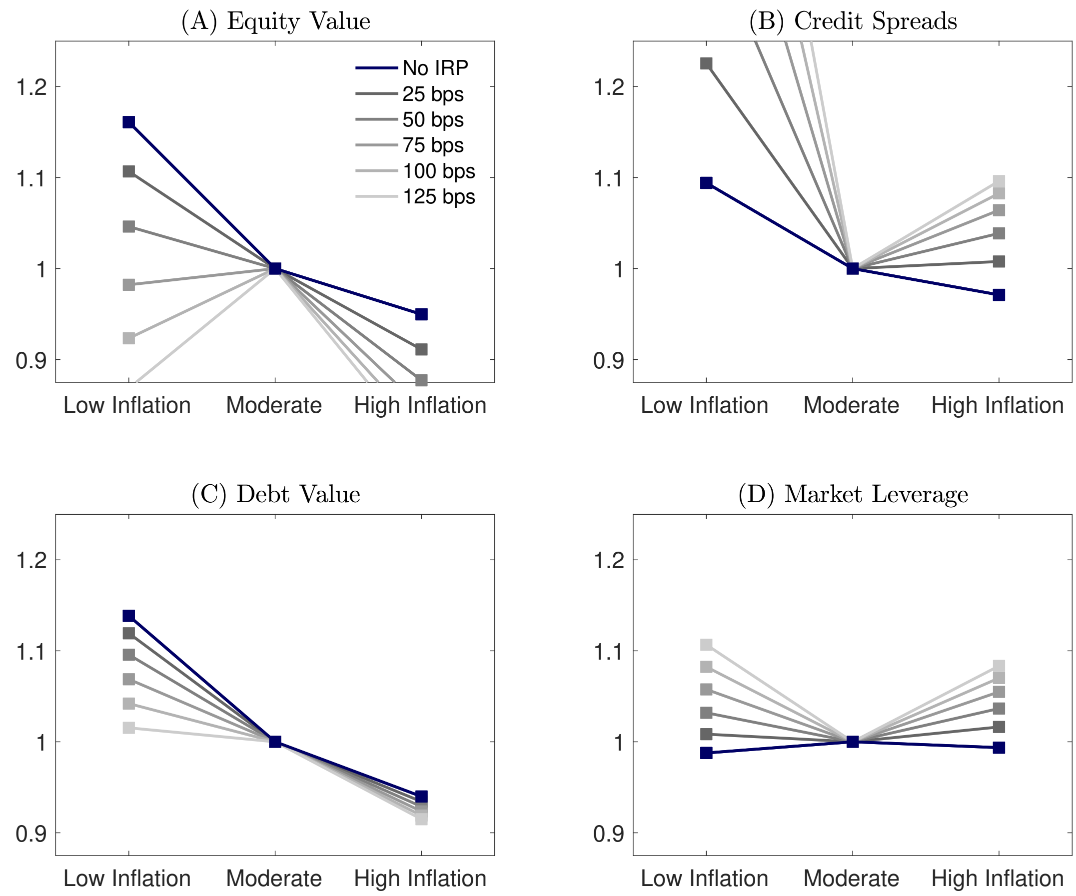
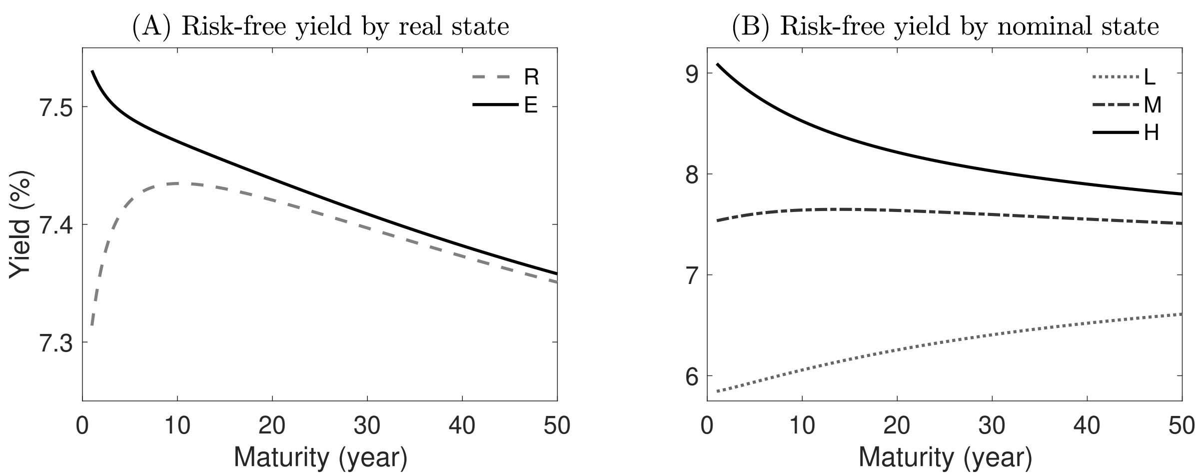


Figure C.1: Term structure of risk-free nominal yield

The figure illustrates the term structure of the risk-free nominal yield. The left panel report predictions by real conditions, while the right panel report predictions by nominal conditions. The parameter values of the model are reported in Table 2 and discussed in Section 5.1.



Online Appendix

OA.A Simple Model

Under the physical probability measure $\mathbb{P}$, real cashflow growth is given by

$$\frac{dY_t}{Y_t} = \mu_Y dt + \sigma_Y dW_t,$$

where W is a standard Brownian motion under $\mathbb{P}$. The date t nominal cashflow is given by $X_t = Y_t P_t^{\varphi}$, and so nominal cashflow growth is given by

$$\frac{dX_t}{X_t} = (\mu_Y + \varphi \mu_P) dt + \sigma_Y dW_t.$$

The real SDF is given by

$$\frac{d\pi_t}{\pi_t} = -r dt - \Theta dZ_t,$$

where Z is a standard Brownian motion under $\mathbb{P}$ such that $dZ_t dW_t = \rho dt$. In this section, there is no risk premium associated with sudden shifts in the state of the economy. In contrast with the main model, we therefore assume $\rho > 0$ to ensure that the risk premium is not zero. Consequently, conditional on date t information, the risk-neutral probability of event A occuring at date T is given by

$$E_t^{\mathbb{Q}}[1_A] = E_t \left[\frac{M_T}{M_t} 1_A \right],$$

where M is an exponential martingale under $\mathbb{P}$, defined by

$$\frac{dM_t}{M_t} = -\Theta dZ_t, \; M_0 = 1.$$

The exogenous price index is given by

$$P_t = P_0 e^{\mu_P t},$$

where μ_P is the constant inflation rate.

The nominal SDF is given by $\pi_t^{\$} = \pi_t / P_t$, and so

$$\frac{d\pi_t^{\$}}{\pi_t^{\$}} = -r^{\$} dt - \Theta dZ_t,$$

where

$$r^{\$} = r + \mu_P.$$

The price-index is not stochastic, so there is no inflation risk premium. We can therefore price risk under the risk-neutral measure $\mathbb{Q}$ with no additional adjustment for an inflation risk premium. If there were a risk premium, we would have to define a different probability measure in order to discount nominal cashflows with the nominal interest rate. Using Girsanov's Theorem, we obtain the evolution of X under $\mathbb{Q}$:

$$\frac{dX_t}{X_t} = (\widehat{\mu}_X + \varphi \mu_P) dt + \sigma_X dW_t^{\mathbb{Q}}.$$

where $W^{\mathbb{Q}}$ is a standard Brownian motion under the risk-neutral measure $\mathbb{Q}$ and $\widehat{\mu}_X = \mu_X - \rho \sigma_X \Theta$ is the risk-neutral expected nominal cash flow growth rate and $\sigma_X = \sigma_Y$.

The date-t nominal after-tax abandonment value of the firm is given by

$$A_t^{\$} = A^{\$}(X_t) = (1-\eta)X_t E_t\left[\int_t^{\infty} \frac{\pi_u^{\$}}{\pi_t^{\$}}\frac{X_u}{X_t}\right] = (1-\eta)X_t E_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r^{\$}(u-t)}\frac{X_u}{X_t}du\right],$$

where η is the tax rate, which we set to zero in Section 2.

The date-t nominal price of the corporate bond is given by

$$\begin{aligned} B_t^{\$} &= cE_t\left[\int_t^{\tau_D} \frac{\pi_u^{\$}}{\pi_t^{\$}}du\right] + \alpha E_t\left[\frac{\pi_{\tau_D}^{\$}}{\pi_t^{\$}}A^{\$}(X_{\tau_D})du\right] \\ &= cE_t\left[\int_t^{\tau_D} e^{-r^{\$}(u-t)}du\right] + \alpha E_t\left[e^{-r^{\$}(\tau_D-t)}A^{\$}(X_{\tau_D})du\right], \end{aligned}$$

where α is the recovery rate which we set to zero in Section 2. Simplifying the expression for the corporate bond price, we obtain

$$\begin{aligned} B_t^{\$} &= cE_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r^{\$}(u-t)}du - e^{-r^{\$}(\tau_D-t)}E_{\tau_D}^{\mathbb{Q}}\int_{\tau_D}^{\infty} e^{-r^{\$}(u-\tau_D)}du\right] + \alpha E_t^{\mathbb{Q}}\left[e^{-r^{\$}(\tau_D-t)}A^{\$}(X_{\tau_D})du\right] \\ &= cE_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r^{\$}(u-t)}du - e^{-r^{\$}(\tau_D-t)}E_{\tau_D}^{\mathbb{Q}}\int_{\tau_D}^{\infty} e^{-r^{\$}(u-\tau_D)}du\right] + \alpha E_t^{\mathbb{Q}}\left[e^{-r^{\$}(\tau_D-t)}\right]A^{\$}(X_D) \\ &= \frac{c}{r^{\$}}E_t^{\mathbb{Q}}\left[1-e^{-r^{\$}(\tau_D-t)}\right] + \alpha E_t^{\mathbb{Q}}\left[e^{-r^{\$}(\tau_D-t)}\right]A^{\$}(X_D) \end{aligned}$$

since $\tau_D = \inf_{t\geq 0}\{X_t \leq X_D\}$. Therefore

$$B_t^{\$} = \frac{c}{r^{\$}}(1-q_{D,t}^{\$}) + \alpha A^{\$}(X_D)q_{D,t}^{\$},$$

where

$$q_{D,t}^{\$} = E_t^{\mathbb{Q}}\left[e^{-r^{\$}(\tau_D-t)}\right]$$

is the date-t price of the Arrow-Debreu default claim, which pays off 1 unit of the numeraire (1 dollar) at the time of default τ_D.

From the principle of no arbitrage, the price of the Arrow-Debreu default claim, $q_D^{\$}(x)$, (where $x = \ln X$) satisfies

$$E_t^{\mathbb{Q}}[dq_D^{\$}(x) - q_D^{\$}(x)r^{\$}dt] = 0.$$

Applying Ito's Lemma gives the ordinary differential equation

$$\frac{1}{2}\sigma_Y^2 q_D^{\$\,\prime\prime}(x) + \left(\widehat{\mu}_Y + \varphi\mu_P - \frac{1}{2}\sigma_Y^2\right)q_D^{\$\,\prime}(x) - r^{\$}q_D(x) = 0.$$

The general solution of the above ordinary differential equation is given by $q_D^{\$}(x) = k_-e^{a_-x} + k_+e^{a_+x}$, where a_- and a_+ are the roots of the following quadratic in a

$$\frac{1}{2}\sigma_Y^2 a^2 + \left(\widehat{\mu}_Y + \varphi\mu_P - \frac{1}{2}\sigma_Y^2\right)a - r^{\$} = 0.$$

It follows that

$$a_{\pm} = -\frac{\widehat{\mu}_Y + \varphi\mu_P - \frac{1}{2}\sigma_Y^2}{\sigma_Y^2} \pm \sqrt{\left(\frac{\widehat{\mu}_Y + \varphi\mu_P - \frac{1}{2}\sigma_Y^2}{\sigma_Y^2}\right)^2 + \frac{r+\mu_P}{\frac{1}{2}\sigma_Y^2}}.$$

We know that $a_- a_+ = -r^\$$, so a_- and a_+ are of opposite sign if $r^\$ > 0$. We can also see that $a_+ > 0$ if $r^\$ > 0$. Therefore, $a_- < 0$ if $r^\$ > 0$. From the no-bubble condition $\lim_{x\to\infty} |q_D(x)| < \infty$, we see that $c_+ = 0$ and so $q_D^\$(x) = k_- e^{a_- x}$. The boundary condition $q_D(x_D) = 1$ (where $x_D = \ln X_D$) implies that $q_D^\$(x) = e^{a_-(x-x_D)}$, and so

$$q_{D,t}^\$ = e^{-a(\mu_P)(x_t - x_D)},$$

where

$$a(\mu_P) = \frac{\widehat{\mu}_Y + \varphi\mu_P - \frac{1}{2}\sigma_Y^2}{\sigma_Y^2} + \sqrt{\left(\frac{\widehat{\mu}_Y + \varphi\mu_P - \frac{1}{2}\sigma_Y^2}{\sigma_Y^2}\right)^2 + \frac{r + \mu_P}{\frac{1}{2}\sigma_Y^2}},$$

and of course $a(\mu_P) > 0$, if $r^\$ > 0$.

We now prove that

$$\frac{\partial s_t}{\partial \mu_P} < 0$$

if $r^\$ > 0$, where s_t is defined in (1). Observe that

$$\frac{\partial \ln q_{D,t}^\$}{\partial \mu_P} = -(x_t - x_D)\frac{\partial a(\mu_P)}{\partial \mu_P} - a(\mu_P)\frac{\partial (x_t - x_D)}{\partial \mu_P}.$$

We now show that $\frac{\partial a(\mu_P)}{\partial \mu_P} > 0$ if $\varphi > -1/a(\mu_P)$ (provided $r^\$ > 0$). Observe that

$$\frac{1}{2}\sigma_Y^2 a(\mu_P)^2 - \left(\widehat{\mu}_Y + \varphi\mu_P - \frac{1}{2}\sigma_Y^2\right) a(\mu_P) - r^\$ = 0.$$

Differentiating with respect to μ_P gives

$$\sigma_Y^2 \left[a(\mu_P) - \left(\widehat{\mu}_Y + \varphi\mu_P - \frac{1}{2}\sigma_Y^2\right)\right] \frac{\partial a(\mu_P)}{\partial \mu_P} = 1 + \varphi a(\mu_P).$$

Hence

$$\sigma_Y^2 \sqrt{\left(\frac{\widehat{\mu}_Y + \varphi\mu_P - \frac{1}{2}\sigma_Y^2}{\sigma_Y^2}\right)^2 + \frac{r + \mu_P}{\frac{1}{2}\sigma_Y^2}}\, \frac{\partial a(\mu_P)}{\partial \mu_P} = 1 + \varphi a(\mu_P),$$

and so $\frac{\partial a(\mu_P)}{\partial \mu_P} > 0$ if $\varphi > -1/a(\mu_P)$ (provided $r^\$ > 0$). Hence, holding the distance to default, $x_t - x_D$, fixed, the price of the Arrow-Debreu default claim, $q_{D,t}^\$$, decreases with expected inflation if $r^\$ > 0$. For the credit spread, as defined in (1), observe that

$$s_t = \frac{r^\$}{1 - q_{D,t}^\$} - r^\$.$$

Since $1/(1 - q_{D,t}^\$)$ decreases with expected inflation, it follows that

$$\frac{\partial}{\partial \mu_P}\left(\frac{r^\$}{1 - q_{D,t}^\$}\right) < \frac{\partial r^\$}{\partial \mu_P}$$

and hence

$$\frac{\partial s_t}{\partial \mu_P} < 0$$

if $r^{\$} > 0$.

The date-t value of levered equity (in nominal units) after taxes is given by

$$S_t^{\$} = (1-\eta)E_t^{\mathbb{Q}}\left[\int_t^{\tau_D} e^{-r^{\$}(u-t)}(X_u - c)du\right].$$

Therefore, we obtain

$$\begin{aligned}
S_t^{\$} &= (1-\eta)\left(X_t E_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r^{\$}(u-t)}\frac{X_u}{X_t}du - e^{-r^{\$}(\tau_D - t)}\frac{X_D}{X_t}E_{\tau_D}^{\mathbb{Q}}\int_{\tau_D}^{\infty} e^{-r^{\$}(u-\tau_D)}\frac{X_u}{X_D}du\right]\right. \\
&\left. - cE_t^{\mathbb{Q}}\left[\int_t^{\tau_D} e^{-r^{\$}(u-t)}du\right]\right) \\
&= (1-\eta)X_t\left(\frac{1}{r^{\$} - \widehat{\mu}_X} - E_{\tau_D}^{\mathbb{Q}}[e^{-r^{\$}(\tau_D - t)}]\frac{X_D}{X_t}\frac{1}{r^{\$} - \widehat{\mu}_X} - \frac{c}{r^{\$}}(1 - q_{D,t}^{\$})\right) \\
&= (1-\eta)\left(\frac{X_t - X_D q_{D,t}^{\$}}{r^{\$} - \widehat{\mu}_X} - \frac{c}{r^{\$}}(1 - q_{D,t}^{\$})\right)
\end{aligned}$$

Within Section 2, we assume an exogenously fixed default boundary, but with an endogenous default policy, the default time τ_D is chosen to maximize the value of levered equity. The smooth pasting condition $\frac{\partial S_t^{\$}}{\partial X_t}|_{X_t = X_D} = 0$ determines the default boundary X_D, i.e.

$$\frac{1 - X_D \left.\frac{\partial q_{D,t}^{\$}}{\partial X_t}\right|_{X_t = X_D}}{r^{\$} - \widehat{\mu}_X} + \frac{c}{r^{\$}}\left.\frac{\partial q_{D,t}^{\$}}{\partial X_t}\right|_{X_t = X_D} = 0,$$

and so we obtain the optimal default policy

$$X_D = c\frac{r + (1-\varphi)\mu_P - \widehat{\mu}_Y}{r + \mu_P}\frac{a(\mu_P)}{1 + a(\mu_P)}.$$

It follows that, for a fixed nominal coupon, c, we have

$$\frac{\partial x_D}{\partial \mu_P} = \frac{(1-\varphi)r - (r - \widehat{\mu}_Y)}{(r + (1-\varphi)\mu_P - \widehat{\mu}_Y)(r + \mu_P)} + \frac{1}{a(\mu_P)(1 + a(\mu_P))}\frac{\partial a(\mu_P)}{\partial \mu_P}.$$

With an endogenous default policy, the distance to default is impacted by inflation. A priori, it is possible that equity holders will choose to default later when when inflation is higher, that is the distance to default will increase. However, for the calibration we have chosen, equityholders default earlier when inflation is higher, because the present value of the coupons they have to pay to bondholders is increased. Even if this were not the case, $\frac{\partial a(\mu_P)}{\partial \mu_P}$ is much larger than $\frac{\partial (x_t - x_D)}{\partial \mu_P}$, so any increase in distance to default would not change the overall sign of $\frac{\partial \ln q_{D,t}^{\$}}{\partial \mu_P}$.

OA.B The Economy

First, we introduce some notation related to jumps in the state of the economy. Suppose that during the small time-interval $[t-\Delta t, t)$ the economy is in state i and that at time t the state changes, so that during the next small time interval $[t, t+\Delta t)$ the economy is in state $j \neq i$. We then define the left-limit of s at time t as

$$s_{t-} = \lim_{\Delta t \to 0} s_{t-\Delta t},$$

and the right-limit as

$$s_t = \lim_{\Delta t \to 0} s_{t+\Delta t}.$$

Therefore $s_{t-} = i$, whereas $s_t = j$, so the left- and right limits are not equal. If some function E depends on the current state of the economy i.e. $E_t = E(s_t)$, then E is a jump process which is right continuous with left limits, i.e. RCLL. If a jump from state i to $j \neq i$ occurs at date t, then we abuse notation slightly and denote the left limit of E at time t by E_i, where i is the index for the state. i.e. $E_{t-} = \lim_{s \uparrow t} E_s = E_i$. Similarly $E_t = \lim_{s \downarrow t} E_s = E_j$. We shall use the same notation for all processes that jump, because of their dependence on the state of the economy.

Using simple algebra we can write the normalized Kreps-Porteus aggregator in the following compact form:

$$f(c, v) = \beta \left(h^{-1}(v)\right)^{1-\gamma} u\left(c/h^{-1}(v)\right), \tag{OA.1}$$

where

$$\begin{aligned} u(x) &= \frac{x^{1-\frac{1}{\psi}} - 1}{1-\frac{1}{\psi}}, \psi > 0, \\ h(x) &= \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma \geq 0, \gamma \neq 1. \\ \ln x, & \gamma = 1. \end{cases} \end{aligned}$$

The representative agent's value function is given by

$$J_t = E_t \int_t^\infty f(C_t, J_t)\, dt. \tag{OA.2}$$

Proposition OA.1 *The SDF of a representative agent with the continuous-time version of Epstein-Zin-Weil preferences is given by*

$$\pi_t = \begin{cases} \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t} e^{\int_0^t p_{C,s}^{-1} ds}\right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, & \psi \neq 1 \\ \beta e^{-\beta \int_0^t \left[1+(\gamma-1)\ln\left(V_s^{-1}\right)\right] ds} C_t^{-\gamma} V_t^{-(\gamma-1)}, & \psi = 1 \end{cases}. \tag{OA.3}$$

When $\psi \neq 1$, the price-consumption ratio in state i, $p_{C,i}$, satisfies the nonlinear equation system:

$$p_{C,i}^{-1} = \bar{r}_i + \gamma \sigma_{C,i}^2 - \mu_{C,i} - \left(1 - \frac{1}{\psi}\right) \sum_{j \neq i} \lambda_{ij} \left(\frac{\left(p_{C,j}/p_{C,i}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} - 1}{1-\gamma} \right), \; i,\, j \in \{1, \ldots, N\},\, j \neq i. \tag{OA.4}$$

where

$$\bar{r}_i = \beta + \frac{1}{\psi}\mu_{C,i} - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma_{C,i}^2,\; i \in \{1, \ldots, N\}. \tag{OA.5}$$

When $\psi = 1$, define V_i via

$$J = \ln(CV). \quad \text{(OA.6)}$$

Then V_i satisfies the nonlinear equation system:

$$\beta \ln V_i = \mu_{C,i} - \frac{\gamma}{2}\sigma_{C,i}^2 + \sum_{j \neq i} \lambda_{ij} \frac{(V_j/V_i)^{1-\gamma} - 1}{1-\gamma}\ i \in \{1, \ldots, N\},\ j \neq i. \quad \text{(OA.7)}$$

OA.C Derivation of the Real SDF

In this section, we derive the state-price density shown in Proposition OA.1.

