Sectoral Heterogeneity in Nominal Price Rigidity and the Origin of Aggregate Fluctuations*

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Abstract

We derive conditions under which heterogeneity in nominal price rigidity amplifies the aggregate fluctuations from idiosyncratic shocks and demonstrate its quantitative implications: (1) GDP volatility from sectoral productivity shocks doubles when nominal price rigidity is heterogeneous rather than homogeneous; (2) sectoral productivity shocks can jointly rationalize the volatility of sectoral prices, aggregate prices, and GDP, unlike aggregate productivity shocks; (3) heterogeneity in nominal rigidity changes the sectors from which aggregate fluctuations originate. We also discuss implications for aggregate price dynamics and the role of alternative monetary policy rules.

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I Introduction

How does nominal price rigidity affect the aggregate propagation of idiosyncratic productivity shocks? The role of nominal price rigidity is well-understood when productivity shocks are aggregate: It dampens the response of GDP and aggregate prices to these shocks and gives room for stabilization policy. We show that the "average" degree of nominal price rigidity operates in an identical way when productivity shocks are idiosyncratic, but its heterogeneity plays a distinct role.[1] A recent literature postulates idiosyncratic shocks as drivers of aggregate fluctuations, the "granular hypothesis"; under empirically plausible conditions, shocks to specific firms or industries, either the biggest ones or the most interconnected ones in the input-output network, can drive aggregate fluctuations. We show heterogeneity in nominal price rigidity may fundamentally change this result along several dimensions.

Theoretically, we find heterogeneous price rigidity can amplify or mute the aggregate volatility from sectoral shocks, depending on the exact interaction with other heterogeneous features of the economy. Quantitatively, however, heterogeneity in nominal price rigidity adds "potency" to the granular hypothesis in two ways. First, it provides amplification: The size of aggregate fluctuations originating from idiosyncratic shocks doubles relative to an otherwise identical economy with homogeneous nominal price rigidity. Second, it adds plausibility: Aggregate productivity shocks cannot *jointly* explain aggregate volatility and sectoral price volatility, whereas idiosyncratic productivity shocks can. Hence, unlike aggregate shocks, idiosyncratic shocks can provide a unified source of variation at the sectoral and aggregate levels. In addition, we find sectoral heterogeneity in nominal price rigidity changes the identity of sectors from which aggregate fluctuations originate. This result is important for stabilization purposes to identify the underlying reasons behind fluctuations. Heterogeneous price rigidity can even change the sign of aggregate fluctuations when they originate from disaggregated shocks.[2]

To establish these results, we study a multi-sector New-Keynesian model in which production requires labor and intermediate inputs and productivity shocks are idiosyncratic at the sectoral level. Sectors are heterogeneous in their production of final goods (in short, GDP), their input-output linkages, and the degree of their nominal price rigidity which we model in two alternative ways. A simple information friction allows us to build intuition through a series of analytical results in a transparent way by paralleling the analysis of Gabaix (2011) once adapted to sectoral shocks and of Acemoglu et al. (2012). Because no direct empirical counterpart exists

[1] A standard result in the literature is that one needs to take into account the cross-sectional dispersion of price rigidities when evaluating the role of the "average" degree of nominal price rigidity.

[2] This is a corollary of the distortion in the identity of the most important sectors for aggregate fluctuations and the idiosyncratic nature of sectoral shocks.

for this friction, we develop quantitative results modeling pricing frictions a la Calvo, which we calibrate to frequencies of price changes for 341 sectors computed from the micro data underlying the Producer Price Index (PPI) from the Bureau of Labor Statistics (BLS); we calibrate sectoral GDP and input-output linkages using BEA's Input-Output Tables. Our baseline setup assumes linear disutility of labor and a passive monetary policy rule. These assumptions add discipline to the analysis by shutting down the effect of active monetary policy and allowing the use of standard benchmarks in the literature. We later relax these assumptions, as we describe below.

The paper starts by revisiting key insights of the granularity literature by using "sectoral multipliers" as objects of analysis – the contribution of volatity of shocks in each single sector to the volatility of either GDP or aggregate prices. Up to a log-linear approximation, sectoral GDP multipliers in a frictionless economy only depend on sectoral GDP shares and the strength of the propagation of shocks through the input-output network. The inverse Leontief matrix weighted by the GDP share of all sectors captures this effect. In this setup, sectoral shocks can generate sizable aggregate fluctuations when some sectors have disproportionally large multipliers, either because they are large producers of final goods, intermediate goods or they are highly interconnected. For a given level of disaggregation, aggregate volatility increases in the cross-sectional dispersion of sectoral multipliers. As disaggregation goes to infinity, the rate of decay of GDP volatility is slower than the square root of the number of sectors, as predicted by the Central Limit Theorem, if sectoral multipliers follow a fat-tailed distribution.

Our main results emerge when we add nominal price rigidity. Our baseline passive monetary policy simplifies the analysis as the response of quantities only depends on prices. Sectoral multipliers now depend on sectors' own degree of price rigidity, but also on the whole distribution of nominal price rigidity, which affects the strength of shock propagation through the input-output network. We show sectoral multipliers are isomorphic to a frictionless economy, in which we focus on *effective* GDP shares, that is, sectors' GDP shares weighted by their own relative degree of price flexibility (the inverse of price rigidity), and the *effective* inverse Leontief matrix, that is, nodes in the input-output network weighted by their degree of price flexibility. Intuitively, as a sector has more flexible prices, its multiplier gets closer to that in a frictionless economy. Sectoral heterogeneity in price rigidity yields a gap with flex-price multipliers, which drives our results in the special case when production does not require intermediate inputs. In a production network, this intuition also applies after taking into account the whole distribution of price rigidity: Even if a sector has perfectly flexible prices, its multiplier depends on price stickiness of upstream and downstream sectors as well as the stickiness of their marginal costs because their intermediate inputs prices are also sticky.

We show idiosyncratic shocks can drive sizable GDP and aggregate price volatility even when all sectors have equal GDP shares and input-output network linkages – a case when idiosyncratic shocks have negligible aggregate effects in a frictionless economy. The mechanism at work is the following: The *average* degree of nominal price rigidity dampens effective sectoral multipliers just as for the case of aggregate shocks. However, the *cross-sectional distribution* of sectoral price rigidity determines whether some multipliers are disproportionally large in relative terms. For a given level of disaggregation, GDP and aggregate price volatility depend on the cross-sectional dispersion of effective sectoral multipliers, which in turn depends on the dispersion of nominal price rigidity, its covariance with sectors' GDP shares, and the price stickiness of suppliers of intermediate inputs. This result highlights the potential of heterogeneous price rigidity to yield a "frictional" origin of aggregate fluctuations conceptually different from size as in Gabaix (2011) or network centrality as in Acemoglu et al. (2012).

In this economy, higher orders of interconnection become less important for the aggregate propagation of shocks when nominal price rigidity increases. Moreover, heterogeneity in nominal price rigidity may affect the rate of decay of GDP and aggregate price volatility as the level of disaggregation goes to infinity by affecting the fat-tailedness of the distribution of effective sectoral multipliers. Overall, these results imply the empirical size distribution and input-output relationships per se are not sufficient to overcome the criticism of Lucas (1977) and Dupor (1999) or to design stabilization policy: It is possible that a calibrated flexible-price model generates a fat-tailed distribution of sectoral multipliers but no aggregate volatility due to sectoral shocks when the calibrated model also matches the distribution of price ridigities, and vice versa. This result goes beyond the violation of Hulten's Theorem highlighted by Baqaee and Farhi (2019) and Bigio and La'O (2020). These papers stress that total sales do not summarize the strength of network propagation, which also holds in our analysis. Yet, network propagation in the presence of frictions generically differs compared to a frictionless economy.

Next we relax several simplifying assumptions. Allowing for a finite Frisch elasticity opens new channels for the propagation of sectoral productivity shocks. Wages are now responsive to labor demand generating upstream effects. In short, the inverse Leontief matrix no longer captures the strength of network propagation regardless of whether nominal prices are rigid or not. However, heterogeneity of nominal price rigidity affects sectoral multipliers in the same way as in the baseline analysis. In another extension, we show results do not change substantively if active monetary policy follows a log-linear rule depending on GDP and/or aggregate prices. In this case, monetary policy responds to sectoral shocks in proportion to effective sectoral multipliers, so it only adds a scale effect. In general, such a scale effect is not the same for

GDP and aggregate prices, and it depends on the sectoral distribution of nominal price rigidity and the exact monetary policy rule. In fact, it may perfectly offset the dampening effect of average nominal price rigidity on GDP under a strict price level targeting rule. We also discuss implications of monetary policy reacting to the output gap or directly to sectoral shocks as well as adding other sources of sectoral heterogeneity. Our model, however, abstracts from capital and other sources of shocks because we do not consider them central for the mechanism we highlight in this paper and leave such extensions for future research.

Although we theoretically show that heterogeneity in nominal price rigidity has ambiguous effects, quantitatively, we find it adds "potency" to the granular hypothesis when we calibrate our model to the finest degree of disaggregation, 341 sectors: (i) GDP volatility increases by a factor of 1.8 to 2.4 relative to an otherwise identical economy with equal nominal price rigidity across sectors. (ii) Idiosyncratic shocks, but not aggregate shocks, can generate volatility of GDP, aggregate prices, and sectoral prices jointly consistent with U.S. data. The volatility of sectoral shocks has to be only one order of magnitude higher than the volatility of aggregate shocks to match macro volatility. The size of sectoral shocks is consistent with estimates from the micro price setting literature that argues large shocks are necessary to rationalize the moments on price setting observed in micro data. From a different angle, heterogeneity in nominal price rigidity substantially changes the identity of the most important sectors for GDP and aggregate price volatility relative to a frictionless economy (or one with identical nominal price rigidity across sectors). In a frictionless economy, the three most important sectors at the six-digits NAICS industry level for GDP volatility are "Retail Trading", "Real Estate" and "Wholesale Trading". However, when we calibrate sectors' average frequency of price changes to match U.S. data, the most important sector is "Monetary Authorities and Depositary Credit Intermediation",[3] followed by "Wholesale" and "Oil and Gas Extraction". The latter ranks 73^{rd} in terms of importance when prices are flexible. Overall, the three most important sectors account for 24% of GDP volatility when prices are flexible but 33% when prices are heterogeneously rigid across sectors. Finally, we confirm our theoretical results in this calibrated model.

Literature review. Sectoral heterogeneity in nominal price rigidity amplifies monetary non-neutrality, see, e.g., Carvalho (2006), Nakamura and Steinsson (2010), and Carvalho and Schwartzman (2015) but it also mutes the response of output and aggregate prices to aggregate productivity shocks.[4] We document a completely new role for this type of heterogeneity: it amplifies the response of aggregate variables to idiosyncratic productivity shocks relative to

[3]This sector includes a variety of financial intermediary services.

[4]In this literature, our setup is most similar to the study of monetary shocks in Pasten et al. (2020).

the case of homogeneous price rigidities. Although our focus is different, we share with this literature the use of micro data on prices (in our case, highly disaggregated sectoral prices) to discipline the strength of the friction and the volatility of idiosyncratic shocks (Midrigan (2011); Nakamura and Steinsson (2008)).

The granular hypothesis provides an answer to Cochrane (1994) pondering whether "we will forever remain ignorant of the fundamental causes of economic fluctuations." Seminal papers are Gabaix (2011) and Acemoglu et al. (2012), although an earlier literature has discussed the role of disaggregated shocks for aggregate fluctuations, such as Lucas (1977), Long and Plosser (1983), Dupor (1999) and Horvath (2000). Recent contributions are, for instance, Barrot and Sauvagnat (2016), Acemoglu et al. (2016), Boehm et al. (2019), Carvalho et al. (2021), Ozdagli and Weber (2016), Foerster et al. (2011), Hornstein et al. (2020) and Cox et al. (2020), while Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) synthesize this literature. Our paper shows that sectoral heterogeneity in nominal price rigidity adds potency to this hypothesis relative to an economy with homogeneous price ridigities.

Our paper is also related to a growing literature studying the effect of frictions in input-output networks but from different perspectives. For instance, Bigio and La'O (2020) study the aggregate effects of the tightening of financial frictions. Baqaee (2018) shows entry and exit of firms coupled with CES preferences may amplify the aggregate effect of disaggregated shocks. Baqaee and Farhi (2020) decompose reduced-form static wedges on aggregate productivity into "direct" and "allocative efficiency" effects. We share with them that Hulten's theorem does not apply in economies with frictions. Rubbo (2020) explores implications for the slope of the Phillips curve and, together with La'O and Tahbaz-Salehi (2020), for optimal monetary policy design. Our work provides analytical and quantitative ground for these explorations by highlighting the role of sectoral heterogeneity in nominal price rigidity for aggregate fluctuations. Our result of a muted role for monetary policy if it reacts to standard macro aggregates is a natural benchmark for these explorations.

II Model

Our multi-sector model has households supplying labor and demanding goods for final consumption, firms operating under monopolistic competition producing varieties of goods using labor and intermediate inputs, and a monetary authority. Sectors are heterogeneous in the amount of final goods they produce, input-output linkages, and the degree of nominal price rigidity.

A. Households

The representative household solves

$$\max_{\{C_t, L_{kt}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{k=1}^{K} g_k \frac{L_{kt}^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

subject to

$$\begin{aligned} \sum_{k=1}^{K} W_{kt} L_{kt} + \sum_{k=1}^{K} \Pi_{kt} + I_{t-1} B_{t-1} - B_t &= P_t^c C_t \\ \sum_{k=1}^{K} L_{kt} &\leq 1. \end{aligned}$$

In this closed economy model with no investment and no government spending, we can interpret C_t either as consumption or GDP and P_t^c as the consumer price index or the GDP deflator. L_{kt} and W_{kt} are labor employed and wages paid in sector $k = 1, ..., K$. Households own firms and receive net income, Π_{kt}, as dividends. Bonds, B_{t-1}, pay a nominal gross interest rate of I_{t-1}. Total labor supply is normalized to 1.

C_t aggregates from sectoral GDP, C_{kt}, and in turn from households' final demand for each good, C_{jkt}, according to

$$C_t \equiv \left[\sum_{k=1}^{K} \omega_{ck}^{\frac{1}{\eta}} C_{kt}^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

$$C_{kt} \equiv \left[n_k^{-1/\theta} \int_{\Im_k} C_{jkt}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

A continuum of goods indexed by $j \in [0, 1]$ exists with total measure 1. Each good belongs to one of the K sectors in the economy. Mathematically, the set of goods is partitioned into K subsets $\{\Im_k\}_{k=1}^{K}$ with associated measures $\{n_k\}_{k=1}^{K}$ such that $\sum_{k=1}^{K} n_k = 1$.[5] We allow the elasticity of substitution across sectors η to differ from the elasticity of substitution within sectors θ.

The first key ingredient of our model is the vector of weights $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]$ in equation (2). Households' sectoral demand

$$C_{kt} = \omega_{ck} \left(\frac{P_{kt}}{P_t^c} \right)^{-\eta} C_t \tag{4}$$

[5]The sectoral subindex is redundant, but it clarifies exposition. We can interpret n_k as sector size or gross output share.

determines the interpretation as sectoral GDP shares because in steady state, when all prices are identical, $\omega_{ck} \equiv \frac{C_k}{C}$ (variables without a time subscript denote steady-state levels.) Thus, the vector Ω_c satisfies $\Omega_c'\iota = 1$, where ι denotes a column-vector of 1s. Away from steady state, sectoral GDP shares depend on sectoral prices relative to the aggregate price index,

$$P_t^c = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{5}$$

Household demand for goods within a sector is given by

$$C_{jkt} = \frac{1}{n_k}\left(\frac{P_{jkt}}{P_{kt}}\right)^{-\theta} C_{kt} \text{ for } k = 1, ..., K. \tag{6}$$

Firms within a sector equally share the production of goods in steady state. Away from steady state, the gap between a firm's price, P_{jkt}, and the sectoral price, P_{kt}, distorts the demand for goods within a sector.

Sector k's price is defined as

$$P_{kt} = \left[\frac{1}{n_k}\int_{\Im_k} P_{jkt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{7}$$

The household first-order conditions determine labor supply and the Euler equation

$$\frac{W_{kt}}{P_t^c} = g_k L_{kt}^{\varphi} C_t^{\sigma} \text{ for all } k, j, \tag{8}$$

$$1 = \mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} I_t \frac{P_t^c}{P_{t+1}^c}\right]. \tag{9}$$

We implicitly assume sectoral segmentation of labor markets, so labor supply in equation (8) holds for a sector-specific wage $\{W_{kt}\}_{k=1}^{K}$. We choose the parameters $\{g_k\}_{k=1}^{K}$ to ensure a symmetric steady state across all firms.

B. Firms

A continuum of monopolistically competitive firms exists, each producing a single good. To facilitate exposition, firms are indexed by the good $j \in [0, 1]$ they produce and the sector, $k = 1, ...K$ they belong to. The production function is

$$Y_{jkt} = e^{a_{kt}} L_{jkt}^{1-\delta} Z_{jkt}^{\delta}, \tag{10}$$

where a_{kt} is an i.i.d. productivity shock to sector k with $\mathbb{E}[a_{kt}] = 0$ and $\mathbb{V}[a_{kt}] = v^2$ for all k, L_{jkt} is labor, and Z_{jkt} is an aggregator of intermediate inputs

$$Z_{jkt} \equiv \left[\sum_{k'=1}^{K} \omega_{kk'}^{\frac{1}{\eta}} Z_{jk} \left(k'\right)^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{11}$$

$Z_{kjt}(k')$ is the amount of goods firm jk demands as inputs at time t from sector k'.

The second key ingredient of our model is heterogeneity in aggregator weights $\{\omega_{kk'}\}_{k,k'}$. We denote these weights in matrix notation as Ω, satisfying $\Omega\iota = \iota$. The demand of firm jk for goods produced in sector k' is given by

$$Z_{jkt}\left(k'\right) = \omega_{kk'} \left(\frac{P_{k't}}{P_t^k} \right)^{-\eta} Z_{jkt}. \tag{12}$$

We interpret $\omega_{kk'}$ as the steady-state share of goods from sector k' in the intermediate input use of sector k, which determines the input-output linkages across sectors in steady state. Away from the steady state, the gap between the price of goods in sector k' and the aggregate price relevant for a firm in sector k, P_t^k, distorts input-output linkages

$$P_t^k = \left[\sum_{k'=1}^{K} \omega_{kk'} P_{k't}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad \text{for } k = 1, ..., K. \tag{13}$$

P_t^k uses the sector-specific steady-state input-output linkages to aggregate sectoral prices.

The aggregator $Z_{jk}(k')$ gives the demand of firm jk for goods produced in sector k'

$$Z_{jk}\left(k'\right) \equiv \left[n_{k'}^{-1/\theta} \int_{\Im_{k'}} Z_{jkt}\left(j', k'\right)^{1-\frac{1}{\theta}} dj' \right]^{\frac{\theta}{\theta-1}}. \tag{14}$$

Firm jk's demand for an arbitrary good j' from sector k' is

$$Z_{jkt}\left(j', k'\right) = \frac{1}{n_{k'}} \left(\frac{P_{j'k't}}{P_{k't}} \right)^{-\theta} Z_{jk}\left(k'\right). \tag{15}$$

In steady state, all firms within a sector share the intermediate input demand of other sectors equally. Away from steady state, the gap between a firm's price and the sectoral price index distorts the firm's share in the production of intermediate inputs. Our economy has $K+1$ different aggregate prices, one for the household sector and one for each of the K sectors. By contrast, the household sector and all sectors face unique sectoral prices.

The third key ingredient of our model is sectoral heterogeneity in price rigidity. For quantitative purposes, we model price rigidity a la Calvo with parameters $\{\alpha_k\}_{k=1}^K$ such that the pricing problem of firm jk is

$$\max_{P_{jkt}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s}\alpha_k^\tau \left[P_{jkt}Y_{jkt+s} - MC_{kt+s}Y_{jkt+s}\right]. \tag{16}$$

Marginal costs are $MC_{kt} = \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta} e^{-a_{kt}} W_{kt}^{1-\delta}\left(P_t^k\right)^\delta$ after imposing the optimal mix of labor and intermediate inputs

$$\delta W_{kt}L_{jkt} = (1-\delta)\, P_t^k Z_{jkt}, \tag{17}$$

and $Q_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$.[6] We assume the elasticities of substitution across and within sectors are the same for households and all firms. This assumption shuts down price discrimination across different customers, and firms choose a single price P_{kt}^*

$$\sum_{\tau=0}^{\infty} Q_{t,t+\tau}\alpha_k^s Y_{jkt+\tau}\left[P_{kt}^* - \frac{\theta}{\theta-1}MC_{kt+\tau}\right] = 0, \tag{18}$$

where $Y_{jkt+\tau}$ is the total production of firm jk in period $t+\tau$.

We define idiosyncratic shocks $\{a_{kt}\}_{k=1}^K$ at the sector level, and it follows the optimal price, P_{kt}^*, is the same for all firms in a given sector. Thus, aggregating among all prices within sector yields

$$P_{kt} = \left[(1-\alpha_k)\, P_{kt}^{*1-\theta} + \alpha_k P_{kt-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{19}$$

C. Monetary Policy, Equilibrium Conditions, and Definitions

The choice of monetary policy is crucial for the effects of shocks on the economy.[7] We document our quantitative results in Section V for different variations of a Taylor rule which, in its general expression, can be represented as

$$I_t = \frac{1}{\beta} I_{t-1}^{\rho_i}\left[\left(\frac{P_t^c}{P_{t-1}^c}\right)^{\phi_\pi}\left(\frac{C_t}{\phi^* C_t^* + (1-\phi^*)\, C}\right)^{\phi_y}\left(\frac{C_t}{C_{t-1}}\right)^{\phi_{gc}}\right]^{1-\rho_i}, \tag{20}$$

with a degree of monetary policy smoothing of ρ_i and responses to inflation, P_t^c/P_{t-1}^c, deviations of GDP from a weighted average between the frictionless and steady-state GDP levels, $C_t/[(\phi^* C_t^* + (1-\phi^*)C]$, and GDP growth. When we derive theoretical results in Section

[6] We choose Calvo pricing merely as an expository tool, and for computational reasons. We discuss details of choosing an endogenous price-adjustment technology at the end of Section III and in the Online Appendix.