Proof of Proposition OA.1. Duffie and Skiadas (1994) show that the state-price density for a *general* normalized aggregator f is given by

$$\pi_t = e^{\int_0^t f_v(C_s, J_s)dt} f_c\left(C_t, J_t\right), \quad \text{(OA.8)}$$

where $f_c(\cdot,\cdot)$ and $f_v(\cdot,\cdot)$ are the partial derivatives of f with respect to its first and second arguments, respectively, and J is the value function given in (OA.2). The Feynman-Kac Theorem implies

$$f\left(C_t, J_{t-}\right)\big|_{s_{t-}=i} dt + E_t\left[dJ_t \middle| s_{t-} = i\right] = 0,\ i \in \{1, \ldots, N\}.$$

Using Ito's Lemma we rewrite the above equation as

$$0 = f\left(C, J_i\right) + CJ_{i,C}g_i + \frac{1}{2}C^2 J_{i,CC}\sigma_{C,i}^2 + \sum_{j \neq i} \lambda_{ij}\left(J_j - J_i\right),$$

for $i, j \in \{1, \ldots, N\}$, $j \neq i$. We guess and verify that $J = h(CV)$, where V_i satisfies the nonlinear equation system

$$0 = \beta u\left(V_i^{-1}\right) + g_i - \frac{1}{2}\gamma\sigma_{C,i}^2 + \sum_{j \neq i} \lambda_{ij}\left(\frac{(V_j/V_i)^{1-\gamma} - 1}{1-\gamma}\right),\ i, j \in \{1, \ldots, N\},\ j \neq i.$$

Substituting (OA.1) into (OA.8) and simplifying gives

$$\pi_t = \beta e^{-\beta\int_0^t \left[1+\left(\gamma-\frac{1}{\psi}\right)u\left(V_s^{-1}\right)\right]dt} C_t^{-\gamma} V_t^{-\left(\gamma-\frac{1}{\psi}\right)}. \quad \text{(OA.10)}$$

When $\psi = 1$, the above equation gives the second expression in (OA.3). We rewrite (OA.9) as

$$\beta\left[1 + \left(\gamma - \frac{1}{\psi}\right)u\left(V_i^{-1}\right)\right] = \overline{r}_i - \left(\gamma - \frac{1}{\psi}\right)\sum_{j \neq i}\lambda_{ij}\left(\frac{(V_j/V_i)^{1-\gamma} - 1}{1-\gamma}\right) - \left[\gamma g_i - \frac{1}{2}\gamma\left(1+\gamma\right)\sigma_{C,i}^2\right],\ i, j \in \{1, \ldots, N\},\ j \neq i, \quad \text{(OA.11)}$$

where $\overline{r}_i$ is given in (OA.5). Setting $\psi = 1$ in (OA.11) gives (OA.7). To derive the first expression in (OA.3) from (OA.10) we prove that

$$V_i = \left(\beta p_{C,i}\right)^{\frac{1}{1-\frac{1}{\psi}}},\ \psi \neq 1. \quad \text{(OA.12)}$$

We proceed by considering the optimization problem for the representative agent. She chooses her optimal consumption, C^*, and risky asset portfolio, φ, to maximize her expected utility

$$J_t^* = \sup_{C^*,\varphi} E_t \int_t^{\infty} f\left(C_t^*, J_t^*\right)dt.$$

Observe that J^* depends on optimal consumption-portfolio choice, whereas the J defined previously in (OA.6) depends on exogenous aggregate consumption. The optimization is carried out subject to the dynamic budget constraint, which we now describe. If the agent consumes at the rate, C^*, invests a proportion, φ, of her remaining financial wealth in risky assets, and puts the remainder in the locally risk-free asset, then her financial wealth, W, evolves according to the dynamic budget constraint:

$$\frac{dW_t}{W_{t-}} = \varphi_{t-}\left(dR_{p,t} - r_{t-}dt\right) + r_{t-}dt - \frac{C^*_{t-}}{W_{t-}}dt,$$

where $dR_{p,t}$ is the cum-dividend return on the risky asset portfolio over the time interval $[t, t+dt)$. We define $N_{ij,t}$ as the Poisson process which jumps upward by one whenever the state of the economy switches from i to $j \neq i$. The compensated version of this process is the Poisson martingale

$$N^P_{ij,t} = N_{ij,t} - \lambda_{ij}t.$$

Note that C^* is the consumption to be chosen by the agent, i.e. it is a control, and at this stage we cannot rule out the possibility that it jumps with the state of the economy. In contrast, C is aggregate consumption, and since it is continuous, its left and right limits are equal, i.e. $C_{t-} = C_t$.

The system of Hamilton-Jacobi-Bellman partial differential equations for the agent's optimization problem is

$$\sup_{C^*,\varphi} f\left(C^*_{t-}, J^*_{t-}\right)\Big|_{s_{t-}=i} dt + E_t\left[dJ^*_t \mid s_{t-} = i\right] = 0, \; i \in \{1,\ldots,N\}.$$

Applying Ito's Lemma to $J^*_t = J^*(W_t, s_t)$ allows us to write the above equation as

$$\begin{aligned} 0 \;=\; & \sup_{C^*_i,\varphi_i} f\left(C^*_i, J^*_i\right) + W_i J^*_{i,W}\left(\varphi_i\left(E_t\left[\frac{dR_{p,t}}{dt}|s_{t-}=i\right] - r_i\right) + r_i - \frac{C^*_i}{W_i}\right) + \frac{1}{2}W_i^2 J^*_{i,WW}\varphi_i^2\frac{1}{dt}E_t[(dR_{p,t})^2|s_{t-}=i] + \\ & + \sum_{j\neq i}\lambda_{ij}\left(J^*_j - J^*_i\right), \; i \in \{1,\ldots,N\},\; j \neq i. \end{aligned}$$

We guess and verify that $J^*_t = h(W_t F_t)$, where F_i satisfies the nonlinear equation system

$$0 = \sup_{C^*_i,\varphi_i} \beta u\left(\frac{C^*_i}{W_i F_i}\right) + \left(\varphi_i\left(E_t\left[\frac{dR_{p,t}}{dt}|s_{t-}=i\right] - r_i\right) + r_i - \frac{C^*_i}{W_i}\right) - \frac{1}{2}\gamma\varphi_i^2 E_t\left[\frac{(dR_{p,t})^2}{dt}|s_{t-}=i\right] + \sum_{j\neq i}\lambda_{ij}\left(\frac{(F_j/F_i)^{1-\gamma}-1}{1-\gamma}\right),$$

$i \in \{1,\ldots,N\}$, $j \neq i$.

From the first order condition with respect to consumption, we obtain the optimal consumption policy

$$C^*_i \;=\; \beta^{\psi}F_i^{-(\psi-1)}W_i, \; i \in \{1,\ldots,N\},$$

The market for the consumption good must clear, so $W_i = P_i$, $C^*_i = C$ (and thus $J = J^*$). Note that this forces the optimal consumption policy to be continuous. Hence,

$$p_{C,i} = \beta^{-\psi}F_i^{1-\psi}. \tag{OA.13}$$

The above equation implies that for $\psi = 1$, $p_{C,i} = 1/\beta$. The equality, $J = J^*$, implies that $CV_i = WF_i$. Hence, $F_i = p_{C,i}^{-1}V_i$. Using this equation to eliminate F_i from (OA.13) gives (OA.12). Substituting (OA.12) into (OA.10) and (OA.11) gives the expression in (OA.3) for $\psi \neq 1$ and (OA.4). ■

OA.D The Evolution of the Real SDF

In this section we derive the evolution of the real SDF, as given in (3).

We start by proving that the real SDF satisfies the stochastic differential equation

$$\left.\frac{d\pi_t}{\pi_{t-}}\right|_{s_{t-}=i} = -r_i dt + \left.\frac{dM_t}{M_{t-}}\right|_{s_{t-}=i}, \tag{OA.14}$$

where M is a martingale under $\mathbb{P}$ such that

$$\left.\frac{dM_t}{M_{t-}}\right|_{s_{t-}=i} = -\Theta_i^B dZ_t + \Theta_{ij}^P dN_{ij,t}^P,\ j \in \{1, \ldots, N\},\ j \neq i, \tag{OA.15}$$

r_i is the risk-free rate in state i given by

$$r_i = \overline{r}_i + \sum_{j \neq i} \lambda_{ij} \left[\frac{\gamma - \frac{1}{\psi}}{\gamma - 1} \left(\omega_{ij}^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right) - (\omega_{ij} - 1) \right], \tag{OA.16}$$

where

$$\omega_{ij} = \frac{\omega_j}{\omega_i},\ i, j \in \{1, \ldots, N\}, \tag{OA.17}$$

and $\omega_2, \omega_3, \ldots, \omega_N$ are determined by the following system of $N-1$ nonlinear algebraic equations:

$$0 = \omega_j^{-\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}} - \frac{k_1 + \frac{1-\frac{1}{\psi}}{\gamma-1} \sum_{k \neq 1} \lambda_{1k} \left(\omega_k^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right)}{k_j + \frac{1-\frac{1}{\psi}}{\gamma-1} \sum_{k \neq j} \lambda_{jk} \left(\frac{\omega_k}{\omega_j}^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right)},\ j \in \{2, \ldots, N\},\ \psi \neq 1, \tag{OA.18}$$

$$0 = \ln \omega_j^{\frac{1}{\gamma-1}} - \frac{\mu_{C,i} - \frac{1}{2}\gamma\sigma_{C,i}^2 + \sum_{k \neq i} \lambda_{ik}(\omega_{ik} - 1)}{\mu_{C,j} - \frac{1}{2}\gamma\sigma_{C,j}^2 + \sum_{k \neq j} \lambda_{jk}(\omega_{jk} - 1)},\ j \in \{2, \ldots, N\},\ \psi = 1, \tag{OA.19}$$

where

$$k_i = \overline{r}_i + \gamma\sigma_{C,i}^2 - \mu_{C,i}. \tag{OA.20}$$

Observe that if we define the $N \times N$ matrix $\mathbf{\Omega}$ via $\Omega_{ij} = \omega_{ij}$, the $N \times N$ matrix physical generator matrix for the Markov chain driving the economy, $\mathbf{\Lambda}$, via $\Lambda_{ij} = \lambda_{ij}$, where $\lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$, then

$$\mathbf{\Omega} = \begin{pmatrix} 1 & \omega_2 & \omega_3 & \cdots & \omega_N \\ \omega_2^{-1} & 1 & \frac{\omega_3}{\omega_2} & \cdots & \frac{\omega_N}{\omega_2} \\ \omega_3^{-1} & \frac{\omega_2}{\omega_3} & 1 & \cdots & \frac{\omega_N}{\omega_3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_N^{-1} & \frac{\omega_2}{\omega_N} & \frac{\omega_3}{\omega_N} & \cdots & 1 \end{pmatrix} \tag{OA.21}$$

and

$$\mathbf{\Lambda} = \begin{pmatrix} -\sum_{j \neq 1} \lambda_{1j} & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & -\sum_{j \neq 2} \lambda_{1j} & \cdots & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N1} & \lambda_{N2} & \cdots & -\sum_{j \neq N} \lambda_{Nj} \end{pmatrix}$$

Θ_i^B is the market price of risk due to Brownian shocks in state i, given by

$$\Theta_i^B = \gamma\sigma_{C,i},\ i \in \{1, \ldots, N\}, \tag{OA.22}$$

and Θ_{ij}^P is the market price of risk due to Poisson shocks when the economy switches out of state i into state j:

$$\Theta_{ij}^P = \omega_{ij} - 1,\ i, j \in \{1, \ldots, N\},\ j \neq i. \tag{OA.23}$$

We begin the proof by noting that if we define

$$\omega_{ij} = \left.\frac{\pi_t}{\pi_{t-}}\right|_{s_{t-}=i, s_t=j},\ i, j \in \{1, \ldots, N\},\ j \neq i, \tag{OA.24}$$

then (OA.3) implies that

$$\omega_{ij} = \begin{cases} \left(\frac{p_{C,j}}{p_{C,i}}\right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, & \psi \neq 1; \\ \left(\frac{V_j}{V_i}\right)^{-(\gamma-1)}, & \psi = 1. \end{cases} \tag{OA.25}$$

Equation (OA.17) follows from the above. We thus see that we need only determine the $N-1$ unknowns $\omega_2, \ldots, \omega_N$.

Using (OA.25) we rewrite (OA.4) and (OA.7) as

$$p_{C,i} = \frac{1}{k_i + \frac{1-\frac{1}{\psi}}{\gamma-1}\sum_{j\neq i}\lambda_{ij}\left(\omega_{ij}^{\frac{\gamma-1}{\gamma-1/\psi}} - 1\right)},\ i, j \in \{1, \ldots, N\},\ j \neq i, \tag{OA.26}$$

where k_i is defined in (OA.20) and

$$\beta \ln V_i = \mu_{C,i} - \frac{1}{2}\gamma\sigma_{C,i}^2 + \sum_{j\neq i}\lambda_{ij}\frac{\omega_{ij}-1}{1-\gamma},\ i, j \in \{1, \ldots, N\},\ j \neq i, \tag{OA.27}$$

respectively. Therefore, from (OA.25) and the above two equations it follows that $\omega_2, \ldots, \omega_N$ is the solution of equation system (OA.18) when $\psi \neq 1$ and (OA.19) when $\psi = 1$.

We now derive expressions for the risk-free rate and risk prices. Ito's Lemma implies that the state-price density evolves according to

$$\begin{aligned} \frac{d\pi_t}{\pi_{t-}} &= \frac{1}{\pi_{t-}}\frac{\partial \pi_{t-}}{\partial t}dt + \frac{1}{\pi_{t-}}C_t\frac{\partial \pi_{t-}}{\partial C_t}\frac{dC_t}{C_t} + \frac{1}{2}\frac{1}{\pi_{t-}}C_t^2\frac{\partial^2\pi_{t-}}{\partial C_t^2}\left(\frac{dC_t}{C_t}\right)^2 \\ &\quad + \sum_{s_t\neq s_{t-}}\lambda_{s_{t-},s_t}\frac{\Delta\pi_t}{\pi_{t-}}dt + \frac{\Delta\pi_t}{\pi_{t-}}dN_{s_{t-},s_t,t}^P, \end{aligned}$$

where $\Delta\pi_t = \pi_t - \pi_{t-}$. The definition (OA.24) implies

$$\frac{\Delta\pi_t}{\pi_{t-}} = \omega_{s_{t-},s_t} - 1.$$

Together with some standard algebra that allows us to rewrite (OA.28) as

$$\frac{d\pi_t}{\pi_{t-}} = -\left(\kappa_{s_{t-}} + \gamma\mu_{C,s_{t-}} - \frac{1}{2}\gamma\left(1+\gamma\right)\sigma^2_{C,s_{t-}} + \sum_{s_t\neq s_{t-}} \lambda_{s_{t-},s_t}\left(1-\omega_{s_{t-},s_t}\right)\right)dt - \gamma\sigma_{C,s_{t-}}dZ_t + \left(\omega_{s_{t-},s_t}-1\right)dN^P_{s_{t-},s_t,t}.$$

Comparing the above equation with (OA.14), which is standard in an economy with jumps, gives (OA.22) and (OA.23), in addition to

$$r_i = \kappa_i + \gamma\mu_{C,i} - \frac{1}{2}\gamma\left(1+\gamma\right)\sigma^2_{C,i} + \sum_{j\neq i}\lambda_{ij}\left(1-\omega_{ij}\right),\ i,j\in\{1,\dots,N\},\ j\neq i,$$

where

$$\kappa_i = \begin{cases} \beta\left[1+\left(\gamma-\frac{1}{\psi}\right)\frac{(\beta p_{C,i})^{-1}-1}{1-\frac{1}{\psi}}\right], & \psi\neq 1,\ i,j\in\{1,\dots,N\},\ j\neq i; \\ \beta\left[1+(\gamma-1)\ln\left(V_i^{-1}\right)\right], & \psi=1,\ i,j\in\{1,\dots,N\},\ j\neq i, \end{cases} \tag{OA.29}$$

We use Equations (OA.26) and (OA.27) to eliminate $p_{C,i}$ and V_i from (OA.29) to obtain

$$\kappa_i = \begin{cases} \overline{r}_i - \left(\gamma-\frac{1}{\psi}\right)\sum_{j\neq i}\lambda_{ij}\left(\frac{\omega_{ij}^{\frac{\gamma-1}{\gamma-1/\psi}}-1}{1-\gamma}\right) - \left[\gamma\mu_{C,i} - \frac{1}{2}\gamma\left(1+\gamma\right)\sigma^2_{C,i}\right], & \psi\neq 1,\ i,j\in\{1,\dots,N\},\ j\neq i; \\ \overline{r}_i + \sum_{j\neq i}\lambda_{ij}\left(\omega_{ij}-1\right) - \left[\gamma\mu_{C,i} - \frac{1}{2}\gamma\left(1+\gamma\right)\sigma^2_{C,i}\right], & \psi=1,\ i,j\in\{1,\dots,N\},\ j\neq i, \end{cases}$$

so

$$r_i = \begin{cases} \overline{r}_i - \left(\gamma-\frac{1}{\psi}\right)\sum_{j\neq i}\lambda_{ij}\left(\frac{\omega_{ij}^{\frac{\gamma-1}{\gamma-1/\psi}}-1}{1-\gamma}\right) + \sum_{j\neq i}\lambda_{ij}\left(1-\omega_{ij}\right), & \psi\neq 1,\ i,j\in\{1,\dots,N\},\ j\neq i,; \\ \overline{r}_i, & \psi=1,\ i\in\{1,\dots,N\}. \end{cases}$$

Taking the limit of the upper expression in the above equation gives the lower expression, so (OA.16) follows. The total market price of consumption risk in real state i accounts for both Brownian and Poisson shocks, and is thus given by

$$\Theta_i = \sqrt{\left(\Theta_i^B\right)^2 + \sum_{j\neq i}\lambda_{ij}\left(\Theta_{ij}^P\right)^2},\ i,j\in\{1,\dots,N\},\ j\neq i.$$

Because the Poisson and Brownian shocks in (OA.15) are independent and their respective prices of risk are bounded, M is a martingale under the actual measure $\mathbb{P}$. Thus, M defines the Radon-Nikodym derivative $\frac{d\mathbb{Q}}{d\mathbb{P}}$ via $M_t = E_t\left[\frac{d\mathbb{Q}}{d\mathbb{P}}\right]$. It is a standard result (see Elliott (1982)) that the risk-neutral switching probabilities per unit time are given by

$$\widehat{\lambda}_{ij} = \lambda_{ij}E_t\left[\left.\frac{M_t}{M_{t-}}\right| s_{t-}=i, s_t=j\right],\ j\neq i.$$

The jump component in $d\pi$ comes purely from dM. Thus, using (OA.24), we can simplify the above expression to obtain (OA.30).

The risk-neutral generator matrix for the Markov chain driving the economy is given by the $N\times N$ matrix $\hat{\mathbf{\Lambda}}$, where $\hat{\Lambda}_{ij} = \hat{\lambda}_{ij}$ where $\hat{\lambda}_{ij}dt$ is the risk-neutral probability of switching from state i to state j over a time interval of length dt, where

$$\begin{aligned} \hat{\lambda}_{ij} &= \lambda_{ij}\omega_{ij},\ i\neq j, \\ \hat{\lambda}_{ij} &= -\sum_{j\neq i}\hat{\lambda}_{ij}. \end{aligned} \tag{OA.30}$$

For the special case, where $\lambda_{ij} = pf_j$, $j \neq i$, f_i is the long-run physical probability that the state of the economy is i, and

$$\mathbf{\Lambda} = \begin{pmatrix} -p\sum_{j\neq 1} f_j & pf_2 & \cdots & pf_N \\ pf_1 & -p\sum_{j\neq 2} f_j & \cdots & pf_N \\ \vdots & \vdots & \ddots & \vdots \\ pf_1 & pf_2 & \cdots & -p\sum_{j\neq N} f_N \end{pmatrix}$$

Observe that, $\underline{f} = (f_1, \ldots, f_N)^\top$ is the long-run physical distribution of the state of the economy. The state of the economy converges to its long-run physical distribution exponentially at the rate p.

OA.D.1 Prices of Risk: Omega Matrices

For convenience, we show the values used for the omega matrix, (OA.21), within our numerical work. We stress that the omega matrices are endogenous and are stated here for the sake of reference.

In the baseline calibration, given that we assume that real and nominal regimes are independent, we obtain the following omega matrix:

	RL	**RM**	**RH**	**EL**	**EM**	**EH**
RL	1.0000	1.0000	1.0000	0.6959	0.6959	0.6959
RM	1.0000	1.0000	1.0000	0.6959	0.6959	0.6959
RH	1.0000	1.0000	1.0000	0.6959	0.6959	0.6959
EL	1.4369	1.4369	1.4369	1.0000	1.0000	1.0000
EM	1.4369	1.4369	1.4369	1.0000	1.0000	1.0000
EH	1.4369	1.4369	1.4369	1.0000	1.0000	1.0000

When we relax this assumption, the following omega matrix yields a 25bps uncondtional inflation risk premium:

	RL	**RM**	**RH**	**EL**	**EM**	**EH**
RL	1.0000	0.9387	0.9419	0.6712	0.6472	0.6643
RM	1.0653	1.0000	1.0034	0.7151	0.6895	0.7077
RH	1.0617	0.9966	1.0000	0.7126	0.6871	0.7053
EL	1.4898	1.3985	1.4033	1.0000	0.9642	0.9897
EM	1.5451	1.4504	1.4553	1.0371	1.0000	1.0264
EH	1.5053	1.4130	1.4178	1.0104	0.9742	1.0000

The following omega matrix yields a 50bps unconditional inflation risk premium:

	RL	**RM**	**RH**	**EL**	**EM**	**EH**
RL	1.0000	0.8719	0.8778	0.6458	0.5954	0.6261
RM	1.1469	1.0000	1.0067	0.7407	0.6829	0.7180
RH	1.1392	0.9933	1.0000	0.7357	0.6783	0.7132
EL	1.5484	1.3501	1.3592	1.0000	0.9219	0.9694
EM	1.6795	1.4644	1.4743	1.0847	1.0000	1.0514
EH	1.5973	1.3927	1.4021	1.0316	0.9511	1.0000

The following omega matrix yields a 75bps unconditionalinflation risk premium:

	RL	**RM**	**RH**	**EL**	**EM**	**EH**
RL	1.0000	0.8026	0.8106	0.6202	0.5428	0.5835
RM	1.2460	1.0000	1.0100	0.7728	0.6764	0.7270
RH	1.2337	0.9901	1.0000	0.7652	0.6697	0.7198
EL	1.6123	1.2939	1.3068	1.0000	0.8752	0.9407
EM	1.8422	1.4785	1.4932	1.1426	1.0000	1.0748
EH	1.7139	1.3755	1.3892	1.0630	0.9304	1.0000

The following omega matrix yields a 100bps unconditional inflation risk premium:

	RL	**RM**	**RH**	**EL**	**EM**	**EH**
RL	1.0000	0.7393	0.7488	0.5972	0.4959	0.5428
RM	1.3526	1.0000	1.0128	0.8077	0.6708	0.7342
RH	1.3355	0.9873	1.0000	0.7975	0.6623	0.7249
EL	1.6745	1.2380	1.2539	1.0000	0.8304	0.9090
EM	2.0165	1.4908	1.5099	1.2042	1.0000	1.0946
EH	1.8422	1.3620	1.3795	1.1001	0.9136	1.0000

Finally, the following omega matrix yields a 125bps unconditional inflation risk premium:

	RL	**RM**	**RH**	**EL**	**EM**	**EH**
RL	1.0000	0.6806	0.6912	0.5757	0.4532	0.5039
RM	1.4692	1.0000	1.0154	0.8459	0.6659	0.7403
RH	1.4469	0.9848	1.0000	0.8330	0.6557	0.7291
EL	1.7369	1.1822	1.2005	1.0000	0.7872	0.8752
EM	2.2064	1.5018	1.5250	1.2703	1.0000	1.1118
EH	1.9845	1.3508	1.3716	1.1426	0.8994	1.0000

OA.E Nominal SDF

We define the nominal SDF $\pi^{\$}$ by

$$\pi_t^{\$} = \frac{\pi_t}{P_t}.$$

Now we apply Ito's Lemma to obtain

$$\begin{aligned}
\frac{d\pi_t^{\$}}{\pi_t^{\$}} &= \frac{d\pi_t}{\pi_t} - \frac{dP_t}{P_t} - \frac{d\pi_t}{\pi_t}\frac{dP_t}{P_t} + \left(\frac{dP_t}{P_t}\right)^2 \\
&= -r_i dt - \gamma\sigma_{C,i} dZ_t + \sum_{j\neq i}(\omega_{ij}-1)dN^P_{ij,t} - \mu_{P,i}dt - \sigma_{P,i}dZ_{P,t} + \gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i}dt + \sigma^2_{P,i}dt \\
&= -(r_i + \mu_{P,i} - \gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i} - \sigma^2_{P,i})dt - \gamma\sigma_{C,i}dZ_t - \sigma_{P,i}dZ_{P,t} + \sum_{j\neq i}(\omega_{ij}-1)dN^P_{ij,t}.
\end{aligned}$$

We thus obtain the nominal risk-free rate

$$\begin{aligned}
r_i^{\$} &= -E_t\left[\frac{d\pi_t^{\$}}{\pi_t^{\$}}\middle| s_{t-} = i\right] \\
&= r_i + \mu_{P,i} - \gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i} - \sigma^2_{P,i}.
\end{aligned}$$

We define the exponential martingale (wrt to $\mathbb{P}$) $M^{\$}$ via

$$\frac{dM_t^{\$}}{M_{t-}^{\$}} = -\gamma\sigma_{C,s_{t-}}dZ_t - \sigma_{P,s_{t-}}dZ_{P,t} + \sum_{s_t\neq s_{t-}}(\omega_{s_t,s_{t-}}-1)dN^P_{s_t,s_{t-},t},\ M_0^{\$} = 1.$$

We use $M^{\$}$ to define the probability measure $\mathbb{Q}^{\$}$ in the standard way.