[7] See Gali (1999) for productivity, or Woodford (2011) for government spending shocks.

III, we mainly focus on a passive monetary policy such that nominal aggregate demand $P_t^c C_t$ remains invariant to shocks.

Bonds are in zero net supply, $B_t = 0$, labor markets clear and goods markets clear such that

$$Y_{jkt} = C_{jkt} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{j'k't}\,(j,k)\,dj', \tag{21}$$

implying a wedge between gross output Y_t and GDP C_t. The Online Appendix contains the full set of log-linearized equations.

III Theoretical Results in a Simplified Model

We now introduce several simplifying assumptions to derive analytical results using first-order approximations. The goal of this section is to build intuition for the effect of heterogeneous price rigidities on the aggregate propagation of sectoral productivity shocks. These assumptions also allow us to embed the essence of the main results in Gabaix (2011) and Acemoglu et al. (2012) for frictionless economies as special cases. We first focus on the interaction of pricing frictions and GDP shares and later augment the analysis to include input-output linkages. Following the literature, we take two perspectives: for a given, finite degree of disaggregation and when the degree of disaggregation goes to infinity. At the end of this section, we discuss the implications of relaxing the simplifying assumptions for the theoretical results, which we also relax in the quantitative Section V.

A. Simplified Setup

We make the following simplifying assumptions, where all variables in lower cases denote log-linear deviations from steady state.

(i) Households have log utility, $\sigma = 1$, and linear disutility of labor, $\varphi = 0$. Thus,

$$w_{kt} = p_t^c + c_t; \tag{22}$$

that is, the labor market is integrated, labor supply is infinitely elastic and nominal wages are proportional to nominal GDP. This assumption shuts down the effect on wages from the response of sectoral labor demand to shocks. Relaxing this assumption introduces new channels for aggregate propagation, but the interplay of price rigidity with GDP shares and input-output linkages remains fundamentally unchanged.

(ii) Monetary policy targets steady state nominal GDP

$$p_t^c + c_t = 0. \tag{23}$$

This monetary policy rule has two advantages for our analysis. First, assumptions (i) and (ii) combined keep wages invariant at the steady state level. Second, monetary policy affects aggregate prices and quantities just like in any model with nominal rigidities; this passive monetary policy allows us to abstract from this effect. When we relax this assumption below, we show that alternative monetary policy rules only affect the propagation of sectoral productivity shocks by a scaling factor if the rule is log-linear and responds to standard aggregate variables.

(iii) We replace Calvo price stickiness by a simple information friction: all prices are flexible, but with probability λ_k, a firm in sector k has to set its price before observing shocks. Thus,

$$P_{jkt} = \begin{cases} \mathbb{E}_{t-1}\left[P_{jkt}^*\right] & \text{with probability } \lambda_k, \\ P_{jkt}^* & \text{with probability } 1-\lambda_k, \end{cases} \tag{24}$$

where $\mathbb{E}_{t-1}$ is the expectation operator conditional on the $t-1$ information set. This modeling device is a simple way to capture sectoral heterogeneity in the responsiveness of prices to shocks.

(iv) Shocks are i.i.d. with $\mathbb{E}\left[a_{kt}\right] = 0$ and $\mathbb{V}\left[a_{kt}\right] = v^2$ for all $k = 1, ..., K$, so $\mathbb{E}_{t-1}\left[P_{jkt}^*\right] = P_{jk}^*$, the steady-state optimal reset price for firm j, k. Hence, the pricing problem of firms is static. $\mathbb{V}\left[a_{kt}\right] = v^2$ further allows us to isolate the role of nominal price rigidity by abstracting from sectoral heterogeneity in the volatility of shocks.

Solution The Online Appendix shows that under these assumptions, GDP solves

$$c_t = \chi' a_t, \tag{25}$$

where

$$\chi \equiv \left(\mathbb{I} - \Lambda\right)\left[\mathbb{I} - \delta\Omega'\left(\mathbb{I} - \Lambda\right)\right]^{-1}\Omega_c. \tag{26}$$

Λ is a diagonal matrix with price-rigidity probabilities $[\lambda_1, ..., \lambda_K]$ as entries, vector Ω_c and matrix Ω, respectively, are steady-state sectoral GDP shares and input-output linkages, and δ is the intermediate input share. In turn, the passive monetary policy implies

$$p_t^c = -c_t = -\chi' a_t. \tag{27}$$

Up to a first-order approximation, a linear combination of sectoral shocks describes the log-deviation of GDP and the aggregate price level. Thus, volatility of GDP and the aggregate price level are

$$v_c = v_p = v\sqrt{\sum_{k=1}^{K} \chi_k^2} = \|\chi\|_2 \, v, \tag{28}$$

where $\|\chi\|_2$ denotes the Euclidean norm of χ. We refer to χ as the *vector of sectoral multipliers* mapping the volatility sectoral shocks into the volatility of GDP and aggregate prices, which are the focus of this paper. In principle, sectoral multipliers are different for GDP and aggregate prices but they coincide under our assumptions in this section, which facilitates exposition.

B. Nominal Price Rigidity and Sectoral GDP

We start by abstracting from input-ouput linkages, setting $\delta = 0$, so the vector of sectoral multipliers χ in equation (26) solves

$$\chi = (\mathbb{I} - \Lambda)\,\Omega_c, \tag{29}$$

with elements $\chi_k = (1 - \lambda_k)\,\omega_{ck}$ for all k and $\omega_{ck} \equiv C_k/C$.

This case is a useful starting point as the steady-state sectoral contribution in consumption, interpreted as real sales of final goods or simply GDP, is the only source of heterogeneity in sectoral size. Despite its simplicity, this case already contains our main results for the role of nominal price rigidity. First, the average level of price rigidity dampens the response of GDP to sectoral productivity shocks, exactly as it does when shocks are aggregate. Second, the heterogeneity of price rigidity alters the main result of the granular hypothesis in Gabaix (2011): what is key for the aggregate propagation of idiosyncratic shocks is not sectoral size alone, but its convolution with nominal price rigidity. Hence, sales are no a longer sufficient statistic for the importance of sectors for aggregate volatility, which also applies to aggregate prices.

B.1 Homogeneous Nominal Price Rigidity and Sectoral GDP

We now formally introduce the dampening effect of the average level of price rigidity. In particular, we assume that $\lambda_k = \lambda$ for all sectors $k = 1, ..., K$.

Lemma 1 *When all prices have the same degree of rigidity, $\lambda_k = \lambda \; \forall k$ and no input-output linkages are present ($\delta = 0$)*

$$v_c = v_p = \frac{(1-\lambda)\,v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\,(C_k) + \overline{C}_k^2},$$

where $\overline{C}_k$ *and* $\mathbb{V}(\cdot)$ *are the sample mean and sample variance of* $\{C_k\}_{k=1}^{K}$.[8]

This lemma states that, for a given level of disaggregation, the cross-sectional dispersion of sectoral final goods sales determines the volatility of GDP and aggregate prices. In this context, homogeneous nominal price rigidity decreases volatility by a constant factor. This lemma shows that a standard result in the literature carries over to a multi-sector setting: The more flexible prices are, the more prices and quantities can adjust after a productivity shock. In the extreme case of fully rigid prices, no effect on aggregate volatility exists regardless of the volatility of sectoral productivity shocks up to a first-order approximation. Intuitively, as nominal price rigidity implies that real aggregate variables are demand-determined and aggregate nominal demand remains invariant to shocks due to assumption (ii), output only responds to the extent that aggregate prices respond. Thus, productivity shocks only generate variations in markups, which feeds into household income with second-order effects on GDP and aggregate prices, so they do not show up in a log-linear approximation around a zero inflation steady state.

Although nominal prices in this economy are rigid, the next proposition allows to formally introduce the main result of the granular hypothesis of Gabaix (2011), because flexible prices are just a special case of homogeneous nominal price rigidity across sectors.

Proposition 1 *If* $\delta = 0$*,* $\lambda_k = \lambda$ *for all* k*, and* $\{C_k\}_{k=1}^{K}$ *follows a power-law distribution with shape parameter* $\beta_c \geq 1$*, then*

$$v_c = v_p \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c,1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} v & \text{for } \beta_c = 1, \end{cases}$$

where u_0 *is a random variable independent of* K *and* v*.*

Proof. See Online Appendix. ■

Proposition 1 states that, under the assumption of no input-output linkages and homogeneous price rigidity across sectors, the Central Limit Theorem does not readily apply if the size distribution of sectors is Pareto fat-tailed. The rate of decay of GDP is slower than the standard rate, $K^{-1/2}$, and depends on the shape parameter β_c. Intuitively, at any degree of disaggregation, some sectors are disproportionally large in relative terms, so idiosyncratic shocks to these sectors do not cancel out with shocks to smaller sectors. Homogeneous nominal price rigidity across sectors is irrelevant for this result, because the shape of the distribution of sectoral multipliers χ only depends on β_c. However, the dampening effect in Lemma 1 of the average degree of price ridigidity arises.

[8]We define $\mathbb{V}(X_k)$ of a sequence $\{X_k\}_{k=1}^{K}$ as $\mathbb{V}(X_k) \equiv \frac{1}{K}\sum_{k=1}^{K}\left(X_k - \overline{X}\right)^2$.

B.2 Heterogeneous Nominal Price Rigidity and Sectoral GDP

We next consider the effect of heterogeneity in nominal price rigidity. First, we study the cross-sectional effect of heterogeneous price rigidity for a given degree of disaggregation.

Lemma 2 *When price rigidity is heterogeneous across sectors and no input-output linkages are present* ($\delta = 0$)*, then*

$$v_c = v_p = \frac{v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\left(\left(1-\lambda_k\right) C_k\right) + \left[\left(1-\overline{\lambda}\right) \overline{C}_k - \mathbb{COV}\left(\lambda_k, C_k\right)\right]^2},$$

where $\overline{\lambda}$ *is the sample mean of* $\{\lambda_k\}_{k=1}^K$ *and* $\mathbb{COV}(\cdot)$ *is the sample covariance of* $\{\lambda_k\}_{k=1}^K$ *and* $\{C_k\}_{k=1}^K$.[9]

Lemma 2 states the volatility of output and prices depends on the sectoral dispersion of the product of price flexibility and sector size and their covariance terms. While trivial statistically, this lemma points to the key insight of the paper: Heterogeneity of price rigidity has the power to increase or decrease the volatility of aggregate output and prices. Intuitively, nominal price rigidity creates a gap between the size of sectors (here depending only on sectoral GDP) and their *effective* sectoral multiplier adjusted by their heterogeneous capacity to propagate shocks (which in turn depends on their degree of price flexibility). Thus, sectors that contribute the most to the variance of GDP may not be those with the largest contributions to GDP.

With idiosyncratic shocks, it is even possible that the sign of aggregate fluctuations changes, for instance if the most important sectors in a flexible-price economy are experiencing positive productivity shocks while the most important sectors in an economy with nominal price rigidity are experiencing negative productivity shocks. This observation is important given the emphasis of the granularity literature on identifying the microeconomic origin of aggregate fluctuations. Depending on the cross-sectional dispersion of price rigidity, sectoral shocks may even generate sizable GDP volatility when all sectors have equal physical size. To see this point, consider $C_k = C/K$ for all k. Lemma 2 implies

$$v_c = v_p = \frac{v\left(1-\overline{\lambda}\right)}{K^{1/2}} \sqrt{\mathbb{V}\left(\frac{1-\lambda_k}{1-\overline{\lambda}}\right) + 1}. \tag{30}$$

Thus, for a given level of disaggregation, aggregate volatility is increasing in the cross-sectoral dispersion of nominal rigidity which is the only source of heterogeneity in sectoral multipliers. This result highlights the potential of heterogeneous price rigidity to be a "frictional"

[9] We define $\mathbb{COV}(X_k, Q_k)$ of sequences $\{X_k\}_{k=1}^K$ and $\{Q_k\}_{k=1}^K$ as $\mathbb{COV}(X_k, Q_k) \equiv \frac{1}{K}\sum_{k=1}^K \left(X_k - \overline{X}\right)\left(Q_k - \overline{Q}\right)$.

origin of aggregate fluctuations conceptually different from size as in Gabaix (2011) or network centrality as Acemoglu et al. (2012).

Lemma 2 is a result for finite samples, so variance and covariance terms are sample and not population statistics. Thus, conditions for large effects of sectoral shocks do not need to hold in the whole support of the sector size distribution; it is possible that price rigidity is important for GDP volatility only because of its correlation with size for the largest sectors.

Turning to a different angle, we now investigate the effect of nominal price rigidity on the rate of decay of volatility in GDP and aggregate prices as the economy becomes increasingly more disaggregated. Our first result is that heterogeneity in price rigidity does not have the power by itself to affect this rate of decay.

Proposition 2 *If $\delta = 0$, $C_k/C = 1/K$, and the distribution of λ_k satisfies*

$$\Pr\left[1-\lambda_k > y\right] = \frac{y^{-\beta_\lambda}-1}{y_0^{-\beta_\lambda}-1} \text{ for } y \in \left[y_0, 1\right], \beta_\lambda > 0,$$

then $v_c = v_p \sim v/K^{1/2}$.

Proof. Given the distribution of $(1-\lambda_k), \mathbb{E}[(1-\lambda_k)^2]$ exists, and the Central Limit Theorem applies. ■

If all sectors have equal size, the Central Limit Theorem applies regardless of the distribution of price rigidity. This result is due to the boundedness of price rigidity, which implies the second moment of the distribution of multipliers exists for any shape parameter. This proposition also holds if the distribution of price rigidity is independent of sectoral GDP. Intuitively, heterogeneous nominal price rigidity by itself does not have the power to generate a sufficiently large number of disproportionally large multipliers at any level of disaggregation. A similar result applies if all sectors have the same probability to have very sticky or flexible prices regardless of their flex-price multiplier, which is here determined only by sectoral GDP.

However, the next proposition shows this result does not generally hold.

Proposition 3 *If $\delta = 0$, $\{(1-\lambda_k)\, C_k\}_{k=1}^K$ follows a power-law distribution with shape parameter $\beta_{\lambda c} \geq 1$, then*

$$v_c = v_p \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_{\lambda c}, 1/2\}}} v & \text{for } \beta_{\lambda c} > 1 \\ \frac{u_0}{\log K} v & \text{for } \beta_{\lambda c} = 1, \end{cases}$$

where u_0 is a random variable independent of K and v.

The proof is identical to the proof of Proposition 1. The distribution of effective sectoral multipliers is key for the rate of decay of v_c and v_p as $K \to \infty$. Although bounded, price rigidity can affect this rate of decay, as the following example illustrates. Assume that C_k follows a Pareto distribution with shape parameter β_c and

$$\lambda_k = \max\left\{0, 1 - \phi C_k^{\mu}\right\} \tag{31}$$

for $\mu \in (-1, 0)$ and $\phi > 0$. This relationship implies sectors with larger GDP shares have more rigid prices. In this case, effective sectoral multipliers are Pareto distributed with shape parameter $\beta_{\lambda c} = \beta_c / (1 + \mu)$. It follows from Proposition 3 that price rigidity accelerates the rate of decay of v_c relative to homogeneous price rigidity (with the frictionless economy as a special case). Thus, it is possible the conditions for granularity are satisfied when nominal prices are flexible, but the Central Limit Theorem governs the rate of decay when prices are rigid.

We can show the converse case with $\mu \in (0, 1)$ in equation (31) through another example. Now any sector with GDP higher than $\phi^{-1/\mu}$ has fully flexible prices. In other words, price rigidity and sectoral GDP become independent for sectors larger than $\phi^{-1/\mu}$. As a result, effective sectoral multipliers follow a Pareto distribution with shape parameter $\beta_{\lambda c} = \beta_c / (1 + \mu)$ for $C_k < \phi^{-1/\mu}$ and β_c for $C_k \geq \phi^{-1/\mu}$. Directly applying the proof of Proposition 1, the rate of decay of v_c depends on $\beta_c / (1 + \mu)$ for $K < K^*$ and on β_c for $K \geq K^*$ with $K^* \equiv \left(x_0 \phi^{1/\mu}\right)^{-\beta_c}$. Intuitively, when the number of sectors is large enough, $K > K^*$, sectors with fully flexible prices dominate the upper tail of the sectoral GDP distribution, so the rate of decay of aggregate volatility is the same as in the frictionless economy. Although weaker, this example still applies if K^* is large or, in other words, when fully flexible sectors do not dominate in the upper tail of the sectoral GDP distribution. Empirically, in our quantitative analysis with 341 sectors, no sector with fully flexible prices exists.

Although these examples assume a deterministic relationship between sectoral GDP and price rigidity for expositional purposes, the same arguments apply when this relationship is stochastic. A stochastic relationship between price rigidity and sectoral GDP shares implies that price rigidity has an effect on the identity of the most important sectors for GDP volatility.

C. Nominal Price Rigidity and Input-Output Linkages

We now introduce input-output linkages by assuming that $\delta \in (0, 1)$ and shutting down the heterogeneity of sectoral GDP shares, that is, $C_k = C/K$. The vector of sectoral multipliers χ in equation (26) now solves

$$\chi \equiv \frac{1}{K} (\mathbb{I} - \Lambda) \left[\mathbb{I} - \delta \Omega' (\mathbb{I} - \Lambda)\right]^{-1} \iota. \tag{32}$$

This expression is identical to the one obtained in Acemoglu et al. (2012) when prices are fully flexible, so Λ is a matrix of zeros.[10] This direct link to their result is due to the simplifying assumptions in this section. If prices are fully flexible, the strength of shock propagation through the input-output network fully governs the distribution of sectoral multipliers. In general, however, sectoral multipliers embed a non-trivial interaction between price rigidity and input-output linkages. Hence, the inverse Leontief matrix does not capture the strength of shock propagation. It is instead the *effective* inverse Leontief matrix, which weights each node by its degree of nominal price rigidity, that determines the shock propagation. To study this interaction, we parallel the analysis in Acemoglu et al. (2012) and use a second-order approximation of the vector of sectoral multipliers

$$\chi \simeq \frac{1}{K}\left(\mathbb{I}-\Lambda\right)\left[\mathbb{I}+\delta\Omega'\left(\mathbb{I}-\Lambda\right)+\delta^{2}\left[\Omega'\left(\mathbb{I}-\Lambda\right)\right]^{2}\right].\iota \tag{33}$$

Again, we first study the case of homogeneous nominal price rigidity across sectors and then introduce heterogeneity. Conceptually, all our results for $\delta=0$ also hold here once adjusted for the interdependence of sectors.

C.1 Homogeneous Nominal Price Rigidity and Input-Output Linkages

When nominal price rigidity is identical across sectors, the approximation of sectoral multipliers becomes

$$\chi \simeq \frac{1}{K}\left(1-\lambda\right)\left[\mathbb{I}+\delta\left(1-\lambda\right)\Omega'\left(\mathbb{I}-\Lambda\right)+\left(\delta\left(1-\lambda\right)\right)^{2}\Omega'^{2}\right]\iota, \tag{34}$$

such that two measures of centrality, $d=\Omega'\iota$ and $q=\Omega'^{2}\iota$ govern the strength of shocks propagation, as in Acemoglu et al. (2012), with elements

$$\begin{aligned} d_k &\equiv \sum_{k'=1}^{K}\omega_{k'k}, \\ q_k &\equiv \sum_{k'=1}^{K}d_{k'}\omega_{k'k}, \end{aligned}$$

where d_k measures the importance of sectors as supplier of intermediate inputs ("first-order outdegree") and q_k measures the importance of sectors as supplier of large suppliers of intermediate inputs ("second-order outdegree").

[10] The only difference is $\chi'\iota=1/\left(1-\delta\right)$, because Acemoglu et al. (2012) normalize the scale of shocks such that the sum of sectoral multipliers equals 1.

Lemma 3 *If price rigidity is homogeneous across sectors, $\lambda_k = \lambda$ for all k, input-output linkages are present, $\delta \in (0,1)$, and all sectors have equal size $\Omega_c = \frac{1}{K}\iota$, then*

$$v_c = v_p \geq \frac{(1-\lambda)\,v}{K^{1/2}} \sqrt{\kappa + \delta'^2 \mathbb{V}(d_k) + 2\delta'^3 \mathbb{COV}(d_k, q_k) + \delta'^4 \mathbb{V}(q_k)}, \tag{35}$$

where $\kappa \equiv 1 + 2\delta' + 3\delta'^2 + 2\delta'^3 + \delta'^4$, $\delta' \equiv \delta(1-\lambda)$, $\mathbb{V}(\cdot)$ and $\mathbb{COV}(\cdot)$ are the sample variance and covariance statistics across sectors.

The inequality in this lemma holds because the exact solution for χ is strictly larger than its approximation. Note upstream effects through demand of intermediate inputs are shut down by our focus on GDP, as opposed to total production, and the simplifying assumptions (i) and (ii) imply wages do not respond to shocks.

Lemma 3 establishes two important insights. First, as in the case with no intermediate inputs, nominal price rigidity dampens GDP and aggregate price volatility. Second, nominal price rigidity penalizes more strongly the quantitative effect of second-order outdegrees than of first-order outdegrees on aggregate volatility. This result is important because the flexible-price analysis of Acemoglu et al. (2012) stresses second-order outdegrees contribute more to the aggregate fluctuations than first-order outdegrees. In general, interconnections of order τ are penalized by a factor $(1-\lambda)^\tau$. This result implies that homogeneous price rigidity can change the relative contribution of sectors to aggregate volatility in addition to the dampening effect.

The next proposition recovers the main result in Acemoglu et al. (2012) for frictionless economies in an economy with homogeneous price rigidity.

Proposition 4 *If $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, $\Omega_c = \frac{1}{K}\iota$, the distribution of outdegrees $\{d_k\}$, second-order outdegrees $\{q_k\}$, and the product of outdegrees $\{z_k = d_k q_k\}$ follow power-law distributions with respective shape parameters $\beta_d, \beta_q, \beta_z > 1$ such that $\beta_z \geq \frac{1}{2}\min\{\beta_d, \beta_q\}$, then*

$$v_c = v_p \geq \begin{cases} \frac{u_3}{K^{1/2}} v & \text{for } \min\{\beta_d, \beta_q\} \geq 2, \\ \frac{u_3}{K^{1-1/\min\{\beta_d,\beta_q\}}} v & \text{for } \min\{\beta_d, \beta_q\} \in (1,2), \end{cases}$$

where u_3 is a random variable independent of K and v.