OA.F Sticky Cash Flows

Applying Ito's Lemma to (4), we obtain

$$\frac{dX_t}{X_t} = \mu_{X,t}dt + \sigma_Y dW_t + \varphi\sigma_{P,t}dZ_{P,t},$$

where

$$\mu_{X,t} = \mu_{Y,t} + \varphi(\mu_{P,t} + \rho_{PY,t}\sigma_{Y,t}\sigma_{P,t}) - \frac{1}{2}\varphi(1-\varphi)\sigma^2_{P,t}.$$

We can use the martingale component of $\pi^{\$}$ to define a new probability measure $\mathbb{Q}^{\$}$, under which

$$\hat{\mu}^{\$}_{X,t} = E^{\mathbb{Q}^{\$}}_t\left[\frac{dX_t}{X_t}\right] = \mu_{X,t} - \gamma\sigma_{C,t}(\sigma_Y\rho_{YC,t} + \varphi\sigma_{P,t}\rho_{PC,t}) - \sigma_{P,i}(\sigma_Y\rho_{PY,t} + \varphi\sigma_{P,t}), \tag{OA.31}$$

via Girsanov's Theorem, where $\rho_{YC,t}dt = E_t[dW_t dZ_t]$, $\rho_{PC,t}dt = E_t[dZ_{P,t}dZ_t]$, and $\rho_{PY,t}dt = E_t[dZ_{P,t}dW_t]$.

Defining $x_t = \ln X_t$, under $\mathbb{Q}^{\$}$, we have

$$dx_t = \hat{\mu}^{\$}_{x,t}dt + \sigma_{x,t}dZ^{\mathbb{Q}^{\$}}_{x,t},$$

where

$$\sigma_{x,t} = \sqrt{\sigma^2_{Y,t} + 2\varphi\sigma_{Y,t}\sigma_{P,t}\rho_{PY,t} + \varphi^2\sigma^2_{P,t}}, \tag{OA.32}$$

$$\begin{aligned}\hat{\mu}^{\$}_{x,t} &= \hat{\mu}^{\$}_{X,t} - \frac{1}{2}\sigma^2_{x,t},\\ &= \mu_{X,t} - \gamma\sigma_{C,t}(\sigma_{Y,t}\rho_{YC,t} + \varphi\sigma_{P,t}\rho_{PC,t}) - \sigma_{P,i}(\sigma_Y\rho_{PY,t} + \varphi\sigma_{P,t}) - \frac{1}{2}\sigma^2_{x,t}.\end{aligned} \tag{OA.33}$$

OA.G Liquidation Value

The abandonment or liquidation value of a firm is just its unlevered value, i.e. the present value of future cashflows, ignoring coupon payments to debtholders and default risk. Small, but frequent shocks to a firm's real cashflow growth are modelled by changes in the standard Brownian motion W_t. Small, but frequent shocks to the real SDF are modelled by changes in the standard Brownian motion Z_t. The assumption that $dZ_t dW_t = 0$ means that small, but frequent shocks to cashflow growth are not priced. However, changes in the expected real cashflow growth rate are driven by the same Markov chain as those driving jumps in the SDF. Hence, changes in unlevered firm value driven by changes in the expected real cashflow growth rate will be priced.

Suppose the economy is currently in state i. Then, the risk-neutral probability of the economy switching into a different state $j \neq i$ during a small time interval of length Δt is $\widehat{\lambda}_{ij}\Delta t$ and the risk-neutral probability of not switching is $1 - \widehat{\lambda}_{ij}\Delta t$. We can therefore write the unlevered nominal firm value in state i as

$$A^{\$}_{i,t} = (1-\eta)X_t\Delta t + e^{-(r^{\$}_i - \mu^{\$}_{X,i})\Delta t}\left[\left(1 - \widehat{\lambda}_{ij}\Delta t\right)A^{\$}_i + \sum_{j\neq i}\widehat{\lambda}_{ij}\Delta t A^{\$}_j\right],\ i,j \in \{1,\ldots,N\},\ j \neq i, \tag{OA.34}$$

where $N = 6$ is the number of states in the economy.

The first term in (OA.34) is the after-tax cash flow received in the next instant and the second term is the discounted continuation value. The discount rate is just the standard discount rate for a perpetuity. Observe that the volatility of cashflow growth does not appear in the discount rate, because $dZ_t dW_t = 0$. The continuation value is the average of $A^{\$}_{i,t}$ and $A^{\$}_{j,t}$, weighted by the risk-neutral probabilities of being in states i and $j \neq i$ a small instant Δt from now. For example, with risk-neutral probability $\widehat{\lambda}_{ij}\Delta t$ the economy will be in state $j \neq i$ and the unlevered nominal firm value will be value will be $A^{\$}_j$. The continuation value is discounted back at a rate reflecting the nominal interest rate rate $r_i^\$$ and the expected nominal earnings growth rate over that instant which is $\mu_{X,i}$ – observe that there is no difference between the physical and risk-neutral nominal earnings growth rates, because $dZ_t dW_t = 0$.

We take the limit of (OA.34) as $\Delta t \to 0$, to obtain

$$0 = (1-\eta)\, X - (r_i^\$ - \mu_{X,i}^\$) A_i^\$ + \sum_{j \neq i} \widehat{\lambda}_{ij} \left(A_j^\$ - A_i^\$ \right), \; i \in \{1,\ldots,N\}, j \neq i.$$

To obtain the solution of the above linear equation system, we define

$$v_{A,i} = \frac{1}{(1-\eta)} \frac{A_i^\$}{X},$$

the before-tax nominal price-earnings ratio in state i. Therefore

$$\left(\operatorname{diag}\left(r_1^\$ - \mu_{X,1}^\$, \ldots, r_N^\$ - \mu_{X,N}^\$\right) - \widehat{\Lambda}\right) \begin{pmatrix} v_{A,1} \\ \vdots \\ v_{A,N} \end{pmatrix} = 1_{N\times 1}, \tag{OA.35}$$

where $1_{N\times1}$ is a $N \times 1$ vector of ones, $\operatorname{diag}\left(r_1^\$ - \mu_{X,1}^\$, \ldots, r_N^\$ - \mu_{X,N}^\$\right)$ is a $N \times N$ diagonal matrix, with the quantities $r_1^\$ - \mu_{X,1}^\$, \ldots, r_N^\$ - \mu_{X,N}^\$$ along the diagonal and $\widehat{\Lambda}$, defined by $[\widehat{\Lambda}]_{ij} = \widehat{\lambda}_{ij}$, $i, j \in \{1,\ldots,N\}$, where

$$\widehat{\lambda}_{ij} = \omega_{ij}\lambda_{ij}, j \neq i$$
$$\widehat{\lambda}_{ii} = -\sum_{j\neq i} \omega_{ij}\lambda_{ij}, j \neq i$$

is the generator matrix of the Markov chain for the combined state of the economy under the risk-neutral measure. Solving (OA.35) gives (5), if $\det\left(\operatorname{diag}\left(r_1^\$ - \mu_{X,1}^\$, \ldots, r_N^\$ - \mu_{X,N}^\$\right) - \widehat{\Lambda}\right) \neq 0$.

Similarly, we can show that the before-tax value of the claim to the real earnings stream Y, when the current state is i is given by $P_{i,t}^Y = p_i Y_t$, where

$$(p_1, \ldots, p_N)^\top = (\operatorname{diag}\left(r_1 - \mu_{Y,1}, \ldots, r_N - \mu_{Y,N}\right) - \widehat{\Lambda})^{-1} 1_{6\times 1}$$

Hence, from the basic asset pricing equation

$$E_t\left[\frac{dP_t^Y + Y dt}{P_t^Y} - r_{s_{t-}} dt \middle| s_{t-} = i \right] = -E_t\left[\frac{d\pi_t}{\pi_t} \frac{dP_t^Y}{P_t^Y} \middle| s_{t-} = i \right],$$

we obtain the unlevered risk premium:

$$E_t\left[\frac{dP_t^Y + Y_t dt}{P^Y} - r_{s_{t-}} dt \middle| s_{t-} = i \right] = -\sum_{j\neq i} \left(\widehat{\lambda}_{ij} - \lambda_{ij}\right) \left(\frac{p_j}{p_i} - 1\right) dt, i, j \in \{1,\ldots,N\}.$$

Applying Ito's Lemma,

$$dP_{i,t}^X = p_i dX_t + \sum_{j\neq i} \lambda_{ij} \left(p_j - p_i\right) dt + \sum_{j\neq i} \left(p_j - p_i\right) dN_{ij,t}^P, \; i, j \in \{1,\ldots,N\},$$

Thus, the volatility of returns on unlevered equity in state i is given by

$$\sigma_{R,i} = \sqrt{\sum_{j\neq i} \lambda_{ij} \left(\frac{p_j}{p_i} - 1\right)^2}, \ i, j \in \{1, \ldots, N\}.$$

OA.H Arrow-Debreu Securities – Default

The Arrow-Debreu default claim denoted by $q^{\$}_{D,ij}$ is the value of a dollar paid if default occurs in state j and the current state is i. In a static capital structure model, these are the only Arrow-Debreu claims needed. Given the initial state (in which the firm selected its capital structure, there are N^2 such claims: $\{q^{\$}_{D,ij}\}_{i,j\in\{1,\ldots,N\}}$. We assume, without loss of generality, that the regimes are labelled so that the default boundaries respect a monotonic ordering $X_{D,1} > \ldots > X_{D,N}$.

We say that a firm's earnings are in the default region $\mathcal{D}_k$, $k = 0, \ldots, N-1$, when they fall in the interval $(X_{D,k+1}, X_{D,k}]$, assuming that $X_{D,0} \to \infty$. Region $\mathcal{D}_N$ is $(-\infty, X_{D,N}]$.

Proposition OA.2 *Let A_k be*

$$A_k = \begin{pmatrix} 0_{N-k\times N-k} & -I_{N-k\times N-k} \\ 2S^{-1}_{x,k}(\widehat{\Lambda}_k - R^{\$}_k) & 2S^{-1}_{x,k} M_{x,k} \end{pmatrix},$$

where $0_{n\times m} \in \mathbb{R}^{n\times m}$ denotes a matrix of zeros, $I_{n\times n} \in \mathbb{R}^{n\times n}$ denotes the n-dimensional identity matrix, $\widehat{\Lambda}_k$, $R^{\$}_k$, $M_{x,k}$, and $S_{x,k}$ are the $N-k$ by $N-k$ matrices obtained by removing the first k rows and columns of $\widehat{\Lambda}$,

$$R^{\$} = \text{diag}(r^{\$}_1, \ldots, r^{\$}_N), \qquad M_x = \text{diag}(\hat{\mu}^{\$}_{x,1}, \ldots, \hat{\mu}^{\$}_{x,N}), \quad \textit{and} \quad S_x = \text{diag}(\sigma^2_{x,1}, \ldots, \sigma^2_{x,N}),$$

with $\widehat{\mu}^{\$}_{x,i} = \widehat{\mu}^{\$}_{X,i} - \frac{1}{2}\sigma^2_{X,i}$ and $\sigma_{x,i} = \sigma_{X,i}$ the drift and diffusion coefficient of $x = \log X$ under $\mathbb{Q}^{\$}$.

Given the integration constants $h_{i,j}(\omega)$, the default Arrow-Debreu in region $\mathcal{D}_k$ are given by

Region $\mathcal{D}_0$:

$$q^{\$}_{D,ij}(x) = \sum_{l=1}^{N} h_{ij}(\omega_{0,l}) e^{-\omega_{0,l} x},$$

where $\omega_{0,1} > \ldots > \omega_{0,N} > 0$ are the N positive eigenvalues of A_0.

Region $\mathcal{D}_k, k \in \{1, \ldots, N-1\}$:

$$q^{\$}_{D,ij}(x) = \delta_{ij}, \qquad i \in \{1, \ldots, k\}, j \in \{1, \ldots, N\},$$

$$q^{\$}_{D,ij}(x) = \sum_{l=1}^{2(N-k)} h_{ij}(\omega_l) e^{-\omega_l x} - [A^{-1}_k B_k]_{i-k,j}, \qquad i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\}.$$

where $\omega_{k,l}$ are the $2(N-k)$ eigenvalues of A_k and

$$B_k = \begin{pmatrix} 0_{N-k\times k} & 0_{N-k\times N-k} \\ B^{\emptyset}_k & 0_{N-k\times N-k} \end{pmatrix}, \qquad B^{\emptyset}_k = \begin{pmatrix} 2\frac{\widehat{\lambda}_{k+1,1}}{\sigma^2_{k+1}} & 2\frac{\widehat{\lambda}_{k+1,2}}{\sigma^2_{x,k+1}} & \cdots & 2\frac{\widehat{\lambda}_{k+1,k}}{\sigma^2_{x,k+1}} \\ 2\frac{\widehat{\lambda}_{k+2,1}}{\sigma^2_{x,k+2}} & 2\frac{\widehat{\lambda}_{x,k+2,2}}{\sigma^2_{x,k+2}} & \cdots & 2\frac{\widehat{\lambda}_{x,k+2,k}}{\sigma^2_{x,k+2}} \\ \vdots & \vdots & \cdots & \vdots \\ 2\frac{\widehat{\lambda}_{N,1}}{\sigma^2_{x,N}} & 2\frac{\widehat{\lambda}_{N,2}}{\sigma^2_{x,N}} & \cdots & 2\frac{\widehat{\lambda}_{N,k}}{\sigma^2_{x,N}} \end{pmatrix}.$$

Region $\mathcal{D}_N$:

$$q^{\$}_{D,k,ij}(x) = \delta_{ij}, \quad \forall i, j.$$

In each default region, for each ω, the integration constants $h_{k+1,\bullet}(\omega) \equiv [h_{k+1,j}(\omega)]_{j=1,\dots,N} \in \mathbb{R}^{1\times N}$, are identified by the boundary conditions (Section OA.H.3), and the remaining integration constants

$$H_k(\omega) = \begin{pmatrix} h_{k+2,1}(\omega) & \cdots & h_{k+2,N}(\omega) \\ \vdots & \dots & \vdots \\ h_{N,1}(\omega) & \cdots & h_{N,N}(\omega) \end{pmatrix}$$

are given by

$$\boldsymbol{H}_k(\omega) = -\boldsymbol{G}_k^{-1}(\omega)\ \underline{g}_{k+\bullet,1}(\omega)\ h_{k+1,\bullet}(\omega)$$

where $\underline{g}_{k+\bullet,k+1}(\omega) \equiv [g_{i,k+1}(\omega)]_{i=k+2,\dots,N} \in \mathbb{R}^{(N-k-1)\times 1}$ comprises the last $N-k-1$ elements of the first column of

$$\boldsymbol{G}(\omega) = 2S_x^{-1}(\widehat{\Lambda} - R^{\$}) - \omega(2S_x^{-1}M_x - \omega I_{N\times N}).$$

and $G_k(\omega)$ is the $N-k-1$ by $N-k-1$ matrix obtained by removing the first $k+1$ rows and columns of $G(\omega)$, i.e.

$$\boldsymbol{G}_k(\omega) = [\boldsymbol{G}(\omega)]_{i\in\{k+2,\dots,N\},\, j\in\{k+2,\dots,N\}},\ for\, k \in \{0,\dots,N-1\}$$

The next two subsections outline the proof of Proposition OA.2.

OA.H.1 Region $\mathcal{D}_0$: $X_t > X_{D,1}$

We start by analyzing the case where earnings at the current date t are above the highest default boundary, i.e. $X_t > X_{D,1}$. Hence, if earnings hit the boundary $X_{D,j}$ from above for the first time in state j, $\{q^{\$}_{D,ij}\}_{i,j\in\{1,\dots,N\}}$ will pay one dollar; otherwise, the security expires worthless. Starting from (6), we change the probability measure from $\mathbb{P}$ to $\mathbb{Q}^{\$}$ to obtain

$$q^{\$}_{D,ij,t} = E_t^{\mathbb{Q}^{\$}}\left[e^{-\int_t^{\tau_D} r_u^{\$} du} I_{\{s_{\tau_D}=j\}} \middle| s_t = i \right].$$

From the Feynman-Kac Theorem, we hence obtain

$$E_t^{\mathbb{Q}^{\$}}[dq^{\$}_{D,ij} - r_i^{\$} q^{\$}_{D,ij} dt] = 0,\ i,j \in \{1,\dots,N\}. \tag{OA.36}$$

Using Ito's Lemma, the above equation can be rewritten as the following second-order ordinary differential-equation system:[40]

$$\frac{1}{2}\sigma_{x,i}^2 \frac{d^2 q^{\$}_{D,ij}}{dx^2} + \widehat{\mu}^{\$}_{x,i}\frac{dq^{\$}_{D,ij}}{dx} + \sum_{k\neq i}\widehat{\lambda}_{ik}\left(q^{\$}_{D,kj} - q^{\$}_{D,ij}\right) = r_i^{\$} q^{\$}_{D,ij},\ i,j \in \{1,\dots,N\},$$

where $\widehat{\mu}^{\$}_{x,i}$ and $\sigma_{x,i}$ are defined in (OA.33) and (OA.32), respectively.

[40]Note that since the puts are perpetual, $\frac{\partial q^{\$}_{ij,t}}{\partial t} = 0$. Hence, $q^{\$}_{ij,t}$ is a function of the stochastic process $x = \log X$ and the state of the economy.

In order to solve this system of ODEs, define

$$\begin{aligned} z_{ij} &= q_{D,ij}, i,j \in \{1,\ldots,N\} \\ z_{N+i,j} &= \frac{dq_{D,ij}}{dx}, i,j \in \{1,\ldots,N\}. \end{aligned}$$

Then, we obtain the following first order linear system

$$\begin{aligned} \frac{dz_{ij}}{dx} - z_{N+i,j} &= 0,\ i,j \in \{1,\ldots,N\}, \\ \frac{dz_{N+i,j}}{dx} + \frac{2\widehat{\mu}^{\$}_{x,i}}{\sigma^2_{x,i}} z_{N+i,j} + \sum_{k\neq i} \frac{2\widehat{\lambda}_{ik}}{\sigma^2_{x,i}} \left(z_{kj} - z_{ij}\right) - \frac{2r^{\$}_i}{\sigma^2_{x,i}} z_{ij} &= 0,\ i,j \in \{1,\ldots,N\}. \end{aligned}$$

Expressing the above equation system in matrix form gives

$$\boldsymbol{Z}' + \boldsymbol{A}_0 \boldsymbol{Z} = \boldsymbol{0}_{2N\times N}, \tag{OA.40}$$

where the ij'th element of the $2N$ by N matrix, $\boldsymbol{Z}$, is

$$[\boldsymbol{Z}]_{ij} = z_{ij}, i \in \{1,\ldots,2N\}, j \in \{1,\ldots,N\},$$

and $\boldsymbol{Z}' = \frac{d\boldsymbol{Z}}{dx}$.

To solve eq. (OA.40), one first finds the eigenvectors and eigenvalues of $\boldsymbol{A}_0$. Their defining equation is

$$\boldsymbol{A}_0 \underline{e}_i = \omega_i \underline{e}_i,\ i \in \{1,\ldots,2N\}, \tag{OA.41}$$

where ω_i is the i'th eigenvalue and $\underline{e}_i$ is the corresponding eigenvector. Note that $\boldsymbol{A}_0$ has N positive and N negative eigenvalues (Jobert and Rogers (2006)).