Proof. See Online Appendix. ■

If the distribution of first- and second-order outdegrees follow Pareto distributions, the distribution with the fattest tail dominates the rate of decay of aggregate volatility. Similarly to Proposition 2, this proposition establishes that homogeneous price rigidity across sectors has no effect on the rate of decay, because multiplying the distribution of outdegrees with a constant does not change the fatness of the tails.

C.2 Heterogeneous Nominal Price Rigidity and Input-Output Linkages

Next, we allow for heterogeneous nominal price rigidity across sectors. The sectoral multipliers now depend on $\Omega' (\mathbb{I} - \Lambda) \iota$ and $[\Omega' (\mathbb{I} - \Lambda)]^2 \iota$ which we call the vectors of *effective first-order outdegrees* and *effective second-order outdegrees*, respectively

$$\begin{aligned} \widetilde{d}_k &\equiv \left(1 - \overline{\lambda}\right) \sum_{k'=1}^{K} \left(\frac{1 - \lambda_{k'}}{1 - \overline{\lambda}}\right) \omega_{k'k}, \\ \widetilde{q}_k &\equiv \left(1 - \overline{\lambda}\right) \sum_{k'=1}^{K} \left(\frac{1 - \lambda_{k'}}{1 - \overline{\lambda}}\right) \widetilde{d}_{k'} \omega_{k'k}, \end{aligned}$$

where $\widetilde{d}_k$ measures the importance of sectors as large suppliers of sectors with the most flexible prices. Similarly, $\widetilde{q}_k$ measures the importance of sectors as large suppliers of the most flexible sectors that are large suppliers of the most flexible sectors. We interpret these statistics as measures of *effective* centrality because they incorporate the effect of the whole cross-sectional distribution of nominal price rigidity on the capacity of sectors to pass-through shocks into quantities and prices. The analysis is isomorphic to the case with homogeneous nominal price rigidity, just reinterpreting the determinants of the strength of shock propagation in the network.

Our first result concerns aggregate volatility for a given finite degree of disaggregation.

Lemma 4 *If price rigidity and input-output linkages are heterogeneous across sectors, $\delta \in (0, 1)$, and all sectors have equal size $\Omega_c = \frac{1}{K} \iota$, then*

$$v_c = v_p \geq \frac{v}{K^{1/2}} \left[\begin{array}{c} \left(\frac{1}{K}\sum_{k=1}^{K} (1 - \lambda_k)^2\right) \left[\widetilde{\kappa} + \delta^2 \mathbb{V}\left(\widetilde{d}_k\right) + 2\delta'^3 \mathbb{COV}\left(\widetilde{d}_k, \widetilde{q}_k\right) + \delta'^4 \mathbb{V}\left(\widetilde{q}_k\right)\right] \\ -\left(\frac{1}{K}\sum_{k=1}^{K} (1 - \lambda_k)^2\right) \left[2\delta^2 \left(1 + \widetilde{\delta} + \widetilde{\delta}^2\right) \mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) + \delta^4 \mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right)^2\right] \\ + \mathbb{COV}\left((1 - \lambda_k)^2, \left(1 + \delta\widetilde{d}_k + \delta^2 \widetilde{q}_k\right)^2\right) \end{array} \right]^{\frac{1}{2}}, \tag{36}$$

where $\widetilde{\kappa} \equiv 1 + 2\widetilde{\delta} + 3\widetilde{\delta}^2 + 2\widetilde{\delta}^3 + \widetilde{\delta}^4$, $\widetilde{\delta} \equiv \delta\left(1 - \overline{\lambda}\right)$, $\overline{\lambda}$ is the sample mean of $\{\lambda_k\}_{k=1}^{K}$, $\mathbb{V}(\cdot)$ and $\mathbb{COV}(\cdot)$ are the sample variance and covariance statistics across sectors.

The cross-sectional variances of effective centrality measures $\widetilde{d}$ and $\widetilde{q}$ and their sample covariances with nominal price rigidity determine the degree of aggregate volatility in this lemma. Hence, our main result is identical to the one in the economy without intermediate inputs but slightly more intricate: Sectoral heterogeneity of nominal price rigidity affects the level of volatility of GDP and aggregate prices that sectoral shocks can generate as well as the identity of the sectors that contribute the most to such volatility. As in other lemmas, Lemma 4 refers to sample statistics, so the covariance term may be quantitatively important even if they hold only for the most central sectors. However, a feature of this lemma that differs from the economy without intermediate inputs is that the covariance terms do not summarize the whole effect of nominal price rigidity, because it also enters in the definition of effective centrality measures $\widetilde{d}$ and $\widetilde{q}$. Thus, it is possible that the covariance of $\widetilde{d}$ and $\widetilde{q}$ and nominal price rigidity is close to zero, yet the effect of nominal price rigidity on the volatility of GDP and aggregate prices is sizable.

We now explore the implications of heterogeneous nominal price rigidity for the rate of decay of GDP and aggregate price volatility as $K \to \infty$.

Proposition 5 *If* $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, *price rigidity is heterogeneous across sectors, the distribution of modified first-order outdegrees* $\left\{\widetilde{d}_k\right\}$, *modified second-order outdegrees* $\{\widetilde{q}_k\}$, *and the product* $\left\{z_k = \widetilde{d}_k \widetilde{q}_k\right\}$ *follow power-law distributions with respective shape parameter* $\widetilde{\beta}_d, \widetilde{\beta}_q, \widetilde{\beta}_z > 1$ *such that* $\widetilde{\beta}_z \geq \frac{1}{2}\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\}$, *then*

$$v_c \geq \begin{cases} \frac{u_4}{K^{1/2}} v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \geq 2, \\ \frac{u_4}{K^{1-1/\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\}}} v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \in (1,2), \end{cases}$$

where u_4 *is a random variable independent of* K *and* v.

This result is a restatement of Proposition 4 and resembles Proposition 3 in the context of production networks and hence, requires no separate proof. Although heterogeneous price rigidity interacts in a more complicated way with heterogeneous input-output linkages than sector GDP, the fundamental intuition is similar. If sectors with the most rigid (flexible) prices are also the most central in the *effective*, price-rigidity-adjusted production network such that $\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} > (<) \min\left\{\beta_d, \beta_q\right\}$, then GDP volatility may decay at a faster (slower) rate compared to an economy with homogeneous price rigidity or an economy in which price rigidity is independent of network centrality. Similar to the interaction with sectoral GDP in Lemma 4, heterogeneity in price rigidity does have an effect for a finite level of disaggregation. This result holds regardless of whether or not price rigidity affects the rate of decay.

Finally, price rigidity distorts the identity of sectors from which aggregate fluctuations originate when idiosyncratic shocks drive aggregate volatility through the network. Re-weighting sectoral shocks of potentially opposite signs can again easily change the sign of business cycles. What matters is the *effective* not the physical network structure.

D. Relaxing Simplifying Assumptions

We now discuss the implications of relaxing the simplifying modeling assumptions (i)-(iv).

Disutility of labor. Once we relax simplifying assumption (i) by allowing for an inverse-Frisch elasticity $\varphi > 0$, wages are partially determined by labor demand

$$w_{kt} = c_t + p_t^c + \varphi l_{kt}^d. \tag{37}$$

Labor markets become sectorally segmented and the response of labor demand to own sector's and others sectors' productivity shocks yield additional channels for propagation in the production network. In particular, shocks can now propagate upstream as labor demand depends on the demand for inputs from downstream sectors. The vector of sectoral multipliers now solves

$$\chi = \left[\mathbb{I} + (1-\delta)\,\varphi\Theta^{-1}\right](\mathbb{I} - \Lambda)\left[\mathbb{I} - \delta\Omega'(\mathbb{I}-\Lambda) - (1-\delta)\left(\theta_p' - \Omega_c\theta_c'\right)(\mathbb{I}-\Lambda)\,\Theta^{-1}\right]^{-1}\Omega_c, \tag{38}$$

where

$$\begin{aligned}
\Theta &\equiv (1+\delta\varphi)\,\mathbb{I} - (1+\varphi)\,\psi D\Omega D^{-1}, \\
\theta_c &\equiv \left[\mathbb{I} - \psi D^{-1}\Omega' D\right]\iota + \varphi\,(1-\psi)\,D^{-1}\Omega_c, \\
\theta_p &\equiv \left[\mathbb{I} - \psi D^{-1}\Omega' D\right]\iota\Omega_c' - \varphi\eta\left[\mathbb{I} - (1-\psi)\,D^{-1}\Omega_c\Omega_c'\right] \\
&\quad +\varphi\left[(\eta-1)\,\psi D^{-1}\Omega' D\Omega - \delta\Omega\right]
\end{aligned}$$

and D is a diagonal matrix with vector $[n_k]_{k=1}^K$ on its diagonal. As defined above, vector Ω_c and matrix Ω, respectively, are steady-state sectoral GDP shares and input-output linkages, Λ is a diagonal matrix with price-rigidity probabilities as entries, δ is the coefficient for intermediate inputs in the production function, and ψ is the steady-state share of intermediate inputs, Z/Y.

We detail the solution in the Online Appendix. The key difference relative to our analysis above is that now, when prices are fully flexible, the standard inverse Leontief matrix no longer captures the strength of aggregate propagation of sectoral productivity shocks. However, all the intuition for the role of sectoral heterogeneity in nominal price rigidity from the simplified setup still holds: It affects the aggregate propagation of sectoral shocks through its interaction with GDP shares and input-output linkages. A new ingredient is matrix D, which captures sectoral steady-state size. Thus, the same intuition as for the case of flexible prices applies after adjusting each component of sectoral multipliers by its degree of nominal price rigidity.

Active monetary policy. To relax assumption (ii), assume the following holds

$$c_t + p_t^c = m_t, \tag{39}$$

where m_t is simply interpreted as the nominal aggregate demand engineered by a time-varying, potentially state-dependent monetary policy. Then, under simplifying assumptions (iii) and (iv), GDP and aggregate prices solve

$$\begin{aligned} c_t &= \left[1 - (1-\delta)\,\chi'\iota\right] m_t + \chi' a_t, && (40) \\ p_t^c &= \chi' \left[(1-\delta)\,\iota m_t - a_t\right], && (41) \end{aligned}$$

where ι is a K-dimensional vector of ones and χ is the vector of multipliers which solves either equation (26) or equation (38) depending on whether simplifying assumption (i) holds. Two results are immediate from these expressions for c_t and p_t^c. First, the equality in volatility of GDP and aggregate prices we obtain in the main analysis only holds for passive monetary policy. Second, if monetary policy responds in a log-linear fashion to GDP c_t and/or aggregate prices p_t^c, then monetary policy just enters as a re-scaling factor. Under this assumption, monetary policy may mitigate the dampening effect of average price rigidity in Lemmas 1 and 3. But it cannot offset either the effect of heterogeneity in nominal rigidity on GDP and aggregate price volatility, or the distortion of the identity of sectors from which fluctuations originate.

To see this, consider the following arbitrary rule

$$m_t = \phi_c c_t + \phi_p p_t^c, \tag{42}$$

so the solution for c_t and p_t^c becomes

$$\begin{aligned} c_t &= \frac{\chi' a_t}{1 - \left[1 - (1-\delta)\,\chi'\iota\right] \frac{\phi_c - \phi_p}{1-\phi_p}}, \\ p_t^c &= \frac{1-\phi_c}{\phi_p - 1} c_t. \end{aligned}$$

This rule nests many alternatives, such as the passive monetary policy if $\phi_c = \phi_p = 0$ and price level targeting if $\phi_c = 1$ and $\phi_p = 0$. Price-level targeting implements the first best if shocks are aggregate: In this case, $a_t = \iota \times \widetilde{a}_t$ where $\widetilde{a}_t$ is the realization of the aggregate productivity shock, so $c_t = (1-\delta)^{-1}\,\widetilde{a}_t$ independent of the degree of nominal price rigidity.

For any rule with $\phi_p \neq 1$, all results we obtained in this section hold once adjusted by a constant factor. Such a re-scaling effect depends on the responsiveness of monetary policy to fluctuations in GDP and aggregate prices, captured by ϕ_c and ϕ_p, and the weighted average price rigidity, captured by $\chi'\iota$. This weighted average considers sectors' own price rigidity, the distribution of sectoral GDP shares and the shape of the production network adjusted by the degree of nominal price rigidity at each node. This re-scaling factor may offset the scale effect of average price rigidity and exactly offsets it for a strict price level target, $\phi_c = 1$ and $\phi_p = 0$.

The intuition for this re-scaling effect is that monetary policy, by reacting to standard macro aggregates in a log-linear fashion, implicitly responds to sectoral productivity shocks according to their effective sectoral multipliers. This result would differ for instance if monetary policy reacted to the gap of GDP with respect to a flex-price economy, in which case

$$
\begin{aligned}
c_t &= \frac{\chi' a_t - \left[1 - (1-\delta)\,\chi'\iota\right] \frac{\phi_c}{1-\phi_p} c_t^*}{1 - \left[1 - (1-\delta)\,\chi'\iota\right] \frac{\phi_c - \phi_p}{1-\phi_p}}, \\
p_t^c &= \frac{(1-\phi_c)\, c_t + \phi_c c_t^*}{\phi_p - 1},
\end{aligned}
$$

where $c_t^* \equiv \chi^{FB\prime} a_t$ is the first-best GDP. It would also not be the case if central banks observed the realizations of shocks, or if they responded to aggregates constructed using different sectoral weights. We do not pursue these alternatives here given the quantitative focus of this paper.

Alternative pricing friction, sources of heterogeneity, and persistence of sectoral shocks. In practical terms, replacing the pricing friction in simplifying assumption (iii) by Calvo pricing and/or introducing persistence of shocks adds serial correlation in the response of GDP and aggregate prices to shocks. Such serial correlation changes the mapping between the vector of multipliers χ and the volatility of GDP and aggregate prices relative to the simplified setup. Measures of volatility in our quantitative analysis below are adjusted to account for serial correlation. For instance, using the representation of the response of GDP to sectoral shocks,

$$
c_t = \sum_{\tau=0}^{\infty} \sum_{k=1}^{K} \rho_{k\tau} a_{kt-\tau}, \tag{43}
$$

we redefine GDP sectoral multipliers as

$$
\chi_k \equiv \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}, \tag{44}
$$

such that $v_c = \|\chi\|_2\, v$ still holds. We apply similar adjustment for aggregate price volatility.

We also argue that endogenous pricing frictions, for example as modeled as in Golosov and

Lucas Jr (2007), would not conceptually change our findings: the interplay of heterogeneity in the responsiveness of prices to shocks, GDP shares, and input-output linkages still affect the aggregate propagation of sectoral shocks. In such a setting, though, the responsiveness of prices, in addition of being sector-specific, would also depend on the magnitudes of shocks.[11] Because we focus on shocks with the potential of driving business cycles, but not abnormally large shocks, we consider exogenous pricing frictions informative for the goal of this paper. Extending our analysis to include endogenous pricing frictions is unfeasible due to the curse of dimensionality as fine disaggregation of sectors is essential for our quantitative analysis.

Finally, a parallel analysis to the one conveyed in this paper allows for alternative sources of heterogeneity across sectors that affect the responsiveness of markups or, more generally, the strength of sectoral multipliers in log-linear dynamics. For instance, if sectors were heterogeneous in the share of intermediate inputs in the production function, the vector of sectoral multipliers in (26) becomes

$$\chi \equiv (\mathbb{I} - \Lambda) \left[\mathbb{I} - \Omega' (\mathbb{I} - \Lambda) \Delta\right]^{-1} \Omega_c, \tag{45}$$

where Δ is a diagonal matrix with $[\delta_1, ..., \delta_K]$ in its diagnonal. We do not pursue this extension as it requires measuring these heterogeneities for a quantitative assessment. This measurability is actually one appeal of nominal price rigidity. Because it is difficult to argue flexible prices would remain exactly constant through time, a simple observable pricing moment, the frequency of price changes, captures the extent of the friction in a way consistent with the model.

IV Data

This section describes the data we use to construct the input-output linkages and sectoral GDP shares, and the micro price data we use to construct sectoral measures of price stickiness.

A. Input-Output Linkages and Sectoral Consumption Shares

The BEA produces input-output tables detailing the dollar flows between all producers and purchasers in the U.S. Producers include all industrial and service sectors, as well as household production. Purchasers include industrial sectors, households, and government entities. The BEA constructs the input-output tables using Census data that are collected every five years beginning in 1982. The input-output tables are based on NAICS industry codes. Prior to 1997, the input-output tables were based on SIC codes.

[11] Pure state-dependent pricing models encounter some difficulties to match the cross-sectional distribution of price changes, as shown by Costain and Nakov (2011) among others.

The input-output tables consist of two basic national-accounting tables: a "make" table and a "use" table. The make table shows the production of commodities by industry. The use table contains the uses of commodities by intermediate and final users. The rows in the use table contain the commodities, and the columns show the industries and final users that utilize them. We use the input-output tables for 2002 to create an industry network of trade flows. The BEA defines industries at two levels of aggregation: detailed and summary accounts. We use the detailed levels of aggregation to create industry-by-industry trade flows. We also use BEA data to calibrate sectoral GDP shares.

The BEA provides concordance tables between NAICS codes and input-output industry codes. We follow the BEA's input-output classifications with minor modifications to create our industry classifications. We account for duplicates when NAICS codes are not as detailed as input-output codes. In some cases, an identical set of NAICS codes defines different input-output industry codes. We aggregate industries with overlapping NAICS codes to remove duplicates. Online Appendix Section A.4 details how we combine make and use tables to construct the matrix of input-output linkages that consistently maps into our model.

B. Frequencies of Price Adjustments

We use the confidential microdata underlying the PPI from the BLS to calculate the frequency of price adjustment at the industry level.[12] The PPI measures changes in prices from the perspective of producers, and tracks prices of all goods-producing industries, such as mining, manufacturing, and gas and electricity, as well as the service sector. The BLS started sampling prices for the service sector in 2005. The PPI covers about 75% of the service-sector output. Our sample ranges from 2005 to 2011.

The BLS applies a three-stage procedure to determine the sample of goods. First, to construct the universe of all establishments in the U.S., the BLS compiles a list of all firms filing with the Unemployment Insurance system. In the second and third stages, the BLS probabilistically selects sample establishments and goods based on either the total value of shipments or the number of employees. The BLS collects prices from about 25,000 establishments for approximately 100,000 individual items on a monthly basis. The BLS defines PPI prices as "net revenue accruing to a specified producing establishment from a specified kind of buyer for a specified product shipped under specified transaction terms on a specified day of the month." Prices are collected via a survey that is emailed or faxed to participating establishments.

[12] The data have been used before in Nakamura and Steinsson (2008), Goldberg and Hellerstein (2011), Bhattarai and Schoenle (2014), Gorodnichenko and Weber (2016), Gilchrist, Schoenle, Sim, and Zakrajšek (2017), Weber (2015), and D'Acunto, Liu, Pflueger, and Weber (2016), among others.

Individual establishments remain in the sample for an average of seven years until a new sample is selected to account for changes in the industry structure.

We calculate the frequency of price changes at the goods level, FPA, as the ratio of the number of price changes to the number of sample months. For example, if an observed price path is \$10 for two months and then \$15 for another three months, one price change occurs during five months, and the frequency is 1/5. We aggregate goods-based frequencies to the BEA industry classification.

The overall mean monthly frequency of price adjustment is 16.78%, which implies an average duration, $-1/\log(1-FPA)$, of 5.44 months. Substantial heterogeneity is present in the frequency across sectors, ranging from as low as 2.74% for the semiconductor manufacturing sector (duration of 35.96 months) to 96.47% for dairy products (duration of 0.30 months).

V Quantitative Analysis

This section provides a quantitative assessment of the theoretical results of Section III. Instead of assuming a static information friction, which has no direct empirical counterpart, we assume Calvo pricing which we can directly calibrate using average frequencies of price changes.

The main conclusions are: (1) all results in the simplified model of Section III also hold quantitatively; and heterogeneity in sectoral nominal price rigidity (2) amplifies the aggregate effect of sectoral productivity shocks, (3) distorts the identity of the most important sectors for aggregate fluctuations, (4) adds "potency" to the granular hypothesis relative to a model with homogeneous price ridigities across sectors, and (5) it add "plausibility" in the following sense. In our model, the required degree of volatility of sectoral shocks to generate sectoral price volatility consistent with U.S. data also generates GDP and aggregate price volatility broadly consistent with U.S. data. If shocks, instead, are aggregate and calibrated to match the empirical volatility of GDP, volatility of sectoral prices is much lower than its empirical counterpart. Thus, while aggregate shocks can generate aggregate fluctuations, our results suggest that only idiosyncratic sectoral shocks provide a unified source of variation at the aggregate and disaggregated levels.

A. Calibration

Our different model economies below use two alternative calibrations for the steady-state sectoral GDP shares, Ω_c, and three alternative calibrations for sectoral Calvo parameters. "Heterogeneous GDP shares" refers to Ω_c matching sectoral GDP shares from the input-output tables. "Homogeneous GDP shares" means that all sectors have equal contributions to GDP in steady state. "Heterogeneous nominal price rigidity" reflects the calibration of Calvo parameters to the industry-average frequencies of price adjustments. "Homogeneous nominal price rigidity" and "flex-price" refer to identical Calvo parameters across industries, equal to the sales-weighted average frequency in U.S. data and zero, respectively. Steady-state input-output linkages across sectors, Ω, match the U.S. input-output tables.

The 2002 BEA data contains 407 unique industries which we interpret as "sectors." We lose some sectors and end up with $K = 341$ sectors for three reasons. First, some sectors produce almost exclusively final goods, so the PPI microdata do not contain enough observations to compute reliable average frequencies of price adjustment. Second, the goods some sectors produce do not trade in a formal market, so the BLS has no prices to record. Examples of missing sectors are (with industry codes in parentheses) "Military armored vehicle, tank, and tank component manufacturing" (336992) or "Religious organizations" (813100). Third, the BEA data for some sectors are not available at the six-digit level.