It follows from (OA.41) that the eigenvalues of A_0 are the roots of its characteristic polynomial; that is, any eigenvalue ω is a solution to the following $2N$'th-order polynomial:

$$\det(\boldsymbol{A}_0 - \omega \boldsymbol{I}) = 0.$$

To simplify the above expression for the characteristic polynomial, we then use the following identity from Silvester: If $\boldsymbol{F} = \begin{pmatrix} \boldsymbol{F}_{11} & \boldsymbol{F}_{12} \\ \boldsymbol{F}_{21} & \boldsymbol{F}_{22} \end{pmatrix}$, where $\boldsymbol{F}_{ij}\ , i,j \in \{1,2\}$ are N by N matrices, any two of which commute with each other, then

$$\det \boldsymbol{F} = \det(\boldsymbol{F}_{11}\boldsymbol{F}_{22} - \boldsymbol{F}_{12}\boldsymbol{F}_{21}). \tag{OA.42}$$

Since

$$\boldsymbol{A}_0 - \omega \boldsymbol{I} = \begin{pmatrix} -\omega \boldsymbol{I} & -\boldsymbol{I} \\ 2\boldsymbol{S}_x^{-1}(\widehat{\boldsymbol{\Lambda}} - \boldsymbol{R}^{\$}) & 2\boldsymbol{S}_x^{-1}\boldsymbol{M}_x - \omega \boldsymbol{I} \end{pmatrix}$$

and diagonal matrices commute with all other matrices of the same size, any pair of the N submatrices in $\boldsymbol{A}_0$ commute. Therefore, one can apply (OA.42) and

$$0 = \det(\boldsymbol{A}_0 - \omega \boldsymbol{I}) = \det \boldsymbol{G}(\omega), \tag{OA.43}$$

where

$$\boldsymbol{G}(\omega) = \omega(\omega \boldsymbol{I} - 2\boldsymbol{S}_x^{-1}\boldsymbol{M}_x) - 2\boldsymbol{S}_x^{-1}(\boldsymbol{R}^{\$} - \widehat{\boldsymbol{\Lambda}}). \tag{OA.44}$$

Observe that the ω's in the N by N matrix $\boldsymbol{G}(\omega)$ appear only along the diagonal. When $N \leq 2$ the polynomial (OA.43) is of order 4 or less and can be solved exactly in closed-form. When $N \geq 3$, it must be solved numerically. From Jobert and Rogers (2006), we know that N of the solutions to (OA.43) lie in the left half-plane of the Argand diagram and N lie in the right half-plane. Once the eigenvalues have been obtained, the eigenvectors are obtained by solving (OA.41). We then define the $2N$ by $2N$ matrix of eigenvectors, E, by stacking the eigenvectors as follows

$$\boldsymbol{E} = (\underline{e}_1, \ldots, \underline{e}_{2N}).$$

Hence, the ij'th component of $\boldsymbol{E}$ is the i'th element of the j'th eigenvector, i.e.

$$\boldsymbol{E}_{ij} = (\underline{e}_j)_i.$$

Given the $\boldsymbol{E}$ matrix, we can define the $2N$ by N matrix $\boldsymbol{W}$ via

$$\boldsymbol{E}\boldsymbol{W} = \boldsymbol{Z}. \tag{OA.45}$$

We can then rewrite (OA.40) as

$$\begin{aligned} \boldsymbol{E}\boldsymbol{W}' + \boldsymbol{A}_0\boldsymbol{E}\boldsymbol{W} &= \boldsymbol{0}_{2N\times N}, \\ \Leftrightarrow \boldsymbol{E}^{-1}(\boldsymbol{E}\boldsymbol{W}' + \boldsymbol{A}_0\boldsymbol{E}\boldsymbol{W}) = \boldsymbol{W}' + \boldsymbol{E}^{-1}\boldsymbol{A}_0\boldsymbol{E}\boldsymbol{W} &= \boldsymbol{0}_{2N\times N}, \\ \Leftrightarrow \boldsymbol{W}' + \boldsymbol{D}\boldsymbol{W} &= \boldsymbol{0}_{2N\times N}, \end{aligned} \tag{OA.46}$$

where

$$\boldsymbol{D} = \boldsymbol{E}^{-1}\boldsymbol{A}_0\boldsymbol{E} = \mathrm{diag}(\omega_1, \ldots, \omega_{2N}).$$

The first order differential equation system of eq. (OA.46) is similar to that of eq. (OA.40), with the notable difference that $\boldsymbol{D}$ is a diagonal matrix while $\boldsymbol{A}_0$ is not. Making use of this, we premultiply both sides of eq. (OA.46) by the integrating factor $e^{\boldsymbol{D}x}$, which yields

$$e^{\boldsymbol{D}x}\boldsymbol{W}' + e^{\boldsymbol{D}x}\boldsymbol{D}\boldsymbol{W} = (e^{\boldsymbol{D}x}\boldsymbol{W})' = \boldsymbol{0}_{2N\times N}.$$

Integrating the above equation gives

$$e^{\boldsymbol{D}x}\boldsymbol{W} = \boldsymbol{K},$$

where $\boldsymbol{K}$ is a $2N$ by N matrix of constants of integration. Therefore, the general solution of eq. (OA.46) is

$$\boldsymbol{W} = e^{-\boldsymbol{D}x}\boldsymbol{K},$$

which, given eq. (OA.45), implies

$$\boldsymbol{Z} = \boldsymbol{E}e^{-\boldsymbol{D}x}\boldsymbol{K} = e^{-\boldsymbol{D}x}\boldsymbol{E}\boldsymbol{K}, \tag{OA.47}$$

given that $\boldsymbol{D}$ is $2N \times 2N$ and diagonal, and that $\boldsymbol{E}$ is $2N \times 2N$.

Thus,

$$q^{\$}_{D,ij}(x) = \sum_{l=1}^{2N} h_{ij}(\omega_l)e^{-\omega_l x}, \tag{OA.48}$$

where the $h_{ij}(\omega_l)$ are constants of integration that depend on the eigenvalues. Then, to ensure the finiteness of the Arrow-Debreu prices as $x \to \infty$, focus on the N positive eigenvalues of $\boldsymbol{A}_0$. That is, we obtain $q^{\$}_{D,ij}$, the Arrow-Debreu prices in region $\mathcal{D}_0$, where

$X > X_{D,1}$:

$$q^{\$}_{D,ij}(x) = \sum_{m=1}^{N} h_{ij}(\omega_m) e^{-\omega_{0,m} x}, \tag{OA.49}$$

where, without loss of generality, $\omega_{0,1} > \ldots > \omega_{0,N} > 0$ are the N positive eigenvalues of $\boldsymbol{A}_0$, or equivalently the N positive roots of the characteristic polynomial $\det \boldsymbol{G}(\omega) = 0$

Note that, for any eigenvalue ω of $\boldsymbol{A}_0$, the solution $q^{\$}_{D,ij} = h_{ij}(\omega)e^{-\omega x}$, with $h_{ij}(\omega_l) = 0, \forall \omega_l \neq \omega$, solves (OA.48). Indeed, we then have

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{H}(\omega) \\ -\omega \boldsymbol{H}(\omega) \end{bmatrix} e^{-\omega x}, \quad \text{where} \quad \boldsymbol{H}(\omega) = \begin{pmatrix} h_{11}(\omega) & \cdots & h_{1N}(\omega) \\ \vdots & \ldots & \vdots \\ h_{N1}(\omega) & \cdots & h_{NN}(\omega) \end{pmatrix}$$

and

$$\boldsymbol{Z}' = -\omega \boldsymbol{Z}.$$

Hence, (OA.40) implies that

$$(-\omega \boldsymbol{I}_{2N\times 2N} + \boldsymbol{A}_0)\boldsymbol{Z} = \boldsymbol{0}_{2N\times N},$$

or, equivalently,

$$\begin{pmatrix} -\omega \boldsymbol{I}_{N\times N} & -\boldsymbol{I}_{N\times N} \\ 2\boldsymbol{S}_x^{-1}(\widehat{\boldsymbol{\Lambda}} - \boldsymbol{R}^{\$}) & 2\boldsymbol{S}_x^{-1}\boldsymbol{M}_x - \omega \boldsymbol{I}_{N\times N} \end{pmatrix} \begin{bmatrix} \boldsymbol{H}(\omega) \\ -\omega \boldsymbol{H}(\omega) \end{bmatrix} = \boldsymbol{0}_{2N\times N}. \tag{OA.50}$$

This particular solution is important since it allows us to express $N(N-1)$ of the N^2 integation constants in terms of the first N ones. Indeed, simplifying (OA.50) gives

$$\begin{aligned} -\omega \boldsymbol{I}_{N\times N}\boldsymbol{H}(\omega) + \boldsymbol{I}_{N\times N}\omega \boldsymbol{H}(\omega) &= \boldsymbol{0}_{N\times N}, \\ (2\boldsymbol{S}_x^{-1}(\widehat{\boldsymbol{\Lambda}} - \boldsymbol{R}^{\$}) - \omega(2\boldsymbol{S}_x^{-1}\boldsymbol{M}_x - \omega \boldsymbol{I}_{N\times N}))\boldsymbol{H}(\omega) &= \boldsymbol{0}_{N\times N}, \end{aligned}$$

where the first equation is trivial. To solve the second equation, we first consider

$$\boldsymbol{G}(\omega)\left(h_{1j}(\omega), \ldots, h_{Nj}(\omega)\right)^T = \boldsymbol{0}_{N\times 1},$$

where $\boldsymbol{G}(\omega)$ is defined in (OA.44). We denote the ij'th element of $\boldsymbol{G}(\omega)$ by $g_{ij}(\omega)$. Observe that only the diagonal elements of $\boldsymbol{G}(\omega)$ depend on ω. We know from (OA.43) that $\det \boldsymbol{G}(\omega) = 0$. Thus, the equations

$$\sum_{k=1}^{N} g_{ik}(\omega)h_{kj}(\omega),\ i \in \{1, \ldots, N\}$$

are linearly dependent. However, the system

$$\sum_{k=1}^{N} g_{ik}(\omega)h_{kj}(\omega),\ i \in \{2, \ldots, N\}$$

is linearly independent, allowing us to solve for $h_{kj}(\omega_{0,m})$, $k \in \{2, \ldots, N\}$ in terms of $h_{1j,m}$, for $j \in \{1, \ldots, N\}$, that is

$$\left(h_{2j}(\omega), \ldots, h_{Nj}(\omega)\right)^\top = -\begin{pmatrix} g_{22}(\omega) & \cdots & g_{2N}(\omega) \\ \vdots & \ldots & \vdots \\ g_{N2}(\omega) & \ldots & g_{NN}(\omega) \end{pmatrix}^{-1} (g_{21}, \ldots, g_{N1})^\top h_{1j}(\omega).$$

where $(g_{21},\ldots,g_{N1})^\top$ is independent of ω For ease of notation, define

$$\underline{v}^0(\omega) = -\begin{pmatrix} g_{22}(\omega) & \cdots & g_{2N}(\omega) \\ \vdots & \ldots & \vdots \\ g_{N2}(\omega) & \ldots & g_{NN}(\omega) \end{pmatrix}^{-1} (g_{21},\ldots,g_{N1})^\top$$

We see that $\underline{v}^0(\omega) \in \mathbb{R}^{N-1}$ is column vector, i.e.

$$\underline{v}^0(\omega) = (v^0(\omega)_1,\ldots,v^0(\omega)_{N-1})^\top,$$

and so

$$\Big(h_{2j}(\omega),\ldots,h_{Nj}(\omega)\Big)^\top = (v^0(\omega)_1,\ldots,v^0(\omega)_{N-1})^\top h_{1j}(\omega).$$

Therefore,

$$h_{ij}(\omega) = v^0(\omega)_{i-1}h_{1j}(\omega),\ i \in \{2,\ldots,N\},\ j \in \{1,\ldots,N\}.$$

Now, recalling that ω can take N values, given by the N positive eigenvalues of A_0, define

$$v^0_{m,j} = [\underline{v}^0(\omega_{0,m})]_j,\ j \in \{1,\ldots,N-1\},\ \ m \in \{1,\ldots,N\}.$$

Therefore

$$q^{\$,0}_{D,ij}(x) = \begin{cases} \sum_{m=1}^N h^0_{1j,m}e^{-\omega_{0,m}x},\ i=1 \\ \sum_{m=1}^N h^0_{1j,m}v^0_{m,i-1}e^{-\omega_{0,m}x},\ i \in \{2,\ldots,N\} \end{cases}, \tag{OA.53}$$

where the superscript 0 is used to clarify that this is the solution for region $\mathcal{D}_0$ and

$$h^0_{1j,m} = h_{1j}(\omega_{0,m}).$$

It is helpful to define

$$\boldsymbol{H}^0_m = \boldsymbol{H}(\omega_{0,m}).$$

We can thus see that $h^0_{1j,1},\ldots,h^0_{1j,N}$ are from different matrices $\boldsymbol{H}^0_1,\ldots,\boldsymbol{H}^0_N$ – $h^0_{1j,m}$ is the $1j$'th element of the matrix $\boldsymbol{H}^0_m$.

Writing out the N by N matrix $\boldsymbol{q}^{\$,0}_D(x) = [q^{\$,0}_{D,ij}(x)]_{ij}$, we have

$$\boldsymbol{q}^{\$,0}_D(x) = \begin{pmatrix} \sum_{m=1}^N h^0_{11,m}e^{-\omega_{0,m}x} & \sum_{m=1}^N h^0_{12,m}e^{-\omega_{0,m}x} & \ldots & \sum_{m=1}^N h^0_{1N,m}e^{-\omega_{0,m}x} \\ \sum_{m=1}^N h^0_{11,m}v^0_{m,1}e^{-\omega_{0,m}x} & \sum_{m=1}^N h^0_{12,m}v^0_{m,1}e^{-\omega_{0,m}x} & \ldots & \sum_{m=1}^N h^0_{1N,m}v^0_{m,1}e^{-\omega_{0,m}x} \\ \vdots & \vdots & \ldots & \vdots \\ \sum_{m=1}^N h^0_{11,m}v^0_{m,N-1}e^{-\omega_{0,m}x} & \sum_{m=1}^N h^0_{12,m}v^0_{m,N-1}e^{-\omega_{0,m}x} & \ldots & \sum_{m=1}^N h^0_{1N,m}v^0_{m,N-1}e^{-\omega_{0,m}x} \end{pmatrix}$$

Note that (OA.55) contains N^2 constants of integration $h^0_{1j,m}$, $j,m \in \{1,\ldots,N\}$, which will be identified by the value and smooth pasting conditions (Section OA.H.3).

OA.H.2 Region $\mathcal{D}_k$: $X_{D,k+1} < X \leq X_{D,k}$

We now turn to the analysis of Arrow-Debreu securities in region $\mathcal{D}_k$, i.e. when current earnings are above default boundary $k+1$, but below default boundary k. Note that the above analysis in default region $\mathcal{D}_0$ can be seen as a special case of the analysis below, with $X_{D,0} \to \infty$.

First, note that $q^{\$}_{D,ij} = \delta_{ij}, \forall i \leq k$. Indeed, if earnings are currently lower than $X_{D,k} < X_{D,k-1} < \ldots$, and if the current state is $i \leq k$, then the firm is in default and the present value of a dollar when the firm defaults in state j is 1 if $i = j$, and 0 otherwise. In particular, this means that, in region $\mathcal{D}_N$, where $X \leq X_{D,N}$, we have

$$q^{\$}_{D,ij} = \delta_{ij}.$$

Applying (OA.36) to the unknown $q^{\$}_{D,ij}, i > k$, yields the following system of ODEs

$$\begin{aligned}
\frac{dz_{k+1,j}}{dx} - z_{N+k+1,j} &= 0, \\
\frac{dz_{k+2,j}}{dx} - z_{N+k+2,j} &= 0, \\
&\vdots \\
\frac{dz_{N,j}}{dx} - z_{2N,j} &= 0, \\
\frac{dz_{N+k+1,j}}{dx} + \frac{2\widehat{\mu}^{\$}_{x,k+1}}{\sigma^2_{x,k+1}} z_{N+k+1,j} + \sum_{l=1}^{k} \frac{2\widehat{\lambda}_{k+1,l}}{\sigma^2_{x,k+1}} \left(\delta_{lj} - z_{k+1,j}\right) & \\
+ \sum_{l=k+2}^{N} \frac{2\widehat{\lambda}_{k+1,l}}{\sigma^2_{x,k+1}} \left(z_{lj} - z_{k+1,j}\right) - \frac{2r^{\$}_{k+1}}{\sigma^2_{x,k+1}} z_{k+1,j} &= 0, \\
\frac{dz_{N+k+2,j}}{dx} + \frac{2\widehat{\mu}^{\$}_{x,k+2}}{\sigma^2_{x,k+2}} z_{N+k+2,j} + \sum_{l=1}^{k} \frac{2\widehat{\lambda}_{k+2,l}}{\sigma^2_{x,k+2}} \left(\delta_{l,j} - z_{k+2,j}\right) & \\
+ \sum_{l=k+1, l\neq k+2}^{N} \frac{2\widehat{\lambda}_{k+2,l}}{\sigma^2_{x,k+2}} \left(z_{l,j} - z_{k+2,j}\right) - \frac{2r^{\$}_{k+2}}{\sigma^2_{x,k+2}} z_{k+2,j} &= 0 \\
&\vdots \\
\frac{dz_{2N,j}}{dx} + \frac{2\widehat{\mu}^{\$}_{x,N}}{\sigma^2_{x,N}} z_{2N,j} + \sum_{l=1}^{k} \frac{2\widehat{\lambda}_{N,l}}{\sigma^2_{x,N}} \left(\delta_{l,j} - z_{N,j}\right) + \sum_{l=k+1}^{N-1} \frac{2\widehat{\lambda}_{N,l}}{\sigma^2_{x,N}} \left(z_{l,j} - z_{N,j}\right) - \frac{2r^{\$}_{N}}{\sigma^2_{x,N}} z_{N,j} &= 0,
\end{aligned}$$

for $j = \{1, \ldots, N\}$. Rewriting the above equation system in matrix form, we obtain

$$\boldsymbol{Z}'_k + \boldsymbol{A}_k \boldsymbol{Z}_k + \boldsymbol{B}_k \;=\; \boldsymbol{Z}'_k + \boldsymbol{A}_k \left(\boldsymbol{Z}_k + \boldsymbol{A}_k^{-1} \boldsymbol{B}_k\right) \;=\; \widetilde{\boldsymbol{Z}}'_k + \boldsymbol{A}_k \widetilde{\boldsymbol{Z}}_k = 0, \tag{OA.54}$$

where $\widetilde{\boldsymbol{Z}}_k = \left(\boldsymbol{Z}_k + \boldsymbol{A}_k^{-1} \boldsymbol{B}_k\right)$, $\boldsymbol{Z}_k$ is the following $2(N-k)$ by N matrix

$$\boldsymbol{Z}_k = \begin{pmatrix}
z_{k+1,1} & z_{k+1,2} & \cdots & z_{k+1,N} \\
z_{k+2,1} & z_{k+2,2} & \cdots & z_{k+2,N} \\
\vdots & \vdots & \ldots & \vdots \\
z_{N,1} & z_{N,2} & \cdots & z_{N,N} \\
z_{N+k+1,1} & z_{N+k+1,2} & \cdots & z_{N+k+1,N} \\
z_{N+k+2,1} & z_{N+k+2,2} & \cdots & z_{N+k+2,N} \\
\vdots & \vdots & \ldots & \vdots \\
z_{2N,1} & z_{2N,2} & \cdots & z_{2N,N}
\end{pmatrix}.$$

Note that the $\boldsymbol{B}_k$ matrix of constants arises from the δ_{lj}'s appearing in the above differential equations:

(i) These appear only in the last $N-k$ equations. Hence, the first $N-k$ rows of the $\boldsymbol{B}_k$ matrix will comprise of zeros.

(ii) Since the sum in which the δ_{lj}'s appear are from 1 to k, δ_{lj} will be zero for all l whenever $j > k$. Hence, the last $N-k$ columns of the $\boldsymbol{B}_k$ matrix will comprise of zeros.

Thereafter, the development made, in region $\mathcal{D}_0$, between equations (OA.41) and (OA.47) can be applied to $\widetilde{\boldsymbol{Z}}_k$ in (OA.54) to yield

$$\widetilde{\boldsymbol{Z}}_k = e^{-\boldsymbol{D}_k x}\boldsymbol{E}_k\boldsymbol{K}_k,$$

or, equivalently,

$$\boldsymbol{Z}_k = e^{-\boldsymbol{D}_k x}\boldsymbol{E}_k\boldsymbol{K}_k - \boldsymbol{A}_k^{-1}\boldsymbol{B}_k.$$

Therefore,

$$q_{D,ij}^{\$,k}(x) = \delta_{ij}, i \in \{1,\dots,k\}, j \in \{1,\dots,N\}$$
$$q_{D,ij}^{\$,k}(x) = \sum_{m=1}^{2(N-k)} h_{ij}(\omega_m)e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{i-k,j}, i \in \{k+1,\dots,N\}, j \in \{1,\dots,N\}.$$

Note that the $-k$ offset on the rows of the $2(N-k)\times N$ matrix $\boldsymbol{A}_k^{-1}\boldsymbol{B}_k$ simply accounts for the fact the first row of this matrix corresponds to the $(k+1)^{\text{th}}$ Arrow-Debreu security. Once more, for each eigenvalue ω of $\boldsymbol{A}_k$, the particular solution

$$\boldsymbol{Z}_k = \begin{pmatrix} \boldsymbol{H}_k(\omega) \\ -\omega\boldsymbol{H}_k(\omega) \end{pmatrix} e^{-\omega y} - \boldsymbol{A}_k^{-1}\boldsymbol{B}_k, \quad \text{where} \quad \boldsymbol{H}_k(\omega) = \begin{pmatrix} h_{k+1,1}(\omega) & \cdots & h_{k+1,N}(\omega) \\ \vdots & \dots & \vdots \\ h_{N,1}(\omega) & \cdots & h_{N,N}(\omega) \end{pmatrix},$$

can be used to express the elements in final $N-k-1$ rows of $\boldsymbol{H}_k$ but in terms of $h_{k+1,1}(\omega),\dots,h_{k+1,N}(\omega)$ as follows:

$$\Big(h_{k+2,j}(\omega),\dots,h_{Nj}(\omega)\Big)^\top = -\boldsymbol{G}_k^{-1}(\omega)\ (g_{k+2,1},\dots,g_{N,1})^\top h_{k+1,j}(\omega),$$

where the $N-k-1$ by $N-k-1$ matrix $G_k(\omega)$ is given by

$$\boldsymbol{G}_k(\omega) = \begin{pmatrix} g_{k+2,k+2}(\omega) & \cdots & g_{k+2,N}(\omega) \\ \vdots & \dots & \vdots \\ g_{N,k+2}(\omega) & \cdots & g_{N,N}(\omega) \end{pmatrix}.$$

For ease of notation, we define

$$\underline{v}^k(\omega) = -\boldsymbol{G}_k^{-1}(\omega)(g_{k+2,1},\dots,g_{N,1})^\top$$

We see that $\underline{v}^k(\omega) \in \mathbb{R}^{N-k-1}$ is a column vector, i.e.

$$\underline{v}^k(\omega) = (v^k(\omega)_1,\dots,v^k(\omega)_{N-k-1})^\top,$$

and so

$$\Big(h_{k+2,j}(\omega),\dots,h_{N,j}(\omega)\Big)^\top = (v^k(\omega)_1,\dots,v^k(\omega)_{N-k-1})^\top h_{k+1,j}(\omega).$$

Therefore,

$$h_{ij}(\omega) = v^k(\omega)_{i-k-1} h_{k+1,j},\ i \in \{k+2,\dots,N\},\ j \in \{1,\dots,N\}.$$

This leaves us with N free constants for each of the $2(N-k)$ eigenvalues, giving $2N(N-k)$ constants of integration in total for region $\mathcal{D}_k$. Hence,

$$q_{D,ij}^{\$,k}(x) = \begin{cases} \delta_{ij},\ i \in \{1,\dots,k\} \\ \sum_{m=1}^{2(N-k)} h_{k+1\,j,m}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{1,j},\ i = k+1 \\ \sum_{m=1}^{2(N-k)} h_{k+1\,j,m}^k v_{m,i-k-1}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{i-k,j},\ i \in \{k+2,\dots,N\} \end{cases}, \tag{OA.55}$$

where $h_{k+1\,j,m}^k = h_{k+1\,j}(\omega_m^k)$. Observe that $h_{k+1\,j,m}^k$ is the j'th entry in the first row of $\boldsymbol{H}_m^k = \boldsymbol{H}^k(\omega_m^k)$.

In matrix form, we have

$$\boldsymbol{q}_D^{\$,k}(x) = \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{0}_{N-k\times N-k} \\ \boldsymbol{q}_{D,L}^{\$,k}(x) & \boldsymbol{q}_{D,R}^{\$,k}(x) \end{pmatrix}$$

where $\boldsymbol{q}_{D,L}^{\$,k}(x)$ is the following $N-k$ by k matrix function

$$\boldsymbol{q}_{D,L}^{\$,k}(x) = \begin{pmatrix} \sum_{m=1}^{2(N-k)} h_{k+1\,1,m}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{1,1} & \cdots & \sum_{m=1}^{2(N-k)} h_{k+1\,k,m}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{1,k} \\ \sum_{m=1}^{2(N-k)} h_{k+1\,1,m}^k v_{m,1}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{2,1} & \cdots & \sum_{m=1}^{2(N-k)} h_{k+1\,k,m}^k v_{m,1}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{2,k} \\ \vdots & \ddots & \vdots \\ \sum_{m=1}^{2(N-k)} h_{k+1\,1,m}^k v_{m,N-k-1}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{N-k,1} & \cdots & \sum_{m=1}^{2(N-k)} h_{k+1\,k,m}^k v_{m,N-k-1}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{N-k,k} \end{pmatrix},$$

and $\boldsymbol{q}_{D,R}^{\$,k}(x)$ is the following $N-k$ by $N-k$ matrix function

$$\boldsymbol{q}_{D,R}^{\$,k}(x) = \begin{pmatrix} \sum_{m=1}^{2(N-k)} h_{k+1\,k+1,m}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{1,k+1} & \cdots & \sum_{m=1}^{2(N-k)} h_{k+1\,N,m}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{1,N} \\ \sum_{m=1}^{2(N-k)} h_{k+1\,k+1,m}^k v_{m,1}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{2,k+1} & \cdots & \sum_{m=1}^{2(N-k)} h_{k+1\,N,m}^k v_{m,1}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{2,N} \\ \vdots & \ddots & \vdots \\ \sum_{m=1}^{2(N-k)} h_{k+1\,k+1,m}^k v_{m,N-k-1}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{N-k,k+1} & \cdots & \sum_{m=1}^{2(N-k)} h_{k+1\,N,m}^k v_{m,N-k-1}^k e^{-\omega_m x} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{N-k,N} \end{pmatrix},$$

$\boldsymbol{I}_{k\times k}$ is the k by k identity matrix and $\boldsymbol{0}_{N-k\times N-k}$ is the $N-k$ by $N-k$ matrix of zeros.

OA.H.3 Boundary conditions for Arrow-Debreu default claims

Given the above development, we are left with N^2 free integration constants in region $\mathcal{D}_0$, and $2(N-k)N$ free constants in region $\mathcal{D}_k, k \in \{1,\dots,N-1\}$. Hence, we still have to solve for the N^3 constants,

$$N^2 + \sum_{k=1}^{N-1} 2(N-k)N = N^2 + 2N \sum_{k=1}^{N-1}(N-k) = N^3,$$

that satisfy the N^3 boundary conditions (value matching & smooth pasting) of the problem at hand. For each default boundary $X_{D,k}, k \in \{1,\dots,N\}$, we have that:

(VM) The value of the $N+(N-k)N$ Arrow-Debreu securities with $i \geq k$, must be the same on both sides of the default boundary, i.e.

$$q_{D,ij}^{\$,k-1}(x_{D,k}) = q_{ij}^{\$,k}(x_{D,k}), \text{ where } , i \in \{k, \ldots, N\}, j \in \{1, \ldots, N\};$$

(SP) The first derivatives of the $(N-k)N$ Arrow-Debreu securities with $i > k$, must be the same on both side of each default boundary, i.e.

$$\left.\frac{dq_{ij}^{\$,k-1}}{dx}\right|_{x_{D,k}} = \left.\frac{dq_{ij}^{\$,k}}{dx}\right|_{x_{D,k}}, \text{ where } , i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\};$$

where the k superscript in q^k notation highlights that the Arrow-Debreu claim is computed in the region $\mathcal{D}_k$.

We can write the N^3 boundary conditions in vector-matrix form as

$$\boldsymbol{T}_N \underline{\phi}_N = \underline{\chi}_N.$$

The N^3 by N^3 sparse matrix $\boldsymbol{T}_N$ is defined by

$$\boldsymbol{T}_N = \begin{pmatrix} \overline{\boldsymbol{L}}_{N,1} & -\overline{\boldsymbol{M}}_{N,1} & & & & \\ & \overline{\boldsymbol{L}}_{N,2} & -\overline{\boldsymbol{M}}_{N,2} & & & \\ & & \overline{\boldsymbol{L}}_{N,3} & -\overline{\boldsymbol{M}}_{N,3} & & \\ & & & \ddots & \ddots & \\ & & & & \overline{\boldsymbol{L}}_{N,N-1} & -\overline{\boldsymbol{M}}_{N,N-1} \\ & & & & & \overline{\boldsymbol{L}}_{N,N} \end{pmatrix},$$

where the unspecified entries are matrices of zeros and

$$\overline{\boldsymbol{L}}_{N,k} = \text{diag}\underbrace{(\boldsymbol{L}_{N,k}, \ldots, \boldsymbol{L}_{N,k})}_{N \text{ repetitions}},$$

$$\overline{\boldsymbol{M}}_{N,k} = \text{diag}\underbrace{(\boldsymbol{M}_{N,k}, \ldots, \boldsymbol{M}_{N,k})}_{N \text{ repetitions}},$$

where

$$\boldsymbol{L}_{N,1} = \left(\begin{array}{ccc} e^{-\omega_{0,1} x_{D,1}} & \cdots & e^{-\omega_{0,N} x_{D,1}} \\ \hdashline v_{1,1}^{0} e^{-\omega_{0,1} x_{D,1}} & \cdots & v_{N,1}^{0} e^{-\omega_{0,N} x_{D,1}} \\ \vdots & \cdots & \vdots \\ v_{1,N-1}^{0} e^{-\omega_{0,1} x_{D,1}} & \cdots & v_{N,N-1}^{0} e^{-\omega_{0,N} x_{D,1}} \\ \hdashline v_{1,1}^{0} \omega_{0,1} e^{-\omega_{0,1} x_{D,1}} & \cdots & v_{N,1}^{0} \omega_{0,N} e^{-\omega_{0,N} x_{D,1}} \\ \vdots & \cdots & \vdots \\ v_{1,N-1}^{0} \omega_{0,1} e^{-\omega_{0,1} x_{D,1}} & \cdots & v_{N,N-1}^{0} \omega_{0,N} e^{-\omega_{0,N} x_{D,1}} \end{array}\right) \in \mathbb{R}^{1+2(N-1) \times N},$$

and

$$\boldsymbol{L}_{N,k} = \left(\begin{array}{ccc} e^{-\omega_1^{k-1} x_{D,k}} & \dots & e^{-\omega_{2(N-(k-1))}^{k-1} x_{D,k}} \\ \hline v_{1,1}^{k-1} e^{-\omega_1^{k-1} x_{D,k}} & \dots & v_{2(N-(k-1)),1}^{k-1} e^{-\omega_{2(N-(k-1))}^{k-1} x_{D,k}} \\ \vdots & \dots & \vdots \\ v_{1,N-k}^{1} e^{-\omega_1^{k-1} x_{D,k}} & \dots & v_{2(N-(k-1)),N-k}^{k-1} e^{-\omega_{2(N-(k-1))}^{k-1} x_{D,k}} \\ \hline v_{1,1}^{k-1} \omega_1^{k-1} e^{-\omega_1^{k-1} x_{D,k}} & \dots & v_{2(N-(k-1)),1}^{k-1} \omega_{2(N-(k-1))}^{k-1} e^{-\omega_{2(N-(k-1))}^{k-1} x_{D,k}} \\ \vdots & \dots & \vdots \\ v_{1,N-k}^{1} \omega_1^{k-1} e^{-\omega_1^{k-1} x_{D,k}} & \dots & v_{2(N-(k-1)),N-k}^{k-1} \omega_{2(N-(k-1))}^{k-1} e^{-\omega_{2(N-(k-1))}^{k-1} x_{D,k}} \end{array}\right) \in \mathbb{R}^{[1+2(N-k)]\times 2[N-(k-1)]},$$

$$k \in \{2, \dots, N-1\},$$

and

$$\boldsymbol{L}_{N,N} = \left(\begin{array}{cc} e^{-\omega_1^{N-1} x_{D,N}} & e^{-\omega_2^{N-1} x_{D,N}} \end{array} \right),$$

and

$$\boldsymbol{M}_{N,k} = \left(\begin{array}{ccc} 0 & \dots & 0 \\ \hline e^{-\omega_1^{k} x_{D,k}} & \dots & e^{-\omega_{2(N-k)}^{k} x_{D,k}} \\ v_{1,1}^{k} e^{-\omega_1^{k} x_{D,k}} & \dots & v_{2(N-k),1}^{k} e^{-\omega_{2(N-k)}^{k} x_{D,k}} \\ \vdots & \dots & \vdots \\ v_{1,N-k-1}^{1} e^{-\omega_1^{k} x_{D,k}} & \dots & v_{2(N-k),N-k-1}^{k} e^{-\omega_{2(N-k)}^{k} x_{D,k}} \\ \hline \omega_1^{k} e^{-\omega_1^{k} x_{D,k}} & \dots & \omega_{2(N-k)}^{k} e^{-\omega_{2(N-k)}^{k} x_{D,k}} \\ v_{1,1}^{k} \omega_1^{k} e^{-\omega_1^{k} x_{D,k}} & \dots & v_{2(N-k),1}^{k} \omega_{2(N-k)}^{k} e^{-\omega_{2(N-k)}^{k} x_{D,k}} \\ \vdots & \dots & \vdots \\ v_{1,N-k-1}^{1} \omega_1^{k} e^{-\omega_1^{k} x_{D,k}} & \dots & v_{2(N-k),N-k-1}^{k} \omega_{2(N-k)}^{k} e^{-\omega_{2(N-k)}^{k} x_{D,k}} \end{array}\right) \in \mathbb{R}^{[1+2(N-k)]\times 2(N-k)},$$

$$k \in \{1, \dots, N-1\}.$$

Observe that

$$\boldsymbol{M}_{N,N-1} = \left(\begin{array}{cc} 0 & 0 \\ \hline e^{-\omega_1^{N-1} x_{D,N-1}} & e^{-\omega_2^{N-1} x_{D,N-1}} \\ \hline \omega_1^{N-1} e^{-\omega_1^{N-1} x_{D,N-1}} & \omega_2^{N-1} e^{-\omega_2^{N-1} x_{D,N-1}} \end{array}\right) \in \mathbb{R}^{3\times 2}.$$

The N^3-dimensional column vector $\underline{\phi}_N$ contains the N^3 constants of integration as follows:

$$\underline{\phi}_N = (\underline{h}_{11}^0, \dots, \underline{h}_{1N}^0, \underline{h}_{21}^1, \dots, \underline{h}_{2N}^1, \dots, \underline{h}_{N1}^{N-1}, \dots, \underline{h}_{NN}^{N-1})^\top \in \mathbb{R}^{N^3}.$$

where

$$\underline{h}_{1j}^0 = (h_{1j,1}^0, \dots, h_{1j,N}^0)^\top$$

$$\underline{h}_{k+1,j}^k = (h_{k+1,j,1}^k, \dots, h_{k+1,j,2(N-k)}^k)^\top,\ k \in \{1, \dots, N-1\}.$$

The N^3-dimensional column vector $\underline{\chi}_N$ is given by

$$\underline{\chi}_N = (\underline{\chi}(N,1)^\top, \ldots, \underline{\chi}(N,N)^\top)^\top,$$

where

$$\underline{\chi}(N,k) = (\underline{\chi}_1(N,k)^\top, \ldots. \underline{\chi}_N(N,k)^\top, \underline{0}_{1\times N(N-k)})^\top,$$

and

$$\underline{\chi}_j(N,k) = (\chi_{1j}(N,k), \ldots, \chi_{N-k+1,j}(N,k))^\top,$$

where

$$\begin{aligned}
&\chi_{1j}(N,1) = \delta_{1j} \\
&\chi_{ij}(N,1) = -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{i-1,j},\, i \in \{2,\ldots,N\},\, j \in \{1,\ldots,N\},
\end{aligned}$$

and

$$\begin{aligned}
&\chi_{1j}(N,k) = [\boldsymbol{A}_{k-1}^{-1}\boldsymbol{B}_{k-1}]_{1j} + \delta_{kj} \\
&\chi_{ij}(N,k) = [\boldsymbol{A}_{k-1}^{-1}\boldsymbol{B}_{k-1}]_{ij} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{i-1,j},\, i \in \{2,\ldots,N-k+1\},\, j \in \{1,\ldots,N\},
\end{aligned}$$

and

$$\chi_{1j}(N,N) = [\boldsymbol{A}_{N-1}^{-1}\boldsymbol{B}_{N-1}]_{1j} + \delta_{Nj},\, j \in \{1,\ldots,N\}.$$

To understand how to implement the above scheme, observe that for $N = 3$, we have

$$\boldsymbol{T}_3 \underline{\phi}_3 = \underline{\chi}_3,$$

where

$$\boldsymbol{T}_3 = \begin{pmatrix} \overline{\boldsymbol{L}}_{3,1} & -\overline{\boldsymbol{M}}_{3,1} & \\ & \overline{\boldsymbol{L}}_{3,2} & -\overline{\boldsymbol{M}}_{3,2} \\ & & \overline{\boldsymbol{L}}_{3,3} \end{pmatrix}$$

$$= \left(\begin{array}{ccc|ccc|ccc} \boldsymbol{L}_{3,1} & \mathbf{0}_{5\times 3} & \mathbf{0}_{5\times 3} & -\boldsymbol{M}_{3,1} & \mathbf{0}_{5\times 4} & \mathbf{0}_{5\times 4} & \mathbf{0}_{5\times 2} & \mathbf{0}_{5\times 2} & \mathbf{0}_{5\times 2} \\ \mathbf{0}_{5\times 3} & \boldsymbol{L}_{3,1} & \mathbf{0}_{5\times 3} & \mathbf{0}_{5\times 4} & -\boldsymbol{M}_{3,1} & \mathbf{0}_{5\times 4} & \mathbf{0}_{5\times 2} & \mathbf{0}_{5\times 2} & \mathbf{0}_{5\times 2} \\ \mathbf{0}_{5\times 3} & \mathbf{0}_{5\times 3} & \boldsymbol{L}_{3,1} & \mathbf{0}_{5\times 4} & \mathbf{0}_{5\times 4} & -\boldsymbol{M}_{3,1} & \mathbf{0}_{5\times 2} & \mathbf{0}_{5\times 2} & \mathbf{0}_{5\times 2} \\ \hline \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \boldsymbol{L}_{3,2} & \mathbf{0}_{3\times 4} & \mathbf{0}_{3\times 4} & -\boldsymbol{M}_{3,2} & \mathbf{0}_{3\times 2} & \mathbf{0}_{3\times 2} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 4} & \boldsymbol{L}_{3,2} & \mathbf{0}_{3\times 4} & \mathbf{0}_{3\times 2} & -\boldsymbol{M}_{3,2} & \mathbf{0}_{3\times 2} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 4} & \mathbf{0}_{3\times 4} & \boldsymbol{L}_{3,2} & \mathbf{0}_{3\times 2} & \mathbf{0}_{3\times 2} & -\boldsymbol{M}_{3,2} \\ \hline \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 4} & \boldsymbol{L}_{3,3} & \mathbf{0}_{1\times 2} & \mathbf{0}_{1\times 2} \\ \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 2} & \boldsymbol{L}_{3,3} & \mathbf{0}_{1\times 2} \\ \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 2} & \mathbf{0}_{1\times 2} & \boldsymbol{L}_{3,3} \end{array}\right),$$

and

$$\boldsymbol{L}_{3,1} = \begin{pmatrix} e^{-\omega_{0,1}x_{D,1}} & e^{-\omega_{0,2}x_{D,1}} & e^{-\omega_{0,3}x_{D,1}} \\ v^0_{1,1}e^{-\omega_{0,1}x_{D,1}} & v^0_{2,1}e^{-\omega_{0,2}x_{D,1}} & v^0_{3,1}e^{-\omega_{0,3}x_{D,1}} \\ v^0_{1,2}e^{-\omega_{0,1}x_{D,1}} & v^0_{2,2}e^{-\omega_{0,2}x_{D,1}} & v^0_{3,2}e^{-\omega_{0,3}x_{D,1}} \\ v^0_{1,1}\omega_{0,1}e^{-\omega_{0,1}x_{D,1}} & v^0_{2,1}\omega_{0,2}e^{-\omega_{0,2}x_{D,1}} & v^0_{3,1}\omega_{0,3}e^{-\omega_{0,3}x_{D,1}} \\ v^0_{1,2}\omega_{0,1}e^{-\omega_{0,1}x_{D,1}} & v^0_{2,2}\omega_{0,2}e^{-\omega_{0,2}x_{D,1}} & v^0_{3,2}\omega_{0,3}e^{-\omega_{0,3}x_{D,1}} \end{pmatrix},$$

$$\boldsymbol{M}_{3,1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ e^{-\omega^1_1 x_{D,1}} & e^{-\omega^1_2 x_{D,1}} & e^{-\omega^1_3 x_{D,1}} & e^{-\omega^1_4 x_{D,1}} \\ v^1_{1,1}e^{-\omega^1_1 x_{D,1}} & v^1_{2,1}e^{-\omega^1_2 x_{D,1}} & v^1_{3,1}e^{-\omega^1_3 x_{D,1}} & v^1_{4,1}e^{-\omega^1_4 x_{D,1}} \\ \omega^1_1 e^{-\omega^1_1 x_{D,1}} & \omega^1_2 e^{-\omega^1_2 x_{D,1}} & \omega^1_3 e^{-\omega^1_3 x_{D,1}} & \omega^1_4 e^{-\omega^1_4 x_{D,1}} \\ v^1_{1,1}\omega^1_1 e^{-\omega^1_1 x_{D,1}} & v^1_{2,1}\omega^1_2 e^{-\omega^1_2 x_{D,1}} & v^1_{3,1}\omega^1_3 e^{-\omega^1_3 x_{D,1}} & v^1_{4,1}\omega^1_4 e^{-\omega^1_4 x_{D,1}} \end{pmatrix},$$

$$\boldsymbol{L}_{3,2} = \begin{pmatrix} e^{-\omega^1_1 x_{D,2}} & e^{-\omega^1_2 x_{D,2}} & e^{-\omega^1_3 x_{D,2}} & e^{-\omega^1_4 x_{D,2}} \\ v^1_{1,1}e^{-\omega^1_1 x_{D,2}} & v^1_{2,1}e^{-\omega^1_2 x_{D,2}} & v^1_{3,1}e^{-\omega^1_3 x_{D,2}} & v^1_{4,1}e^{-\omega^1_4 x_{D,2}} \\ v^1_{1,1}\omega^1_1 e^{-\omega^1_1 x_{D,2}} & v^1_{2,1}\omega^1_2 e^{-\omega^1_2 x_{D,2}} & v^1_{3,1}\omega^1_3 e^{-\omega^1_3 x_{D,2}} & v^1_{4,1}\omega^1_4 e^{-\omega^1_4 x_{D,2}} \end{pmatrix},$$

$$\boldsymbol{M}_{3,2} = \begin{pmatrix} 0 & 0 \\ e^{-\omega^2_1 x_{D,2}} & e^{-\omega^2_2 x_{D,2}} \\ \omega^2_1 e^{-\omega^2_1 x_{D,2}} & \omega^2_2 e^{-\omega^2_2 x_{D,2}} \end{pmatrix},$$

$$\boldsymbol{L}_{3,1} = \begin{pmatrix} e^{-\omega^2_1 x_{D,3}} & e^{-\omega^2_2 x_{D,3}} \end{pmatrix}$$

$$\underline{\phi}_3 = (h^0_{11,1}, \ldots, h^0_{11,3}, h^0_{12,1}, \ldots, h^0_{12,3}, h^0_{13,1}, \ldots, h^0_{13,3}, h^1_{21,1}, \ldots, h^1_{21,4}, h^1_{22,1}, \ldots, h^1_{22,4}, h^1_{23,1}, \ldots, h^1_{23,4}, h^2_{31,1}, h^2_{31,2}, h^2_{32,1}, h^2_{32,2}, h^2_{33,1}, h^2_{33,2})^\top$$

The 3^3-dimensional column vector $\underline{\chi}_3$ is given by

$$\underline{\chi}_3 = (\underline{\chi}(3,1)^\top, \underline{\chi}(3,2)^\top, \underline{\chi}(3,3)^\top)^\top,$$

where

$$\underline{\chi}(3,k) = (\underline{\chi}_1(3,k)^\top, \ldots, \underline{\chi}_3(3,k)^\top, \underline{0}_{1\times 3(3-k)})^\top,$$

and

$$\underline{\chi}_j(3,k) = (\chi_{1j}(3,k), \dots, \chi_{3-k+1,j}(3,k))^\top,$$

where

$$\begin{aligned}
\chi_{1j}(3,1) &= \delta_{1j} \\
\chi_{ij}(3,1) &= -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{i-1,j},\ i \in \{2,3\},\ j \in \{1,\dots,3\},
\end{aligned}$$

and

$$\begin{aligned}
\chi_{1j}(3,k) &= [\boldsymbol{A}_{k-1}^{-1}\boldsymbol{B}_{k-1}]_{1j} + \delta_{kj} \\
\chi_{ij}(3,k) &= [\boldsymbol{A}_{k-1}^{-1}\boldsymbol{B}_{k-1}]_{ij} - [\boldsymbol{A}_k^{-1}\boldsymbol{B}_k]_{i-1,j},\ i \in \{2,\dots,3-k+1\},\ j \in \{1,\dots,3\},
\end{aligned}$$

and

$$\chi_{1j}(3,3) = [\boldsymbol{A}_2^{-1}\boldsymbol{B}_2]_{1j} + \delta_{3j},\ j \in \{1,\dots,N\}.$$

We can thus visualize the construction of $\underline{\chi}_3 = (\underline{\chi}(3,1), \underline{\chi}(3,2), \underline{\chi}(3,3))^\top$ as follows:

$$\begin{array}{cllll}
 & \underline{\chi}_1(3,1) \to & (\chi_{11}(3,1), & \chi_{21}(3,1), & \chi_{31}(3,1))^\top \\
\underline{\chi}(3,1) \to & \underline{\chi}_2(3,1) \to & (\chi_{12}(3,1), & \chi_{22}(3,1), & \chi_{32}(3,1))^\top \\
 & \underline{\chi}_3(3,1) \to & (\chi_{13}(3,1), & \chi_{23}(3,1), & \chi_{33}(3,1))^\top \\
 & \quad \underline{0}_{1\times 6} & & & \\
\hline
 & \underline{\chi}_1(3,2) \to & (\chi_{11}(3,2), & \chi_{21}(3,2))^\top & \\
\underline{\chi}(3,2) \to & \underline{\chi}_2(3,2) \to & (\chi_{12}(3,2), & \chi_{22}(3,2))^\top & \\
 & \underline{\chi}_3(3,2) \to & (\chi_{13}(3,2), & \chi_{23}(3,2))^\top & \\
 & \quad \underline{0}_{1\times 3} & & & \\
\hline
 & \underline{\chi}_1(3,3) \to & \chi_{11}(3,3) & & \\
\underline{\chi}(3,3) \to & \underline{\chi}_2(3,2) \to & \chi_{12}(3,3) & & \\
 & \underline{\chi}_3(3,3) \to & \chi_{13}(3,3) & &
\end{array},$$

which can be written more explicitly as

$$\begin{array}{cllll}
 & \underline{\chi}_1(3,1) \to & (1, & -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{11}, & -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{21})^\top \\
\underline{\chi}(3,1) \to & \underline{\chi}_2(3,1) \to & (0, & -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{12}, & -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{22})^\top \\
 & \underline{\chi}_3(3,1) \to & (0, & -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{13}, & -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{23}^\top \\
 & \quad \underline{0}_{1\times 6} & & & \\
\hline
 & \underline{\chi}_1(3,2) \to & ([\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{11}, & [\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{21} - [\boldsymbol{A}_2^{-1}\boldsymbol{B}_2]_{11})^\top & \\
\underline{\chi}(3,2) \to & \underline{\chi}_2(3,2) \to & ([\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{12} + 1, & [\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{22} - [\boldsymbol{A}_2^{-1}\boldsymbol{B}_2]_{12})^\top & \\
 & \underline{\chi}_3(3,2) \to & ([\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{13}, & [\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{23} - [\boldsymbol{A}_2^{-1}\boldsymbol{B}_2]_{13})^\top & \\
 & \quad \underline{0}_{1\times 3} & & & \\
\hline
 & \underline{\chi}_1(3,3) \to & [\boldsymbol{A}_2^{-1}\boldsymbol{B}_2]_{11} & & \\
\underline{\chi}(3,3) \to & \underline{\chi}_2(3,2) \to & [\boldsymbol{A}_2^{-1}\boldsymbol{B}_2]_{12} & & \\
 & \underline{\chi}_3(3,3) \to & [\boldsymbol{A}_2^{-1}\boldsymbol{B}_2]_{13} + 1 & &
\end{array},$$

and so

$$\underline{\chi}_3 = \begin{array}{c}
(1, -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{11}, -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{21}, 0, -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{12}, -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{22}, 0, -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{13}, -[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1]_{23}, \underline{0}_6^\top, -\left[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1\right]_{11}, \\
\left[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1\right]_{21} - \left[\boldsymbol{A}_2^{-1}\boldsymbol{B}_2\right]_{11}, \left[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1\right]_{12} + 1, \left[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1\right]_{22} - \left[\boldsymbol{A}_2^{-1}\boldsymbol{B}_2\right]_{12}, \left[\boldsymbol{A}_1^{-1}\boldsymbol{B}_1\right]_{23} - \left[\boldsymbol{A}_2^{-1}\boldsymbol{B}_2\right]_{13}, \\
0, 0, 0, \left[\boldsymbol{A}_2^{-1}\boldsymbol{B}_2\right]_{11}, \left[\boldsymbol{A}_2^{-1}\boldsymbol{B}_2\right]_{12}, \left[\boldsymbol{A}_2^{-1}\boldsymbol{B}_2\right]_{13} + 1)^\top
\end{array},$$

where $\underline{0}_6$ is the 6×1 column vector of zeros.