Our calibrations are at a monthly frequency, so the discount factor is $\beta = 0.9967$. We show results for an elasticity of substitution across sectors $\eta = 2$. No estimates of this elasticity exist at the level of disaggregation we require. Atalay (2017) reports an average elasticity of substitution of .15 for 30 industries; presumably a higher level of disaggregation implies a higher elasticity of substitution across sectors. The elasticity of substitution within sectors is $\theta = 6$ in our baseline calibration, which implies a markup of 20%. We set $\delta = 0.5$ so the intermediate-input share in steady state is $\delta \times (\theta - 1)/\theta = 0.42$ in the baseline calibration, which matches the 2002 BEA data. In some exercises we ignore input-output linkages by setting $\delta = 0$.

We report results for values of the inverse-Frisch elasticity $\varphi = 0$, $\varphi = 1$ and $\varphi = 2$. We calibrate several alternative monetary policy rules: A Taylor rule reacting to contemporaneous GDP and inflation, respectively, with parameters $\phi_c = 0.33/12 = 0.0275$, $\phi_\pi = 1.34$ without interest rate smoothing ($\rho_i = 0$) and no weight on output growth as in Rudebusch (2002), a variation of the previous rule that responds to the output gap, a passive monetary policy targeting constant nominal GDP in steady-state as in Section III, price-level targeting, and two versions of an augmented Taylor rule, one with the GDP growth rate as an additional target and a parameter of $\phi_{gc} = 1.5$, and another with interest rate smoothing with a persistence parameter of 0.9 following Coibion and Gorodnichenko (2012).[13]

The quantitative assessment of our theoretical results focuses on the computation of sectoral multipliers. These multipliers are independent of the volatility of sectoral or aggregate shocks which is why we do not need to specify them. However, we also examine which magnitudes

[13] For price-level targeting, we by-pass indeterminacy problems by directly restricting the model to set inflation to zero instead of using an interest rate rule that could implement it.

of sectoral and aggregate volatility of productivity shocks are required to match the observed volatility of GDP and prices in Section *C.* below.

B. Quantitative Assessment on Theoretical Results

This section provides a quantitative evaluation of the results in Section III, which often depend on the exact interplay of the distributions of sectoral price rigidity, sectoral GDP shares, and input-output relationships across sectors. To facilitate comparison with Section III, we use a baseline calibration with inverse-Frisch elasticity $\varphi = 0$ and a monetary policy rule that targets constant nominal GDP. Subsequently, we relax these assumptions.

Aggregate Effects Section III shows heterogeneity in nominal price rigidity can shape the amplification of sectoral shocks to generate GDP and aggregate price volatility. Here, we show quantitatively that it generates a strong amplification mechanism relative to an otherwise identical economy with homogeneous price rigidities. Table 1 reports the volatility in the heterogeneous price rigidity economy relative to an identical but homogeneous price rigidity economy, with identical degrees of average price stickiness across economies. We see in row (1) and column (1) that sectoral productivity shocks generate 1.7 times higher GDP volatility for heterogeneous relative to homogeneous nominal rigidity when steady-state sectoral GDP shares and input-output linkages match their empirical counterparts and monetary policy targets nominal GDP. This amplification effect is 2.5 times when we turn off the heterogeneity in GDP shares ($\omega_{ck} = 1/K$ for all k) and 1.9 times without intermediate input demand. Consistent with Section III, the passive monetary policy rule implies these numbers are exactly the same for aggregate price volatility.

Section III also shows the volatility of GDP and aggregate prices differs under non-passive monetary policy rules. Keeping the assumption that the inverse-Frisch elasticity is $\varphi = 0$, GDP volatility is larger for heterogeneous versus homogeneous price rigidity for all active monetary policy rules in Table 1, while aggregate price volatility is smaller for a standard Taylor rule (column (2)) and trivially equal to zero for a strict price level targeting rule (column (3)). For monetary policy rules introducing inertia, such as the augmented Taylor rule with GDP growth (column (4)) or interest rate persistence (column (5)), the amplification effect for heterogeneous price rigidity are similar to those for the passive monetary policy rule. When monetary policy reacts to the output gap (column (6)), results for prices are similar to column (2) while output amplification is smaller but with similar magnitudes than other alternative rules.

Section III also documents the average degree of price rigidity yields a "dampening effect" on both GDP and aggregate price volatility relative to a flex-price economy. This effect is fully operative when monetary policy is passive, but it may be smaller for alternative monetary policy rules or even disappear for price level targeting. Table 2 reports the volatility of GDP and aggregate prices of the fully heterogeneous economy relative to an economy with flexible prices. These numbers are reported in percentage points as opposed to Table 1. Across columns and rows, Table 2 shows that the economy with heterogeneous nominal price rigidity across sectors generates volatility of only around 3% to 9% compared to a flex-price economy for any monetary policy rule except for strict price-level targeting. Under strict price-level targeting, the scale effect is largely mitigated and actually fully offset when intermediate inputs are ignored ($\delta = 0$). This results occurs because we use sectoral GDP shares as weights when we calculate the average frequency of price changes for the calibration.

Sectoral Multipliers Sectoral multipliers refer to the volatility of GDP and aggregate prices generated by productivity shocks of unit volatility in each single sector. A series of Lemmas in Section III show sectoral heterogeneity in nominal price rigidity changes the magnitude and relative importance of sectoral multipliers. A series of propositions show that it may also affect the shape parameters in the Pareto distribution of sectoral multipliers. In the following, we show that these effects are quantitatively sizable.

Because closed-form solutions are not available with Calvo pricing, we cannot use these lemmas to tease out the exact effect of heterogeneity in nominal price rigidity through its interaction with either GDP shares and input-output linkages. Yet, Table 3 reports the correlation between sectoral multipliers when prices are either flexible or heterogeneously rigid across sectors. Again, the inverse-Frisch elasticity is $\varphi = 0$. For GDP, the correlation of sectoral multipliers is between 42% to 68% across different monetary policy rules and slightly lower when computed only for the upper 10% tail in the distribution of sectoral multipliers in the flex-price economy. For aggregate prices, the correlation is between 34% and 83% across all sectors and between 46% and 83% when we only consider sectors in the upper 10%.

For instance, sector 211000 (Oil and Gas Extraction) ranks 73^{rd} when prices are flexible (accounting for 0.2% of GDP volatility) but 3^{rd} when prices are heterogeneously sticky (accounting for 9% of GDP volatility). The most important sector with heterogeneous nominal price rigidity is $52A000$ (Monetary Authorities and Depository Credit Intermediation), which accounts for 13% of total GDP volatility but only 4.4% when prices are flexible. Similarly, the second most important sector with sticky prices is 420000 (Wholesale Trading) which accounts for 11% GDP volatility when prices are heterogeneously sticky but 7% when prices are flexible. In turn, the three most important sectors when prices are flexible are $4A0000$ (Retail Trading), 531000 (Real Estate) and 420000 (Wholesale Trading), while they rank 5^{th}, 7^{th} and 2^{nd} when prices are heterogeneously sticky, respectively.

To provide an overview of the ranking effect across all sectors, Figures 1 shows scatterplots of the ranking of sectoral multipliers from calibrations with heterogeneously sticky and flexible prices for different assumptions on sectoral steady-state GDP shares and input-output linkages for GDP. The inverse-Frisch elasticity is $\varphi = 0$ and monetary policy follows the passive rule in all plots but results are similar for different monetary policy rules. If heterogeneity in price rigidity was irrelevant, all points in the scatter plots would align along the 45^o line. In the left figure, when both steady-state GDP shares and input-output linkages are heterogeneous and match our empirical targets, heterogeneity in price rigidity greatly affects the relative ranking of sectoral multipliers.

The middle and right plots decompose this result. The middle figure shows that dispersion in the ranking of sectoral multipliers is substantial in a calibration without intermediate inputs ($\delta = 0$). In the right figure, the difference in rankings is even larger when all sectors are equal in their GDP shares but input-output linkages match U.S. Input-Output tables. This latter result highlights the importance of intersectoral relations in determining the relevant aggregate degree of nominal price rigidity beyond a sector's own stickiness.

Finally, Table 4 reports estimates and standard deviations for the shape parameter in the Pareto distribution of sectoral GDP multipliers. In all cases, estimates are significant at the 5% level. When price rigidity interacts with both heterogeneity in sectoral GDP and input-output linkages (first column), the point estimate of the shape parameter is $\widehat{\beta} = 0.78$, which compares with $\widehat{\beta} = 1.20$ when prices are flexible. As we show in Section III, the rate of decay of aggregate volatility is $1/K^{1-1/\beta}$. Thus, the effect of sectoral heterogeneity in nominal price rigidity quantitatively increases the fatness of the upper tail of the sectoral multipliers by such an extent that GDP volatility would actually increase as the economy becomes more disaggregated. This strong effect does not hold in more plausible calibrations with positive inverse-Frisch elasticity below.[14]

Monetary Policy Our quantitative results largely confirm the main result in Section III regarding monetary policy rules. For simple log-linear monetary policy rules depending on GDP and aggregate prices, the exact monetary policy rule only has a re-scaling effect and does not change the relative importance of sectoral multipliers. This result no longer holds exactly if monetary policy responds to the output gap. Table 5 reports the correlation of sectoral multipliers when monetary policy follows the standard Taylor rule and all six alternative

[14]We do not report results for alternative monetary policy rules because they all deliver similar results.

monetary rules we study for a calibration in which GDP shares, input-output linkages and frequency of price changes all match empirical targets. The correlations of sectoral multipliers for both GDP and aggregate prices are all larger than 96%, even for a Taylor rule responding to the output gap, because a standard parametrization of a Taylor rule outs little weight on the output gap.

Elasticity of Labor Supply Departing from the baseline assumption of an inverse-Frisch elasticity $\varphi = 0$ opens new channels for the propagation of sectoral productivity shocks, partially due to upstream effects as Section III shows. Thus, heterogeneity in nominal price rigidity may have quantitatively different effects but it conceptually works in the same way. To assess the impact of these additional channels, we briefly go through the main results reported for an inverse-Frisch elasticity $\varphi = 2$ and passive monetary policy. The aggregate multiplier with heterogeneous nominal price rigidity is now 1.5 times the aggregate multiplier with homogeneous nominal price rigidity (for $\varphi = 0$, it is 1.7). The slight dampening occurs because the new forces of sectoral propagation with $\varphi = 2$ imply the "average stickiness" in an economy with heterogeneous nominal price rigidity does not correspond to the weighted average frequency of price changes that the calibration with homogeneous price rigidity targets. For the reason, the scale effect is larger relative to the calibration with $\varphi = 0$.