OA.I Modified Arrow-Debreu Default Claims

Arrow-Debreu securities provide the expected value of a \$ 1 cash flow conditional on the state of the world in which they occur. In particular, in region $\mathcal{D}_k$ at time t, we have the Arrow-Debreu default claim price

$$\begin{aligned} q^{\$,k}_{D,ij,t} &= E_t \left[\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t} 1_{\{s_{\tau_D}=j\}} \Big| s_t = i \right] \\ &= E^{\mathbb{Q}^{\$}}_t \left[e^{-\int_t^{\tau_D} r^{\$}_u du} 1_{\{s_{\tau_D}=j\}} \Big| s_t = i \right]. \end{aligned}$$

For cash flows that do not depend on the level of earnings when the firm defaults, $X_{\tau_D} = e^{x_{\tau_D}}$, these Arrow-Debreu securities yield a straightforward approach to derive the cash flows' expected values. If a cash flow does depend on X_{τ_D}, the Arrow-Debreu securities may not be as useful.

In a model with no jumps, the earnings always approaches the default boundary from above along a continuous path and default occurs when $X_{\tau_D} = X_D$; that is, there is no uncertainty with respect to the level of earnings upon default and Arrow-Debreu securities can readily be used to compute expected cash flows. In our economy, however, "deep defaults" can occur immediately when the state of the economy jumps from its current state to a worse state.

We order default boundaries such that $X_{D,1} > \ldots > X_{D,N}$; hence, state N is the best state of the economy, state 1 is the worst. When the state of the economy jumps toward a better state, the default boundary decreases as growth opportunities improve; hence, if the firm was not in default, it is even further away from default after the jump. However, if the level of earnings is $X_{\tau_D^-} \in (X_{D,j+1}, X_{D,j}]$ prior to a jump into state j at time τ_D, the firm automatically defaults. The level of earnings X_{τ_D} is then only a fraction of the default boundary $X_{D,j}$, and the firm will thus be able to honor its obligations to the debtholders, for instance, only partially.

We thus introduce "modified" Arrow-Debreu securities

$$\begin{aligned}\widetilde{q}^{\$}_{D,ij,t} &= E_t\left[\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\frac{X_{\tau_D}}{X_{D,j}}1_{s_{\tau_D}=j}\middle|s_t=i\right] \\ &= \frac{X_t}{X_{D,j}}E_t\left[\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\frac{X_{\tau_D}}{X_t}1_{s_{\tau_D}=j}\middle|s_t=i\right] \\ &= e^{x_t-x_{D,j}}E_t\left[\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\frac{X_{\tau_D}}{X_t}1_{s_{\tau_D}=j}\middle|s_t=i\right].\end{aligned}$$

We now change the probability measure from $\mathbb{P}$ to $\widetilde{\mathbb{Q}}$, using the exponential martingale $\widetilde{M}$ (with respect to $\mathbb{P}$), given by

$$\begin{aligned}\left.\frac{d\widetilde{M}_t}{\widetilde{M}_t}\right|_{s_{t-}=i} &= \frac{d(\pi^{\$}_t X_t)}{\pi^{\$}_t X_t} - E_t\left[\frac{d(\pi^{\$}_t X_t)}{\pi^{\$}_t X_t}\middle| s_{t-}=i\right] \\ &= -\gamma\sigma_{C,i}dZ_t - (\varphi-1)\sigma_{P,i}dZ_{P,t} + \sigma_{Y,i}dW_t + \sum_{j\neq i}(\omega_{ij}-1)dN^P_{ij,t}.\end{aligned}$$

Therefore

$$\widetilde{q}^{\$}_{D,ij,t} = e^{x_t-x_{D,j}}\overline{q}_{ij,t},$$

where

$$k_t|_{s_{t-}=i} = r^{\$}_i + \gamma\rho_{YC,i}\sigma_{Y,i}\sigma_{C,i} + \varphi\gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i} + \rho_{PY,i}\sigma_{P,i}\sigma_{Y,i} + \varphi\sigma^2_{P,i} - \mu_{X,i}$$

and

$$\overline{q}_{ij,t} = E^{\widetilde{\mathbb{Q}}}_t\left[e^{-\int_t^{\tau_D}k_u du}1_{s_{\tau_D}=j}\middle|s_t=i\right].$$

We have N^3 boundary conditions (value matching & smooth pasting) for the modified Arrow-Debreu securities. For each default boundary $X_{D,k}, k \in \{1,\ldots,N\}$, we have that:

(VM) The value of the $N+(N-k)N$ modified Arrow-Debreu securities with $i \geq k$, must be the same on both sides of the default boundary, i.e.

$$\widetilde{q}^{k-1}_{D,ij}(x_{D,k}) = \widetilde{q}^{k}_{ij}(x_{D,k}), \text{ where } , i \in \{k,\ldots,N\}, j \in \{1,\ldots,N\};$$

(SP) The first derivatives of the $(N-k)N$ modified Arrow-Debreu securities with $i > k$, must be the same on both side of each default boundary, i.e.

$$\left.\frac{d\widetilde{q}^{k-1}_{ij}}{dx}\right|_{x_{D,k}} = \left.\frac{d\widetilde{q}^{k}_{ij}}{dx}\right|_{x_{D,k}}, \text{ where } , i \in \{k+1,\ldots,N\}, j \in \{1,\ldots,N\}.$$

Observe that the above N^3 boundary conditions imply the following N^3 boundary conditions for $\overline{q}_{ij,t}$:

(VM)

$$\overline{q}^{k-1}_{D,ij}(x_{D,k}) = \overline{q}^{k}_{ij}(x_{D,k}), \text{ where } , i \in \{k,\ldots,N\}, j \in \{1,\ldots,N\};$$

(SP)

$$\left.\frac{d\overline{q}^{k-1}_{ij}}{dx}\right|_{x_{D,k}} = \left.\frac{d\overline{q}^{k}_{ij}}{dx}\right|_{x_{D,k}}, \text{ where } , i \in \{k+1,\ldots,N\}, j \in \{1,\ldots,N\};$$

We can therefore evaluate $\overline{q}_{ij,t}$ in the same way as $q_{ij,t}$, but replacing $r^{\$}$ with k and $\hat{\mu}^{\$}_{x,i}$ with

$$\hat{\mu}^{\widetilde{\mathbb{Q}}}_{x,i} = \hat{\mu}^{\$}_{x,i} - \left(\sigma^2_{Y,i} + 2\varphi\rho_{PY,i}\sigma_{P,i}\sigma_{Y,i} + \sigma^2_{P,i}\right)$$

Technically, the (standard) Arrow-Debreu securities are special cases of their modified counterparts, with $\frac{x_{\tau_D}}{x_{D,j}} = 1$. Moreover, when in region $\mathcal{D}_0$, deep defaults are irrelevant, because the economy is already in the worst possible state.

not a direct concern, the firm would survive even to a jump to the worse state, state 1. Hence, the general solution in (OA.49) holds. However, in region $\mathcal{D}_k, k > 0$, applying (OA.36) to the unknown $\tilde{q}_{D,ij}, i > k$, accounting for deep defaults, yields the following system of ODEs

$$\begin{aligned} \frac{dz_{i,j}}{dx} - z_{N+i,j} &= 0, \\ \frac{dz_{N+i,j}}{dx} + \frac{2\widehat{\mu}_{x,i}}{\sigma^2_{x,i}} z_{N+i,j} + \sum_{l=1}^{k} \frac{2\widehat{\lambda}_{i,l}}{\sigma^2_{x,i}} \left(e^{x-x_{D,j}}\delta_{lj} - z_{i,j}\right) & \\ + \sum_{l=k+1, l\neq i}^{N} \frac{2\widehat{\lambda}_{i,l}}{\sigma^2_{x,i}} \left(z_{l,j} - z_{i,j}\right) - \frac{2r_i}{\sigma^2_{x,i}} z_{i,j} &= 0, \end{aligned}$$

with $i \in \{k+1,\ldots,N\}$ and $j \in \{1,\ldots,N\}$. This can be written equivalently in matrix form as

$$Z'_k + A_k Z_k + \widetilde{B}_k = 0,$$

where

$$\widetilde{B}_k = \begin{pmatrix} 0_{N-k\times k} & 0_{N-k\times N-k} \\ \widetilde{B}_k^{\o} & 0_{N-k\times N-k} \end{pmatrix},$$

and

$$\widetilde{B}_k^{\o} = \begin{pmatrix} 2\frac{\widehat{\lambda}_{k+1,1}}{\sigma_{k+1}^2}e^{x-x_{D,1}} & 2\frac{\widehat{\lambda}_{k+1,2}}{\sigma_{k+1}^2}e^{x-x_{D,2}} & \cdots & 2\frac{\widehat{\lambda}_{k+1,k}}{\sigma_{k+1}^2}e^{x-x_{D,k}} \\ 2\frac{\widehat{\lambda}_{k+2,1}}{\sigma_{k+2}^2}e^{x-x_{D,1}} & 2\frac{\widehat{\lambda}_{k+2,2}}{\sigma_{k+2}^2}e^{x-x_{D,2}} & \cdots & 2\frac{\widehat{\lambda}_{k+2,k}}{\sigma_{k+2}^2}e^{x-x_{D,k}} \\ \vdots & \vdots & \cdots & \vdots \\ 2\frac{\widehat{\lambda}_{N,1}}{\sigma_N^2}e^{x-x_{D,1}} & 2\frac{\widehat{\lambda}_{N,2}}{\sigma_N^2}e^{x-x_{D,2}} & \cdots & 2\frac{\widehat{\lambda}_{N,k}}{\sigma_N^2}e^{x-x_{D,k}} \end{pmatrix}.$$

Now, $\widetilde{B}_k$ is not constant with respect to x anymore, but $\widetilde{B}_k' = \widetilde{B}_k$. Hence, letting $\widetilde{Z}_k = Z_k + (A_k + I)^{-1}\widetilde{B}_k$, we have $\widetilde{Z}_k' = Z_k' + (A_k + I)^{-1}\widetilde{B}_k$

$$\begin{aligned} \widetilde{Z}_k' + A_k\widetilde{Z}_k &= Z_k' + (A_k + I)^{-1}\widetilde{B}_k + A_kZ_k + A_k(A_k + I)^{-1}\widetilde{B}_k \\ &= Z_k' + A_kZ_k + \widetilde{B}_k = 0. \end{aligned} \tag{OA.56}$$

Once more, the development made between equations (OA.41) and (OA.47) can be applied to $\widetilde{Z}_k$ in (OA.56) to yield

$$\widetilde{Z}_k = e^{-D_kx}E_kK_k,$$

or, equivalently,

$$Z_k = e^{-D_kx}E_kK_k - (A_k + I)^{-1}\widetilde{B}_k.$$

Therefore,

$$\begin{aligned} \tilde{q}_{D,ij}(x) &= \delta_{ij}e^{x-x_{Dj}} = \delta_{ij}\frac{X}{X_{Di}}, \quad i \in \{1,\dots,k\}, j \in \{1,\dots,N\}, \\ \tilde{q}_{D,i,j}(x) &= \sum_{l=1}^{2(N-k)} h_{ij}(\omega_l)e^{-\omega_l x} - [(A_k + I)^{-1}\widetilde{B}_k]_{i-k,j}, \quad i \in \{k+1,\dots,N\}, j \in \{1,\dots,N\}. \end{aligned}$$

For $1 \le i \le k$, if the earnings at current time t are $X_t < X_{D,i}$ while the current state is i, it must be that the state just jumped to state i at time t^- and the firm is now in (deep) default, hence the first equation in the above system.

OA.J Bond Prices

In this proof it is not necessary to distinguish between the state of the economy at dates $t-$ and t. The central part of our proof consists of proving that

$$E_t\left[\int_t^{\tau_D} \frac{\pi_s^\$}{\pi_t^\$}ds \,\middle|\, s_t = i\right] = \frac{1}{r_{P,i}^\$} - \sum_{j=1}^N \frac{q_{D,ij}^\$}{r_{P,j}^\$}, \tag{OA.57}$$

where $r_{P,i}^\$$, the discount rate for a fixed nominal perpetuity, when the economy is in state i, is given by

$$r_{P,i}^\$ = \left(E_t\left[\int_t^{\infty} \frac{\pi_s^\$}{\pi_t^\$}ds \,\middle|\, s_t = i\right]\right)^{-1}, \tag{OA.58}$$

and

$$E_t\left[\left.\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\alpha A^{\$}_{\tau_D}\left(X_{\tau_D}\right)\right| s_t=i\right] \quad = \quad \sum_{j=1}^{N}\alpha A^{\$}_j\left(X_{D,j}\right)\tilde{q}^{\$}_{D,ij}.$$

To prove (OA.57), we note that

$$E_t\left[\left.\int_t^{\tau_D}\frac{\pi^{\$}_s}{\pi^{\$}_t}ds\right| s_t=i\right]=E_t\left[\left.\int_t^{\infty}\frac{\pi^{\$}_s}{\pi^{\$}_t}ds\right| s_t=i\right]-E_t\left[\left.\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\int_{\tau_D}^{\infty}\frac{\pi^{\$}_s}{\pi^{\$}_{\tau_D}}ds\right| s_t=i\right],$$

and conditioning on the event $\{s_{\tau_D}=j\}$, we obtain

$$E_t\left[\left.\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\int_{\tau_D}^{\infty}\frac{\pi^{\$}_s}{\pi^{\$}_{\tau_D}}ds\right| s_t=i\right]=\sum_{j=1}^{N}E_t\left[\left.\Pr\left(s_{\tau_D}=j\right|s_t=i\right)\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\int_{\tau_D}^{\infty}\frac{\pi^{\$}_s}{\pi^{\$}_{\tau_D}}ds\right| s_t=i\right].$$

Since consumption is Markovian, so is the state-price density, which implies that

$$\begin{aligned}
& E_t\left[\left.\Pr\left(s_{\tau_D}=j\right|s_t=i\right)\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\int_{\tau_D}^{\infty}\frac{\pi^{\$}_s}{\pi^{\$}_{\tau_D}}ds\right| s_t=i\right] \\
= \quad & E_t\left[\left.\Pr\left(s_{\tau_D}=j\right|s_t=i\right)\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\right| s_t=i\right]E_t\left[\left.\int_{\tau_D}^{\infty}\frac{\pi^{\$}_s}{\pi^{\$}_{\tau_D}}ds\right| s_{\tau_D}=j\right].
\end{aligned}$$

Therefore

$$\begin{aligned}
E_t\left[\left.\int_t^{\tau_D}\frac{\pi^{\$}_s}{\pi^{\$}_t}ds\right| s_t=i\right] \quad = \quad & E_t\left[\left.\int_t^{\infty}\frac{\pi^{\$}_s}{\pi^{\$}_t}ds\right| s_t=i\right] \\
& -\sum_{j=1}^{N}E_t\left[\left.\Pr\left(s_{\tau_D}=j\right|s_t=i\right)\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\right| s_t=i\right]E_t\left[\left.\int_{\tau_D}^{\infty}\frac{\pi^{\$}_s}{\pi^{\$}_{\tau_D}}ds\right| s_{\tau_D}=j\right].
\end{aligned}$$

Conditional on being in state i, the value of a claim which pays one risk-free unit of consumption in perpetuity is $E_t\left[\left.\int_t^{\infty}\frac{\pi^{\$}_s}{\pi^{\$}_t}ds\right| s_t=i\right]$, so the discount rate for this perpetuity, $r^{\$}_{P,i}$, is given by (OA.58). Consequently, (OA.60) implies

$$E_t\left[\left.\int_t^{\tau_D}\frac{\pi^{\$}_s}{\pi^{\$}_t}ds\right| s_t=i\right]=\frac{1}{r^{\$}_{P,i}}-\sum_{j=1}^{N}\frac{E_t\left[\left.\Pr\left(s_t=i\right|s_{\tau_D}=j\right)\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\right| s_t=i\right]}{r^{\$}_{P,j}}.$$

To obtain (OA.57) from the above expression, we note that

$$q^{\$}_{D,ij,t}=E_t\left[\left.\Pr\left(s_{\tau_D}=j\right|s_{\tau_t}=i\right)\frac{\pi^{\$}_{\tau_D}}{\pi^{\$}_t}\right| s_t=i,\right]. \tag{OA.61}$$

To prove (OA.59), we condition on the event $\{s_{\tau_D} = j\}$ to obtain

$$\alpha E_t \left[\frac{\pi^\$_{\tau_D}}{\pi^\$_t} A^\$_{\tau_D} \left(X_{\tau_D} \right) \middle| s_t = i \right] = \alpha \sum_{j=1}^{N} A^\$_j \left(X_{D,j} \right) E_t \left[\frac{X_{\tau_D}}{X_{D,j}} \Pr \left(s_{\tau_D} = s_j \middle| s_t = i \right) \frac{\pi^\$_{\tau_D}}{\pi^\$_t} \middle| s_t = i \right].$$

Using (OA.61) to simplify the above expression we obtain (7).

OA.K Equity Risk Premium

Applying Ito's Lemma to $S^\$_{i,t}$ gives

$$\frac{dS^\$_{i,t} + (X_t - c)dt}{S^\$_{i,t}} = \frac{X_t}{S^\$_{i,t}} \frac{\partial S^\$_{i,t}}{\partial X_t} \frac{dX_t}{X_t} + \frac{1}{2} \frac{X_t^2}{S_{i,t}} \frac{\partial^2 S^\$_{i,t}}{\partial X_t^2} \left(\frac{dX_t}{X_t} \right)^2 + \sum_{j \neq i}^{N} \frac{S^\$_{j,t} - S^\$_{i,t}}{S^\$_{i,t}} dN_{ij,t} + \frac{(X_t - c)dt}{S^\$_{i,t}}, \; i, j \in \{1, \ldots, N\}.$$

Observe that

$$\frac{\partial S^\$_{i,t}}{\partial X_t} = (1 - \eta) \frac{1}{r^\$_{A,i}} - \sum_{j=1}^{N} \left(A^\$_j (X_{D,j}) \frac{\partial \tilde{q}^\$_{D,ij,t}}{\partial X_t} - (1 - \eta) \frac{\partial q_{D,ij,t}}{\partial X_t} \frac{c}{r^\$_{P,j}} \right)$$

$$\frac{\partial^2 S^\$_{i,t}}{\partial X_t^2} = - \sum_{j=1}^{N} \left(A^\$_j (X_{D,j}) \frac{\partial^2 \tilde{q}^\$_{D,ij,t}}{\partial X_t^2} - (1 - \eta) \frac{\partial^2 q_{D,ij,t}}{\partial X_t^2} \frac{c}{r^\$_{P,j}} \right), \; i, j \in \{1, \ldots, N\}.$$

Define the date-t conditional nominal expected return

$$\mu^\$_{R,i,t} = \frac{1}{dt} E_t \left[\frac{dS^\$_{s_{t-},t} + (X_t - c)dt}{S^\$_{s_{t-},t}} \middle| s_{t-} = i \right], \; i \in \{1, \ldots, N\}.$$

The basic asset pricing equation is

$$\mu^\$_{R,i,t} - r^\$_{i,t} = -\frac{1}{dt} E_t \left[\frac{d\pi^\$_t}{\pi^\$_t} \frac{dS^\$_{s_{t-},t}}{S^\$_{s_{t-},t}} \middle| s_{t-} = i \right], \; i \in \{1, \ldots, N\}.$$

Hence

$$\mu^\$_{R,i,t} - r^\$_{i,t} = \sum_{j \neq i} (1 - \omega_{ij}) \frac{S^\$_j - S^\$_i}{S^\$_i} \lambda_{ij} + \frac{X_t}{S^\$_{i,t}} \frac{\partial S^\$_{i,t}}{\partial X_t} E_t \left[\left(\sigma_{Y,t} dW_t + \varphi \sigma_{P,t} dZ_{P,t} \right) \left(\gamma \sigma_{C,i} dZ_t + \sigma_{P,i} dZ_{P,t} \right) \right]$$

$$= \sum_{j \neq i} (1 - \omega_{ij}) \frac{S^\$_j - S^\$_i}{S^\$_i} \lambda_{ij} + \frac{X_t}{S^\$_{i,t}} \frac{\partial S^\$_{i,t}}{\partial X_t} \left(\rho_{YC,i} \sigma_{Y,i} \gamma \sigma_{C,i} + \rho_{YP,i} \sigma_{Y,i} \sigma_{P,i} + \gamma \varphi \rho_{PC,i} \sigma_{P,t} \sigma_{C,i} + \varphi \sigma^2_{P,i} \right)$$

$$, \; i, j \in \{1, \ldots, N\}.$$

The unexpected nominal stock return in state i is given by

$$\sum_{j \neq i} \sigma^P_{R,ij} dN^P_{ij,t} + \frac{X_t}{S^\$_{i,t}} \frac{\partial S^\$_{i,t}}{\partial X_t} \left(\sigma_{Y,i} dW_t + \varphi \sigma_{P,i} dZ_{P,t} \right), \; i, j \in \{1, \ldots, N\},$$

where

$$\sigma^P_{R,ij} = \frac{S^\$_j}{S^\$_i} - 1,\; i,j \in \{1,\ldots,N\}.$$

If we define

$$S_{i,t} = \frac{S^\$_{i,t}}{P_t},$$

then Ito's Lemma implies that

$$\begin{aligned}
\frac{dS_{i,t}}{S_{i,t}} &= \frac{dS^\$_{i,t}}{S^\$_{i,t}} - \frac{dP_t}{P_t} + \sigma^2_{P,i}dt - \frac{dS^\$_{i,t}}{S^\$_{i,t}}\frac{dP_t}{P_t} \\
&= -\frac{dP_t}{P_t} + \sigma^2_{P,i}dt - \frac{X_t}{S^\$_{i,t}}\frac{\partial S^\$_{i,t}}{\partial X_t}\frac{dX_t}{X_t}\frac{dP_t}{P_t} \\
&= \frac{dS^\$_{i,t}}{S^\$_{i,t}} - \frac{dP_t}{P_t} + \sigma^2_{P,i}dt - \frac{X_t}{S^\$_{i,t}}\frac{\partial S^\$_{i,t}}{\partial X_t}(\sigma_{P,i}\sigma_{Y,i}\rho_{PY,i}dt + \varphi\sigma^2_{P,i})dt.
\end{aligned}$$

Therefore, the real risk premium in state i is

$$\begin{aligned}
\mu_{R,i} - r_i &= \mu^\$_{R,i} - \mu_{P,i} + \sigma^2_{P,i} - \frac{X_t}{S^\$_{i,t}}\frac{\partial S^\$_{i,t}}{\partial X_t}(\sigma_{P,i}\sigma_{Y,i}\rho_{PY,i} + \varphi\sigma^2_{P,i})dt - r_i \\
&= \mu^\$_{R,i} - r^\$_i - \mu_{P,i} + \sigma^2_{P,i} - \frac{X_t}{S^\$_{i,t}}\frac{\partial S^\$_{i,t}}{\partial X_t}(\sigma_{P,i}\sigma_{Y,i}\rho_{PY,i} + \varphi\sigma^2_{P,i})dt + r^\$_i - r_i \\
&= \mu^\$_{R,i} - r^\$_i - \mu_{P,i} + \sigma^2_{P,i} - \frac{X_t}{S^\$_{i,t}}\frac{\partial S^\$_{i,t}}{\partial X_t}(\sigma_{P,i}\sigma_{Y,i}\rho_{PY,i} + \varphi\sigma^2_{P,i})dt + \mu_{P,i} - \gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i} - \sigma^2_{P,i} \\
&= \mu^\$_{R,i} - r^\$_i - \frac{X_t}{S^\$_{i,t}}\frac{\partial S^\$_{i,t}}{\partial X_t}(\sigma_{P,i}\sigma_{Y,i}\rho_{PY,i} + \varphi\sigma^2_{P,i})dt - \gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i} \\
&= \sum_{j\neq i}(1-\omega_{ij})\frac{S^\$_j - S^\$_i}{S^\$_i}\lambda_{ij} + \frac{X_t}{S^\$_{i,t}}\frac{\partial S^\$_{i,t}}{\partial X_t}\left(\rho_{YC,i}\sigma_{Y,i}\gamma\sigma_{C,i} + \gamma\varphi\rho_{PC,i}\sigma_{P,t}\sigma_{C,i}\right) - \gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i},
\end{aligned}$$

where

$$\frac{X_t}{S^\$_{i,t}}\frac{\partial S^\$_{i,t}}{\partial X_t} = \frac{X_t}{S^\$_{i,t}}\left[\frac{A^\$_i(X_t)}{X_t} - \sum_{j=1}^{N}\left(A^\$_j(X_{D,j})\frac{\partial \tilde{q}^\$_{D,ij,t}}{\partial X_t} - (1-\eta)\frac{\partial q^\$_{D,ij,t}}{\partial X_t}v_{B,i}c\right)\right].$$

Observe that if $\rho_{YC,i} = 0$ and $\rho_{YP,i} = 0$, we obtain (9) and

$$\mu_{R,i} - r_i = \sum_{j\neq i}(1-\omega_{ij})\frac{S^\$_j - S^\$_i}{S^\$_i}\lambda_{ij} + \left(\varphi\frac{X_t}{S^\$_{i,t}}\frac{\partial S^\$_{i,t}}{\partial X_t} - 1\right)\gamma\rho_{PC,i}\sigma_{P,t}\sigma_{C,i}.$$

Similarly, the nominal expected risk premium for a corporate bond is

$$-\frac{1}{dt}E_t\left[\frac{d\pi_t^\$}{\pi_t^\$}\frac{dB_{s_{t-},t}^\$}{B_{s_{t-},t}^\$}\middle| s_{t-}=i\right]$$
$$=\sum_{j\neq i}(1-\omega_{ij})\frac{B_j^\$-B_i^\$}{B_i^\$}\lambda_{ij}+\frac{X_t}{B_{i,t}^\$}\frac{\partial B_{i,t}^\$}{\partial X_t}\left(\sigma_{Y,i}\gamma\sigma_{C,i}\rho_{YC,i}+\sigma_{Y,i}\sigma_{P,i}\rho_{YP,i}+\varphi\sigma_{P,i}\gamma\sigma_{C,i}\rho_{PC,i}+\varphi\sigma_{P,i}^2\right),$$

where

$$\frac{\partial B_{i,t}^\$}{\partial X_t}=-c\sum_{j=1}^N\frac{\partial q_{D,ij,t}^\$}{\partial X_t}\frac{1}{r_{P,j}^\$}+\alpha\sum_{j=1}^N A_j^\$(X_{D,j})\frac{\partial\hat{q}_{D,ij,t}^\$}{\partial X_t},$$

and the real expected risk premium for a corporate bond is

$$-\frac{1}{dt}E_t\left[\frac{d\pi_t^\$}{\pi_t^\$}\frac{dB_{s_{t-},t}^\$}{B_{s_{t-},t}^\$}\middle| s_{t-}=i\right]-\gamma\rho_{PC,i}\sigma_{P,i}\sigma_{C,i}-\sigma_{P,i}^2.$$

OA.L Finite Maturity Corporate Bonds

Proposition OA.3 *Conditional on the current state being i, the date-t nominal price of a zero coupon risk-free bond which pays out one USD at date $t+\tau$, is $B_{i,\tau}^\$$, which is the i'th element of the vector*

$$\underline{B}_\tau^\$=(B_{1,\tau}^\$,\ldots,B_{N,\tau}^\$)^\top,$$

where

$$\underline{B}_\tau^\$=\exp[-(\boldsymbol{R}^\$-\hat{\boldsymbol{\Lambda}})\tau]\underline{1}_N,$$

and $\underline{1}_N$ is the $N\times 1$ vector of ones, and

$$\boldsymbol{R}^\$=diag(r_1^\$,\ldots,r_N^\$).$$

Proof of Proposition OA.3. The time-t nominal price of a zero coupon bond paying off 1 USD at time $t+\tau$, conditional on the current state of the economy being i is given by

$$B_{i,\tau}^\$=E_t\left[\frac{\pi_{t+\tau}^\$}{\pi_t^\$}\right]$$
$$=E_t^{\mathbb{Q}^\$}\left[e^{-\int_t^{t+\tau}r_u^\$du}\right].$$

From the no-arbitrage condition

$$E_t\left[dB_{s_{t-},\tau}^\$-r_{s_{t-}}^\$B_{s_{t-},\tau}^\$dt\right]=-E_t\left[dB_{s_{t-},\tau}^\$\frac{d\pi_t^\$}{\pi_{t-}^\$}\right],$$

applying Ito's Lemma gives the following linear system

$$\sum_{j\neq i}(B_{j,\tau}^\$-B_{i,\tau}^\$)\hat{\lambda}_{ij}-r_i^\$B_{i,\tau}^\$=\frac{\partial}{\partial\tau}B_{i,\tau}^\$,\ i\in\{1,\ldots,N\},$$

where $B^{\$}_{i,0} = 1$. Defining

$$\underline{B}^{\$}_{\tau} = (B^{\$}_{1,\tau}, \ldots, B^{\$}_{N,\tau})^{\top},$$

we have

$$\frac{\partial}{\partial \tau}\underline{B}^{\$}_{\tau} = -(\boldsymbol{R}^{\$} - \hat{\boldsymbol{\Lambda}})\underline{B}^{\$}_{\tau},$$

where

$$\boldsymbol{R}^{\$} = \text{diag}(r^{\$}_1, \ldots, r^{\$}_N),$$

and $\underline{B}^{\$}_{\tau=0} = \underline{1}_N$. Therefore

$$\begin{aligned}\underline{B}^{\$}_{\tau} &= \exp[-(\boldsymbol{R}^{\$} - \hat{\boldsymbol{\Lambda}})\tau]\underline{1}_N \\ &= \boldsymbol{E}e^{-\boldsymbol{D}\tau}\boldsymbol{E}^{-1}\underline{1}_N,\end{aligned}$$

where $\boldsymbol{E} = [\underline{e}_1, \ldots, \underline{e}_N]$ and $\boldsymbol{D} = \text{diag}(d_1, \ldots, d_N)$, and $\underline{e}_n$, $n \in \{1, \ldots, N\}$, where $\underline{e}_n$ is an eigenvector of $\boldsymbol{R}^{\$} - \hat{\boldsymbol{\Lambda}}$ with corresponding eigenvalue d_n, ordered such that $d_1 \leq \ldots \leq d_N$. Observe that

$$B^{\$}_{i,\tau} = \sum_{k=1}^{N} E_{ik}e^{-d_k\tau}\sum_{j=1}^{N}[E^{-1}]_{kj}.$$

■

Proposition OA.4 *Finite maturity risk-free debt pays a coupon at the rate c until maturity (time T) and the amount P at maturity. The time-t price of finite maturity risk-free debt when the current state is i is given by*

$$B^{\$}_{f,i,\tau} = c\left(\frac{1}{r^{\$}_{P,i}} - B^{\$}_{i,\tau}\sum_{j=1}^{N}\frac{1}{r^{\$}_{P,j}}\hat{\Pr}(s_T = j|s_t = i)\right) + P^{\$}B^{\$}_{i,\tau}. \tag{OA.62}$$

Proof of Proposition OA.4. The date-t value of finite maturity risk-free debt is

$$B^{\$}_{f,i,T-t} = cE_t\left[\int_t^T \frac{\pi^{\$}_s}{\pi^{\$}_t}ds|s_t = i\right] + P^{\$}E_t\left[\frac{\pi^{\$}_T}{\pi^{\$}_t}|s_t = i\right],$$

conditional on the current state being i. Hence,

$$B^{\$}_{f,i,T-t} = cE_t\left[\int_t^{\infty} \frac{\pi^{\$}_s}{\pi^{\$}_t}ds|s_t = i\right] - E_t\left[\int_T^{\infty} \frac{\pi^{\$}_s}{\pi^{\$}_t}ds|s_t = i\right] + P^{\$}E_t\left[\frac{\pi^{\$}_T}{\pi^{\$}_t}|s_t = i\right].$$

We know that

$$E_t\left[\int_t^{\infty} \frac{\pi^{\$}_s}{\pi^{\$}_t}ds|s_t = i\right] = \frac{1}{r^{\$}_{P,i}}.$$

Furthermore,

$$
\begin{aligned}
E_t\left[\int_T^\infty \frac{\pi_s^\$}{\pi_t^\$} ds | s_t = i\right] &= E_t\left[\frac{\pi_T^\$}{\pi_t^\$}|s_t=i\right]\sum_{j=1}^N E_T\left[\int_T^\infty \frac{\pi_s^\$}{\pi_T^\$} ds|s_T=j\right]\hat{\Pr}(s_T=j|s_t=i) \\
&= \sum_{j=1}^N E_T\left[\int_T^\infty \frac{\pi_s^\$}{\pi_T^\$} ds|s_T=j\right]\hat{\Pr}(s_T=j|s_t=i)E_t\left[\frac{\pi_T^\$}{\pi_t^\$}|s_t=i\right] \\
&= \sum_{j=1}^N \frac{1}{r_{P,j}^\$}\hat{\Pr}(s_T=j|s_t=i)B_{i,\tau}^\$,
\end{aligned}
$$

where $\tau = T - t$ and

$$E_t\left[\frac{\pi_T}{\pi_t}|s_t=i\right] = B_{i,\tau}^\$.$$

Equation (OA.62) follows. ■

Proposition OA.5 *Finite maturity corporate debt pays a coupon at the rate c until default (the random time, τ_D) or maturity (time T), whichever is earlier, and the amount $P^\$$ at maturity, if default has not already occurred. The time-t price of finite maturity corporate debt is*

$$\overline{B}_{i,T-t}^\$ = cE_t\left[\int_t^{\min(\tau_D,T)} \frac{\pi_s^\$}{\pi_t^\$} ds|s_t=i\right] + P^\$ E_t\left[\frac{\pi_T^\$}{\pi_t^\$}1_{\{\tau_D>T\}}|s_t=i\right] + E_t\left[\frac{\pi_{\tau_D}^\$}{\pi_t^\$}1_{\{\tau_D\le T\}}A_{s_{\tau_D}}^\$(X_{\tau_D})|s_t=i\right].$$

Closed-form expressions for $B_{f,ij,T-t}^0$ and $B_{f,i,T-t}^0$ are given in the proposition above, whereas $\hat{p}_{D,i,T-t}$ and $q_{D,ij,T-t}$ are computed by Monte-Carlo simulation.

Proof of Proposition OA.5. The proof follows immediately from the definition of the security. ■

OA.M Dividend Strips

Proposition OA.6 *Conditional on the current state being i, the date-t nominal price of an unlevered dividend strip which pays out the nominal cashflow $X_{t+\tau}$ at date $t+\tau$, is $S_{i,\tau}^\$$, which is the i'th element of the vector*

$$\underline{S}_\tau^\$ = (S_{1,\tau}^\$, \ldots, S_{N,\tau}^\$)^\top,$$

where

$$\underline{S}_\tau^\$ = X_t \exp[-(\boldsymbol{R}^\$ - \widehat{\boldsymbol{M}}^\$ - \hat{\boldsymbol{\Lambda}})\tau]\underline{1}_N,$$

and $\underline{1}_N$ is the $N\times 1$ vector of ones, and

$$
\begin{aligned}
\boldsymbol{R}^\$ &= diag(r_1^\$, \ldots, r_N^\$) \\
\widehat{\boldsymbol{M}}^\$ &= diag(\hat{\mu}_{X,1}^\$, \ldots, \hat{\mu}_{X,N}^\$).
\end{aligned}
$$

and $\hat{\mu}_{X,i}^\$$ is defined in (OA.31).

Proof of Proposition OA.6.

The proof follows the same steps as the Proof of Proposition OA.3.

■

OA.N Markov Chain - Statistics

We now compute the conditional mean of expected consumption growth and the conditional covariance of expected consumption growth with expected inflation. Suppose the economy is in state i at date t. The expression for $E_t[\mu_{C,t+u}|s_t = i]$ is:

$$E_t[\mu_{C,t+u}|s_t = i] = \sum_{j=1}^{N} \mu_{C,j} \Pr(s_{t+u} = j|s_t = i),$$

where

$$\Pr(s_{t+u} = j|s_t = i) = [\exp(\Lambda u)]_{ij},$$

and also for $Cov_t[\mu_{C,t+u}, \mu_{P,t+u}|s_t = i]$:

$$Cov_t[\mu_{C,t+u}, \mu_{P,t+u}|s_t = i] = \sum_{j=1}^{N} (\mu_{C,j} - E_t[\mu_{C,t+u}|s_t = i])(\mu_{P,j} - E_t[\mu_{P,t+u}|s_t = i]) \Pr(s_{t+u} = j|s_t = i).$$

OA.O Sticky Prices

Here, we show that in a continuous-time New Keynesian model with costly price-adjustment, the expected growth rate of nominal profits goes up by less than 1% if expected inflation increases by 1%.

The economy consists of a continuum of firms producing differentiated goods and a continuum of identical households. Financial markets are dynamically complete.

OA.O.1 Households

There is a continuum of households, who make real expenditure flow decisions and decide how many person-hours to work per unit time. We sometimes refer to them as consumer-workers. They obtain utility flows from real consumption expenditure flows and disutility flows from the person-hours they work per unit time. Households are identical, so we maximize the following expected utility function of the representative agent:

$$E_t \int_t^{\infty} e^{-\delta(u-t)} \left(\ln C_u - \frac{N_u^{1+\phi}}{1+\phi} \right) du,$$

where C_t is related to the consumption flows of individual goods via the Dixit-Stiglitz aggregator, i.e.

$$C_t = \left(\int_{f \in [0,1]} C_t(f)^{1-\frac{1}{\epsilon}} \right)^{\frac{1}{1-\frac{1}{\epsilon}}},$$

where $\epsilon > 1$ is the elasticity of substitution between any two goods.

The underlying reason for differentiation across goods could be because of types of goods, branding or quality.

OA.O.2 Firms, imperfect competition and costly price adjustment

There is a continuum of firms, $f \in [0, 1]$. Firms produce differentiated goods, which households can choose to consume. The goods are differentiated, because the elasticity of substitution between any two goods is finite. Hence, firms have monopoly power, which they can use to extract rents from consumers by setting prices to maximize firm value.[41]

We are assuming that the objective of a firm is to maximize the expected present value of profits as opposed to the welfare of households, who own the firm. This is because we are modelling the agency conflict between managers of firms who have different objectives from the firms' owners, i.e. the households.

Firm f produces good f as follows:

$$O_t(f) = A_t N_t(f),$$

where $O_t(f)$ is time t output flow for firm f and $N_t(f)$ is the labor input flow demanded by firm f. The level of technological progress A is common across firms. Technological change takes place exogenously according to

$$\frac{dA_t}{A_t} = \mu dt + \sigma dZ_t,$$

where Z is a standard Brownian motion under the physical probability measure $\mathbb{P}$.

The price of good f is denoted by $P(f)$ and firms pay wages at the nominal wage rate W_t, and so firm f's real profit flow is

$$\Pi_t(f) = \frac{P_t(f)}{P_t} O_t(f) - \frac{W_t}{P_t} N_t(f).$$

When firms adjust prices they face quadratic adjustment costs, such that they choose a price process to maximize date-t firm value net of adjustment costs, i.e.

$$J_t(f) = \sup_{(P_u(f))_{u \geq t}} E_t \int_t^\infty \frac{\Lambda_u}{\Lambda_t} \left[\Pi_u(f) - \frac{1}{2}\theta \left(\frac{dP_u(f)/du}{P_u(f)} \right)^2 O_u \right] du,$$

where the SDF process, Λ, given by

$$\Lambda_t = e^{-\delta t} C_t^{-1}, \tag{OA.63}$$

is determined in equilibrium. Observe that in the special case $\theta = 0$, there are no price adjustment costs, making prices fully flexible.

OA.O.3 Individual firm's problem

We now solve an individual firm's optimal stochastic control problem. An individual firm seeks to exploit its monopoly power to maximize the expected value if its profits at the expense of householders. Firm f's optimal stochastic control problem is

$$J_t(f) = \sup_{(P_u(f))_{u \geq t}} E_t \int_t^\infty \frac{\Lambda_u}{\Lambda_t} \left[\Pi_u(f) - \frac{1}{2}\theta \left(\frac{dP_u(f)/du}{P_u(f)} \right)^2 O_u \right] du,$$

where the SDF process, Λ, given by

$$\Lambda_t = e^{-\delta t} C_t^{-1}.$$

is determined in equilibrium. We assume that price adjustment costs are redistributed to shareholders, so that the date-t value of firm f is given by

$$E_t \int_t^\infty \frac{\Lambda_u}{\Lambda_t} \Pi_u(f) du.$$

[41] In the limiting case of $\epsilon \to \infty$, goods are no longer differentiated and firms have zero monopoly power.

Assuming that the aggregate price index is locally-free, date-t inflation is given by $\mu_{P,t} = \frac{1}{P_t}\frac{dP_t}{dt}$ and we can obtain a relationship between real profits flows, inflation, and the nominal interest rate, as shown below.

Proposition OA.7 *If the nominal interest rate rule is given by*

$$r_t^{\$} = a_0 + a_1 \mu_{P,t},$$

then nominal profit flow for firm f is given by $\Pi_t^{\$}(f)$, where

$$d\ln \Pi_t^{\$}(f) = da_t - [(\epsilon - 1)(1+\phi) - 1](a_0 - r_n)dt + \varphi \mu_{P,t} dt,$$

with

$$\varphi = 1 - [(\epsilon - 1)(1+\phi) - 1](a_1 - 1)],$$

where r_n is the natural rate of interest given in (OA.72)*.*

If $\epsilon > 1 + \frac{1}{1+\phi}$ and $a_1 > 1$ (i.e. the Taylor principle holds), then $\varphi < 1$, and so, expected nominal profit growth increases by less than one percentage point when inflation increases by one percentage point.

Proof of Proposition OA.7

Firm-level optimal price setting problem

We solve the firm-level optimal price setting problem. We have the standard result that

$$C_t(f)/C_t = \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon}.$$

In equilibrium $C_t(f) = O_t(f)$ and $C_t = O_t$, and so,

$$O_t(f)/O_t = \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon}.$$

Since we have $C_t = O_t$, firm f's optimal stochastic control problem is

$$J_t(f) = O_t \sup_{(P_u(f))_{u \geq t}} E_t \int_t^{\infty} e^{-\delta(u-t)} O_u^{-1} \left[D_u(f) - \frac{1}{2}\theta \left(\frac{dP_u(f)/du}{P_u(f)}\right)^2 O_u \right] du,$$

Now observe that

$$\begin{aligned} D_t(f) &= \left(\frac{P_t(f)}{P_t} - \frac{W_t}{P_t A_t}\right) O_t(f) \\ &= \left(\frac{P_t(f)}{P_t} - \frac{W_t}{P_t A_t}\right) \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon} O_t \\ &= \left(\left(\frac{P_t(f)}{P_t}\right)^{1-\epsilon} - \frac{W_t}{P_t A_t} \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon} \right) O_t \end{aligned}$$

Therefore

$$J_t(f) = O_t \sup_{(P_u(f))_{u \geq t}} E_t \int_t^{\infty} e^{-\delta(u-t)} \left[\left(\frac{P_u(f)}{P_u}\right)^{1-\epsilon} - \frac{W_u}{P_u A_u} \left(\frac{P_u(f)}{P_u}\right)^{-\epsilon} - \frac{1}{2}\theta \left(\frac{dP_u(f)/du}{P_u(f)}\right)^2 \right] du.$$

The above problem is equivalent to

$$\hat{J}_t(f) = \sup_{(P_u(f))_{u\geq t}} E_t \int_t^{\infty} e^{-\delta(u-t)} \left[\left(\frac{P_u(f)}{P_u}\right)^{1-\epsilon} - \frac{W_u}{P_u A_u}\left(\frac{P_u(f)}{P_u}\right)^{-\epsilon} - \frac{1}{2}\theta\left(\frac{dP_u(f)/du}{P_u(f)}\right)^2 \right] du.$$

We assume that prices are locally risk-free. The firm uses the control $dP_t(f)/dt$ to alter the state variable $P_t(f)$, and so we define

$$\overline{\mu}_{P(f),t} = \frac{dP_t(f)}{dt}.$$

Firm f's problem is to choose the rate of change for the price of good f, i.e. $\mu_{P(f)}$ to maximize

$$E_t \int_t^{\infty} e^{-\delta(u-t)} \left[\left(\frac{P_u(f)}{P_u}\right)^{1-\epsilon} - \frac{W_u}{P_u A_u}\left(\frac{P_u(f)}{P_u}\right)^{-\epsilon} - \frac{1}{2}\theta\left(\frac{\overline{\mu}_{P(f)}}{P_u(f)}\right)^2 \right] du.$$

To solve the individual firm's problem, we apply the stochastic maximum principle. Define the Hamiltonian

$$\mathcal{H}_t = \left(\frac{P_t(f)}{P_t}\right)^{1-\epsilon} - \frac{W_t}{P_t A_t}\left(\frac{P_t(f)}{P_t}\right)^{-\epsilon} - \frac{1}{2}\theta\left(\frac{\overline{\mu}_{P(f),t}}{P_t(f)}\right)^2 + E_t\left[\frac{d\hat{J}_t}{dt}\right].$$

The HJB equation is

$$\sup_{\overline{\mu}_{P(f),t}} \mathcal{H}_t - \delta \hat{J}_t$$

The FOC is

$$\mathcal{H}_{\overline{\mu}_{P(f),t}} = 0, \tag{OA.64}$$

and so

$$\hat{J}_{P(f)} = \theta \frac{\overline{\mu}_{P(f)}}{P(f)^2} \tag{OA.65}$$

Differentiating the HJB equation with respect to the state variable $P(f)$ gives

$$\mathcal{H}_{P(f)} + \mathcal{H}_{\mu_{P(f)}} \frac{\partial \overline{\mu}_{P(f)}}{\partial P(f)} - \delta \hat{J}_{P(f)} = 0.$$

Using the FOC (OA.64) we obtain

$$\mathcal{H}_{P(f)} - \delta \hat{J}_{P(f)} = 0.$$

Therefore

$$(\epsilon - 1)\frac{1}{P}\left(\frac{P(f)}{P}\right)^{-\epsilon} = \epsilon \frac{W}{PA}\frac{1}{P}\left(\frac{P(f)}{P}\right)^{-\epsilon-1} + \theta\frac{\overline{\mu}_{P(f)}}{P(f)^2} + E_t\left[\frac{d\hat{J}_{P(f)}}{dt}\right] - \delta \hat{J}_{P(f)}.$$

The above equation tells us that at the optimum the marginal loss in revenues from higher prices and the marginal loss from price adjustment costs must be offset by marginal gains from lower labor costs and changes in the present expected value of future profits.

Using the FOC (OA.65), we obtain

$$(\epsilon-1)\frac{1}{P}\left(\frac{P(f)}{P}\right)^{-\epsilon}-\theta\frac{1}{P(f)}\left(\frac{\overline{\mu}_{P(f)}}{P(f)}\right)^2=\epsilon\frac{W}{PA}\frac{1}{P}\left(\frac{P(f)}{P}\right)^{-\epsilon-1}+\theta E_t\left[\frac{d(\overline{\mu}_{P(f)}/P(f)^2)}{dt}\right]-\delta\theta\frac{\overline{\mu}_{P(f)}}{P(f)^2}.$$

Define inflation in good f via

$$\mu_{P,t}(f)=\frac{1}{P_t(f)}\frac{dP_t(f)}{dt}=\frac{\overline{\mu}_{P(f),t}}{P_t(f)}.$$

Hence

$$(\epsilon-1)\frac{1}{P}\left(\frac{P(f)}{P}\right)^{-\epsilon}=\epsilon\frac{W}{PA}\frac{1}{P}\left(\frac{P(f)}{P}\right)^{-\epsilon-1}+\theta\frac{1}{P_t(f)}E_t\left[\frac{d\mu_{P,t}(f)}{dt}\right]-\delta\theta\frac{\mu_{P,t}(f)}{P_t(f)},$$

By symmetry

$$P_t(f)=P_t,$$

and so

$$\mu_{P,t}dt=\mu_{P,t}(f)dt=\frac{dP_t(f)}{P_t(f)}=\frac{dP_t}{P_t}.$$

Therefore, the real wage rate is given by

$$\frac{W_t}{P_t}=A_t\left[\left(1-\frac{1}{\epsilon}\right)-\frac{\theta}{\epsilon}\left(E_t\left[\frac{d\mu_{P,t}}{dt}\right]-\delta\mu_{P,t}\right)\right]. \tag{OA.66}$$

Real profit flow for firm f is hence given by

$$\begin{aligned}\Pi_t(f)&=\left(1-\frac{W_t}{P_tA_t}\right)O_t(f)\\&=\left[\frac{1}{\epsilon}+\frac{\theta}{\epsilon}\left(E_t\left[\frac{d\mu_{P,t}}{dt}\right]-\delta\mu_{P,t}\right)\right]O_t(f).\end{aligned} \tag{OA.67}$$

Representative household's stochastic optimal control problem

We now derive an expression for O_t and hence $O_t(f)$ in terms of the nominal interest rate i. We need to consider the representative household's stochastic optimal control problem, which is

$$V_t=\sup_{(C_u)_{u\geq t},(N_u)_{u\geq t},(\phi_u)_{u\geq t}}E_t\int_t^\infty e^{-\delta(u-t)}\left(\ln C_u-\frac{N_u^{1+\phi}}{1+\phi}\right)du,$$

subject to the dynamic intertemporal budget constraint

$$dH_t=\left(r_tH_t+\frac{W_t}{P_t}N_{h,t}-C_t\right)dt+H_t\phi_t\left(dP_{D,t}+\Pi_tdt\right),$$

where H_t is the time-t financial wealth of the representative household, $P_{D,t}$ is the time-t real present-value value of future real aggregate profits, i.e.

$$P_{D,t}=E_t\left[\int_t^\infty\frac{\Lambda_u}{\Lambda_t}\Pi_u\right],$$

and ϕ_t is the fraction of financial wealth invested in the claim to future profits, i.e. the risky stock. Define

$$a_t = \ln A_t,$$

$$h_t = \ln H_t - a_t,$$

$$c_t = \ln C_t - a_t.$$

Therefore

$$V_t = E_t \int_t^\infty e^{-\delta(u-t)} a_u du + \hat{V}_t,$$

where

$$\hat{V}_t = \sup_{(c_u)_{u\geq t},\,(n_u)_{u\geq t},\,(\phi_u)_{u\geq t},} E_t \int_t^\infty e^{-\delta(u-t)} \left(c_u - \frac{e^{(1+\phi)n_u}}{1+\phi} \right) du,$$

and

$$dh_t = \left\{ [r_t + \phi_t(\mu_{R,t} - r)] - e^{c_t - h_t} + \frac{W_t}{A_t P_t} e^{n_t - h_t} - \frac{1}{2}\phi_t^2 \sigma_R^2 - \mu_a \right\} dt + \phi_t \sigma_{R,t} dZ_{R,t} - \sigma dZ_t,$$

where

$$\mu_a = E_t \left[\frac{da_t}{dt} \right] = \mu - \frac{1}{2}\sigma^2,$$

and

$$\mu_{R,t} = \frac{1}{dt} E_t \left[\frac{dP_{D,t} + \Pi_t dt}{P_{D,t}} \right],$$

$$\sigma_{R,t} dZ_{R,t} = \frac{dP_{D,t} + \Pi_t dt}{P_{D,t}} - E_t \left[\frac{dP_{D,t} + \Pi_t dt}{P_{D,t}} \right],$$

$$\sigma_{R,t} = \frac{1}{dt} E_t \left(\frac{dP_{D,t}}{P_{D,t}} \right)^2.$$

The Hamilton-Jacobi-Bellman equation for the household's stochastic optimal control problem is

$$0 = \sup c_t - \frac{e^{(1+\phi)n_t}}{1+\phi} - \delta \hat{V}_t + \hat{V}_{h,t} \left\{ [r_t + \phi_t(\mu_{R,t} - r_t)] - e^{c_t - h_t} + \frac{W_t}{A_t P_t} e^{n_t - h_t} - \frac{1}{2}\phi_t^2 \sigma_{R,t}^2 - \mu_a \right\}$$
$$+ \frac{1}{2}(\phi_t^2 \sigma_{R,t}^2 - 2\rho_{R,t}\sigma\sigma_{R,t} + \sigma^2)\hat{V}_{hh,t} + \text{terms stemming from other state variables which are not controlled},$$

where

$$\rho_{R,t} dt = E_t[dZ_t dZ_R].$$

The FOCs are

$$1 = e^{c_t - h_t} \hat{V}_t,$$

$$e^{n_t(1+\phi)} = \frac{W_t}{A_t P_t} e^{n_t - h_t} \hat{V}_{h,t},$$

$$\phi_t = \frac{\hat{V}_{h,t}(\mu_{R,t} - r_t) - \hat{V}_{hh,t}\rho\sigma_{R,t}\sigma}{\sigma_{R,t}^2 \left(\hat{V}_{h,t} - \hat{V}_{hh,t} \right)}, \text{ if } \sigma_{R,t} \neq 0. \tag{OA.68}$$

The first two FOCs can be combined to yield

$$\frac{W_t}{A_t P_t} e^n = e^{c_t} e^{n_t(1+\phi)}.$$

In equilibrium, markets clear and so

$$c_t = n_t.$$

Hence

$$e^{n_t(1+\phi)} = \frac{W_t}{A_t P_t}.$$

Since $\ln O_t = y_t = a_t + n_t$ we have

$$\frac{W_t}{e_t^a P_t} = e^{(1+\phi)(y_t - a_t)},$$

which is equivalent to

$$\frac{W_t}{P_t A_t} = \left(\frac{O_t}{A_t}\right)^{1+\phi}.$$

Using the Ansatz (standard for logarithmic preferences)

$$\hat{V}_t = a_h h_t + j_t,$$

where j_t is independent of h_t, we see that

$$\phi_t = \frac{\mu_{R,t} - r_t}{\sigma_{R,t}^2}.$$

The three equation model in continuous-time with aggregate risk

In equilibrium, open interest in the bond market is zero, and so $\phi_t = 1$, yielding

$$\mu_{R,t} - r_t = \sigma_{R,t}^2. \tag{OA.69}$$

It follows from (OA.66) that

$$\left(\frac{O_t}{A_t}\right)^{1+\phi} = 1 - \frac{1}{\epsilon} - \frac{\theta}{\epsilon}\left(E_t\left[\frac{d\mu_{P,t}}{dt}\right] - \delta\mu_{P,t}\right) \tag{OA.70}$$

When the cost of adjusting prices is zero, i.e. $\theta = 0$, then

$$N_n^{1+\phi} = \left(\frac{O_{n,t}}{A_t}\right)^{1+\phi} = 1 - \frac{1}{\epsilon}, \tag{OA.71}$$

where we append a subscript n to equilibrium variables. Hence,

$$O_{n,t} = A_t N_n = A_t \left(1 - \frac{1}{\epsilon}\right)^{\frac{1}{1+\phi}},$$

and so (via (OA.63)), it follows that with $\theta = 0$,

$$\Lambda_t = e^{-\delta t} A_t^{-1} \left(1 - \frac{1}{\epsilon}\right)^{-\frac{1}{1+\phi}}.$$

Applying Ito's Lemma, we obtain

$$\frac{d\Lambda_t}{\Lambda_t} = -r_n dt - \Theta_n dZ_t,$$

where

$$r_n = \delta + \mu - \frac{1}{2}\sigma^2 \tag{OA.72}$$
$$\Theta_n = \sigma.$$

Now, from (OA.70) and (OA.71), we obtain the stochastic and nonlinearised version of the New Keynesian Phillips Curve:

$$e^{(1+\phi)x_t} = 1 - \tau \left(E_t \left[\frac{d\mu_{P,t}}{dt} \right] - \delta \mu_{P,t} \right),$$

where

$$\tau = \frac{\theta}{\epsilon - 1} > 0,$$

and

$$x_t = o_t - o_{n,t}$$

is the output gap, with

$$o_t = \ln O_t,$$
$$o_{n,t} = \ln O_{n,t}.$$

Hence,

$$d\mu_{P,t} = \left[\delta \mu_{P,t} - \frac{1}{\tau} \left(e^{(1+\phi)x_t} - 1 \right) \right] dt + dM_{\pi,t}, \tag{OA.73}$$

where $E_t[dM_{\pi,t}] = 0$. The transversality condition from the firm's optimization problem is

$$\lim_{T\to\infty} E_t[e^{-\delta(T-t)}\mu_{P,T}] = 0. \tag{OA.74}$$

Solving (OA.73) backwards in time from the date-T, we obtain

$$\mu_{P,t} = \frac{1}{\tau} E_t \int_t^T e^{-\delta(u-t)} \left(e^{(1+\phi)x_u} - 1 \right) du + E_t[e^{-\delta(T-t)}\mu_{P,T}].$$

Letting $T \to \infty$ and imposing the transversality condition (OA.74) gives

$$\mu_{P,t} = \frac{1}{\tau} E_t \left[\int_t^\infty e^{-\delta(u-t)} \left(e^{(1+\phi)x_u} - 1 \right) du \right]. \tag{OA.75}$$

From the Feynman-Kac Theorem, we obtain the ode

$$0 = \frac{1}{2}\sigma_{x,t}^2 \mu_P''(x_t) + \mu_{x,t}\mu_P'(x_t) - \delta\mu_P(x_t) + \frac{1}{\tau}\left(e^{(1+\phi)x_t} - 1\right),$$

where

$$\mu_{x,t} = E_t\left[\frac{dx_t}{dt}\right],$$

$$\sigma_{x,t}^2 = E_t\left[\frac{(dx_t)^2}{dt}\right].$$

The equilibrium SDF is given by

$$\Lambda_t = e^{-\delta t} O_t^{-1}. \tag{OA.76}$$

Therefore, the time-t value of the claim to aggregate output flow is

$$P_{O,t} = O_t E_t\left[\int_t^\infty \frac{\Lambda_u}{\Lambda_t}\frac{O_u}{O_t}\right]$$
$$= \frac{O_t}{\delta}.$$

Time-t labor income flow is given by

$$L_t = \frac{W_t}{P_t} N_t = O_t N_t^{1+\phi}.$$

Therefore, the time-t real present-value of labor income flow is given by

$$P_{L,t} = E_t\left[\int_t^\infty \frac{\Lambda_u}{\Lambda_t} L_u du\right] = O_t E_t\left[\int_t^\infty e^{-\delta(u-t)} N_u^{1+\phi} du\right]$$
$$= O_t E_t\left[\int_t^\infty e^{-\delta(u-t)} N_u^{1+\phi} du\right]$$
$$= \left(1 - \frac{1}{\epsilon}\right) O_t E_t\left[\int_t^\infty e^{-\delta(u-t)} e^{(1+\phi)x_u} du\right]$$
$$= \left(1 - \frac{1}{\epsilon}\right) O_t \left(\tau\mu_{P,t} + \frac{1}{\delta}\right)$$

The present value of real profits is given by $P_{D,t}$, where

$$P_{D,t} + P_{L,t} = P_{O,t} = \frac{O_t}{\delta}.$$

Therefore,

$$P_{D,t} = \frac{1}{\epsilon}\frac{O_t}{\delta} - \tau\left(1 - \frac{1}{\epsilon}\right) Y_t \mu_{P,t} = \frac{Y_t}{\epsilon\delta}(1 - \delta\theta\mu_{P,t}). \tag{OA.77}$$

From, (OA.76), we obtain

$$\ln\Lambda = -\delta t - o_{n,t} - x_t,$$

and so, applying Ito's Lemma gives

$$d\ln\Lambda = -\delta dt - da_t - dx_t,$$

where $a_t = \ln A_t$. Hence

$$d\ln\Lambda = -\delta dt - \left(\mu - \frac{1}{2}\sigma^2\right) dt - \sigma dZ_t - dx_t,$$

We know that

$$E_t[d\ln\Lambda_t] = -\left(r_t + \frac{1}{2}\Theta_t^2\right) dt,$$

where r_t is the equilibrium risk-free rate and Θ_t is the equilibrium market price of risk. Hence

$$E_t[dx_t] = \left(-\delta - \mu + \frac{1}{2}\sigma^2 + r_t + \frac{1}{2}\Theta_t^2\right) dt$$

Therefore, we obtain

$$dx_t = \left(-\delta - \mu + \frac{1}{2}\sigma^2 + r_t + \frac{1}{2}\Theta_t^2\right) dt + dM_{x,t},$$

where $M_x = (M_x)_{t\in\mathcal{T}}$ is a stochastic process such that

$$E_t[dM_{x,t}] = 0.$$

This just means that M_x is a local martingale. It follows that

$$d\ln\Lambda_t - E_t[d\ln\Lambda_t] = -\sigma dZ_t - dM_{x,t}.$$

If we assume that M_x is a continuous stochastic process, then (via the Martingale Representation Theorem)

$$dM_t = \sigma_{x,t} dZ_{x,t}$$

where $Z_x = (Z_{x,t})_{t\in\mathcal{T}}$ is a standard Brownian motion and $\sigma_{x,t}$ is stochastic process (which has to be adapted to the relevant filtration). Therefore, by making the sole assumption that M is a continuous process, we obtain

$$dx_t = \left(-\delta - \mu + \frac{1}{2}\sigma^2 + r_t + \frac{1}{2}\Theta_t^2\right) dt + \sigma_{x,t} dZ_{x,t}.$$

If we assume that only fundamental shocks can be present, ruling out sunspot equilibria, then $Z_x = Z_t$, and so

$$dx_t = \left(-\delta - \mu + \frac{1}{2}\sigma^2 + r_t + \frac{1}{2}\Theta_t^2\right) dt + \sigma_{x,t} dZ_t.$$

It follows that

$$d\ln\Lambda_t - E_t[d\ln\Lambda_t] = -\Theta_t dZ_t.$$

where

$$-\Theta_t dZ_t = -\sigma dZ_t - \sigma_{x,t} dZ_t.$$

Therefore

$$\Theta_t = \sigma + \sigma_{x,t},$$

and

$$
\begin{aligned}
dx_t &= \left(-\delta - \mu + \frac{1}{2}\sigma^2 + r_t + \frac{1}{2}\left(\sigma^2 + 2\sigma\sigma_{x,t} + \sigma_{x,t}^2\right)\right)dt + \sigma_{x,t}dZ_t \\
&= \left(r_t - r_n + \frac{1}{2}\sigma_{x,t}\left(2\sigma + \sigma_{x,t}\right)\right)dt + \sigma_{x,t}dZ_t,
\end{aligned}
$$

where r_n is the natural rate of interest given in (OA.72). Inflation is locally risk-free, so the nominal interest rate is given by

$$
r_t^{\$} = r_t + \mu_{P,t},
$$

and so, given x_0, we obtain the stochastic dynamic investment-savings curve:

$$
dx_t = \left(r_t^{\$} - \mu_{P,t} - r_n + \frac{1}{2}\sigma_{x,t}\left(2\sigma + \sigma_{x,t}\right)\right)dt + \sigma_{x,t}dZ_t. \tag{OA.78}
$$

Returning to (OA.75), we see that $\mu_{P,t} = \mu_P(x_t)$, and so (OA.73) can be rewritten as

$$
d\mu_{P,t} = \left[\delta\mu_{P,t} - \frac{1}{\tau}\left(e^{(1+\phi)x_t} - 1\right)\right]dt + \mu_P'(x_t)\sigma_{x,t}dZ_t. \tag{OA.79}
$$

Showing the output gap is locally determinstic

We now show that if the nominal interest rate depends only on the output gap and inflation, then $\sigma_{x,t}$=0. If we apply Ito's Lemma to (OA.77), after some algebra, we see that

$$
\frac{dP_{D,t} + \Pi_t dt}{P_{D,t}} = (r_t + \sigma_{R,t}\sigma_{o,t})dt + \sigma_{R,t}dZ_t, \tag{OA.80}
$$

where

$$
\begin{aligned}
r_t &= \delta + \mu_{o,t} - \frac{1}{2}\sigma_{o,t}^2, \\
\mu_{o,t} &= \mu_a + \mu_{x,t}, \\
\sigma_{o,t} &= \sigma + \sigma_{x,t}, \\
\sigma_{R,t} &= \sigma_{o,t} - \frac{\delta\theta\mu_P'(x_t)\sigma_{x,t}}{1 - \delta\theta\mu_P(x_t)}.
\end{aligned}
$$

Observe also that

$$
E_t\left[\frac{dP_{D,t} + \Pi_t dt}{P_{D,t}} - r_t dt\right] = -E_t\left[\frac{d\Lambda_t}{\Lambda_t}\frac{dP_{D,t}}{P_{D,t}}\right],
$$

and so (OA.80) is consistent with no arbitrage. We can write

$$
\mu_{R,t} = r_t + \sigma_{R,t}\sigma_{o,t},
$$

but we also have (OA.69), and so

$$
\sigma_{R,t}^2 = \sigma_{R,t}\sigma_{o,t}.
$$

Therefore $\sigma_{R,t} = 0$ or $\sigma_{R,t} = \sigma_{o,t}$. The case $\sigma_{R,t} = 0$ is ruled out via (OA.68), and so $\sigma_{R,t} = \sigma_{o,t}$, which implies that $\mu_P'(x_t)\sigma_{x,t} = 0$. Therefore, at least one of the following two equations holds:

$$\mu_P'(x_t) = 0$$
$$\sigma_{x,t} = 0.$$

Observe that if $\mu_P'(x_t) = 0$, then the ode for inflation, , implies that

$$\mu_P'(x_t) = \frac{1}{\tau\delta}(e^{(1+\phi)x_t} - 1),$$

which can only be valid if x_t is constant, which is not the case, so we have a contradiction. Therefore $\sigma_{x,t} = 0$. Hence, if the nominal interest rate does not depend on any shocks in addition to Z, then x and $\mu_{P,t}$ evolve deterministically, and so (OA.78) and (OA.79) reduce to

$$\frac{dx_t}{dt} = (r_t^{\$} - \mu_{P,t} - r_n),$$
$$\frac{d\mu_{P,t}}{dt} = \delta\mu_{P,t} - \frac{1}{\tau}\left(e^{(1+\phi)x_t} - 1\right).$$

Endogenous sticky cash flows

If $\sigma_{x,t} = 0$, then (OA.67) reduces to

$$\Pi_t(f) = \left[\frac{1}{\epsilon} + \frac{\theta}{\epsilon}\left(\frac{d\mu_{P,t}}{dt} - \delta\mu_{P,t}\right)\right] O_t(f).$$

Therefore,

$$\begin{aligned}\ln\Pi_t(f) &= \ln\left[1 - (\epsilon - 1)(e^{(1+\phi)x_t} - 1)\right] + \ln O_t(f) - \ln\epsilon \\ &= -(\epsilon - 1)(1 + \phi)x_t + \ln O_t(f) - \ln\epsilon,\end{aligned}$$

where the last line follows by using an linear approximation. Applying Ito's Lemma gives

$$\begin{aligned}d\ln\Pi_t(f) &= da_t + [1 - (\epsilon - 1)(1 + \phi)]dx_t \\ &= da_t + [1 - (\epsilon - 1)(1 + \phi)](r_t^{\$} - \mu_{P,t} - r_n)dt.\end{aligned}$$

Now, if the nominal interest rate is given by

$$i_t = a_0 + a_1\mu_{P,t},$$

then we have

$$d\ln\Pi_t(f) = da_t - [(\epsilon - 1)(1 + \phi) - 1][a_0 - r_n + (a_1 - 1)\mu_{P,t}]dt.$$

We therefore see that if inflation increases by one percentage point, then real profit growth decreases by $[(\epsilon - 1)(1 + \phi) - 1](a_1 - 1)$ percentage points. Observe that $(\epsilon - 1)(1 + \phi) - 1 > 0$ if and only if

$$\epsilon > 1 + \frac{1}{1 + \phi}.$$

Defining nominal profit flow for firm f via

$$\Pi_t^{\$}(f) = P_t\Pi_t(f),$$

we obtain

$$d \ln \Pi_t^{\$}(f) = da_t - [(\epsilon - 1)(1 + \phi) - 1][a_0 - r_n]dt + \varphi \mu_{P,t} dt,$$

where

$$\varphi = 1 - [(\epsilon - 1)(1 + \phi) - 1](a_1 - 1)].$$

Therefore, if $\epsilon > 1 + \frac{1}{1+\phi}$ and $a_1 > 1$, then $\varphi < 1$, and so, expected nominal profit growth increases by less than one percentage point when inflation increases by one percentage point. ■