From a different angle, the correlation of sectoral multipliers when nominal rigidity is heterogeneous and homogeneous was 85% with $\varphi = 0$ and with $\varphi = 2$, it is 65%. However, heterogeneity in nominal price rigidity continues to have strong effects on the shape parameters of Pareto distributions of sectoral multipliers. The point estimate of the shape parameter is $\widehat{\beta} = 1.11$, which compares with $\widehat{\beta} = 1.22$ when prices are flexible and $\widehat{\beta} = 0.78$ when $\varphi = 0$. Now, the rate of decay with sticky prices is $1/K^{0.1}$ versus $1/K^{0.2}$ for flexible prices and now aggregate volatility indeed decays as the level of disaggregation increases, instead of exploding when $\varphi = 0$. The effect of sectoral heterogeneity in price rigidity on the shape parameter can be decomposed as follows. When we shut down intermediate inputs ($\delta = 0$), $\widehat{\beta} = 0.83$, which compares to $\widehat{\beta} = 1.02$ with flexible prices. If all sectors have equal GDP shares, $\widehat{\beta} = 2.10$, which compares to $\widehat{\beta} = 1.62$ with flexible prices.

Finally, it still holds that alternative monetary policies mainly enter as a scale effect; the correlation of sectoral multipliers is now 98% or higher versus the 96% or higher for $\varphi = 0$.

Alternative calibrations. Appendix Section A.5 shows results for alternative calibrations for a passive monetary policy rule. Column (1) in all tables reproduces results in column (1) of Tables 1 to 4. Column (2) shows the baseline calibration but with an inverse-Frisch elasticity $\varphi = 2$, column (3) shows the baseline calibration with $\varphi = 2$ and an elasticity of substitution within sectors $\theta = 11$ such that the steady-state markup is 10%, column (4) shows the baseline calibration with $\varphi = 2$, $\theta = 11$ and an elasticity of substitution across sectors of $\eta = 1$ and column (5) shows the baseline calibration with $\varphi = 2$, $\theta = 11$, $\eta = 1$ and persistence of idiosyncratic shocks $\rho_k = 0.5$. All results continue to hold qualitatively across these alternative calibrations.[15]

C. Amplification and Implied Volatility of Shocks

We now seek to provide an answer to the following question: Is the level of sectoral volatility shocks needed to generate plausible aggregate volatility consistent with the data? Our answer is "yes" as long as nominal price rigidity is heterogeneous across sectors and shocks are idiosyncratic. Although the model is arguably quite stylized, it is able to provide the correct order of magnitude in terms of fluctuations.

In particular, we now perform a number of exercises using three empirical targets: the standard deviations of real consumption expenditure, the standard deviation of the Consumer Price Index (CPI), and the mean of the standard deviation of subindexes of the PPI. All these series are available at the monthly frequency, seasonally adjusted and de-trended in logs using a HP filter with a smoothing parameter of 14,400. The index of real consumption expenditure comes from the BEA, the CPI and subindexes of the PPI come from the BLS. In particular, we compute as targets 0.56% for the standard deviation of real consumption expenditure, 0.4% for the CPI, and 2.01% for PPI sub-indexes, all between January 1985 and December 2005.

We first want to discuss a few of our empirical choices. Although we interpret C_t in the model as GDP because it equals total value added and the total real demand for final goods, empirically it better fits the definition of real consumption. Similarly, we interpret P_t^c here as the CPI, whereas in the theoretical model we refer to it as GDP deflator. In addition, because the calibration of the model is at the monthly frequency, we aim for empirical targets at the same frequency. But even the standard deviations of GDP and the GDP deflator during this sample period, once divided by $\sqrt{3}$ to approximate a monthly frequency, are quite similar at 0.57% and 0.28%, respectively. In addition, the PPI subindexes correspond to commodities and not industries, but they are defined at a similar level of disaggregation we use in the calibration. This is the reason why we use only the average of the sectoral standard deviation as empirical target; we also follow this approach to isolate the effect of heterogeneity in nominal price rigidity from heterogeneity in volatility of sectoral shocks. Finally, all series are HP filtered in logs to

[15] We do not report Table 5 for these alternative calibrations as results are almost identical.

get close to the log-linear deviations from steady state in the model. We focus on a sample between 1985 and 2005 to avoid special episodes, such as the Volcker disinflation and the Great Recession, while matching the sample period we use to calibrate Calvo parameters, GDP shares and input-output linkages.

We consider three exercises: First, when shocks are idiosyncratic at the sectoral level in a fully heterogeneous model, we vary their standard deviation to match the 0.56% standard deviation of HP-filtered log real consumption expenditures. Second, we vary the standard deviation when shocks are aggregate instead of sectoral. Third, we repeat the first exercise but impose homogeneous pricing frictions. All these targeting exercises use the model calibrations for an economy that matches the empirical steady-state distribution of GDP shares and input-output linkages. In addition, we set the inverse-Frisch elasticity to $\varphi = 1$ while monetary policy follows the augmented Taylor rule with persistence in the nominal interest rate following Coibion and Gorodnichenko (2012). The model with $\varphi = 1$ exhibits similar properties to calibrations with $\varphi = 2$ above, with the only exception being the point estimate of the shape parameter in the Pareto distribution of sectoral multipliers which is now $\widehat{\beta} = 1.02$, i.e., the rate of decay of GDP volatility is now close to $1/\log(K)$.

Table 6 summarizes results. The first column presents the empirical targets. The second column corresponds to model-generated moments when the volatility of sectoral productivity shocks is 18.9% such that the model with heterogeneous nominal price rigidity matches the 0.56% target for consumption. This calibration implies a standard deviation of aggregate prices of 0.63% and a mean of the standard deviations for sectoral prices of 1.58%. These numbers compare to 0.4% and 2% in the data, respectively. The third column shows results when shocks are aggregate instead of sectoral. Now, the required volatility of aggregate shocks is 3.55% to match the target. The implied standard deviations of aggregate and sectoral prices are 0.63% and 0.48%, respectively. Hence, when shocks are aggregate, a calibration consistent with GDP volatility generates too little volatility of sectoral prices (which is 2% in the data as the first column shows). In summary, our model generates slightly more volatility of aggregate prices than aggregate quantities, which is counterfactual although differences are small, while it generates volatility of sectoral prices close to the data only when shocks are sectoral.

The last column in Table 6 shows model-generated moments for the same volatility of sectoral shocks as in Column 2, but assumes nominal price rigidity is homogeneous across sectors and equal to its average level, which amounts to 18% after weighting all sectors by their relative size. The model generates volatility of aggregate prices of 0.43%, which is close to the 0.4% in the data, but the volatility of consumption and sectoral prices are now 0.35% and 1.28%, respectively. To make the model match the empirical volatility of aggregate consumption, the volatility of sectoral shocks must almost double to 30.3%. Alternatively, for the same volatility of 18.9% for sectoral shocks when price rigidity is heterogeneous across sectors, the homogeneous degree of price rigidity must increase a third from 18% to 24% to match the empirical volatility of aggregate consumption. These results are corollaries of our results above that heterogeneity in price rigidity represents a strong amplification mechanism for the aggregate propagation of sectoral shocks. In turn, the counterfactual prediction of the model generating stronger amplification for GDP than for aggregate prices is unrelated to whether nominal price rigidity is heterogeneous or homogeneous across sectors, or whether shocks are defined at the aggregate or sectoral level. Correcting it requires a more elaborate model whose ingredients are not related to the origin of shocks at the aggregate or idiosyncratic level.

VI Concluding Remarks

To date, the importance of idiosyncratic shocks for aggregate fluctuations in the presence of frictions, and pricing frictions in particular, remains largely unexplored. We study the effect of price rigidity on the potential of sectoral shocks to drive aggregate fluctuations. We do so theoretically and quantitatively in a 341-sector New Keynesian model with heterogeneity in sector size, sector input-output linkages, and output-price stickiness. Our analysis suggests price rigidity has direct relevance for the modeling and understanding of business cycles. The interaction between different heterogeneities also has important implications for the conduct of monetary policy because it changes the origins from which aggregate fluctuations arise.

A central bank that aims to stabilize sectoral prices of "big" or "central" sectors might make systematic policy mistakes if it does not take into account the "frictional" origin of aggregate fluctuations that heterogeneity in price stickiness generates. Overall, our results suggest that heterogeneity in nominal price rigidity is a quantitatively strong amplifier of the aggregate effect of idiosyncratic shocks. Our paper further documents that idiosyncratic shocks are able to rationalize fluctuations at the aggregate and sectoral level, whereas aggregate shocks cannot. Thus, our paper suggests that idiosyncratic shocks and not aggregate shocks could be the main driver of aggregate fluctuations in the data. Unfortunately, currently available data is not sufficiently rich for a full quantitative analysis. We leave this question for future work.

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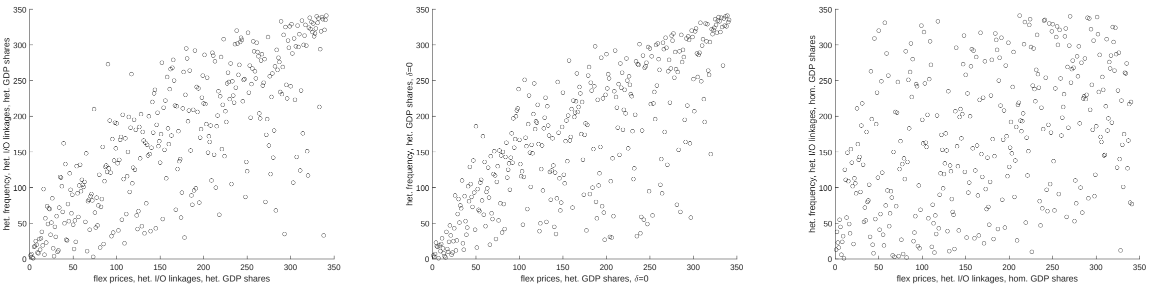
Figure 1: Ranking of Sectors: Heterogeneous versus Homogeneous Price Stickiness



The left figure plots the ranking of sectors based on their contributions to aggregate fluctuations for an economy with heterogeneous price stickiness across sectors (y-axis) and an economy with identical price stickiness for all sectors (x-axis). We assume heterogeneous GDP shares and input-output linkages calibrated to the US in both cases. The center figure plots the ranking of sectors for aggregate fluctuations originating from sectoral shocks for an economy with heterogeneous price stickiness across sectors (y-axis) and an economy with identical price stickiness for all sectors (x-axis). We assume heterogeneous GDP shares calibrated to the US and no intermediate inputs, $\delta = 0$*. The right figure plots the ranking of sectors for aggregate fluctuations originating from sectoral shocks for an economy with heterogeneous price stickiness across sectors (y-axis) and an economy with identical price stickiness for all sectors (x-axis). We assume equal GDP shares across sectors and input-output linkages calibrated to the US.*

Table 1: GDP and Aggregate Price Volatility, Heterogeneous vs. Homogeneous Price Stickiness

This table reports the aggregate monthly GDP volatility from sectoral shocks in a heterogeneous price rigidity economy relative to the volatility in an otherwise identical calibration with homogeneous nominal price rigidity. Ω_c *represents the vector of GDP shares,* Ω *the matrix of input-output linkages, and* δ *the intermediate input share. We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics. Monetary policy differs across columns: (1) Strict nominal GDP targeting, (2) standard Taylor rule, (3) strict price-level targeting, (4) augmented-Taylor rule with output growth, (5) augmented-Taylor rule with persistence in nominal interest rate, and (6) standard Taylor rule with output gap.*

	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
		GDP				
Het Ω_c, het Ω	1.7	2.7	2.4	1.8	2.0	1.8
Hom Ω_c, het Ω	2.5	4.6	3.0	2.7	2.7	1.7
Het Ω_c, δ=0	1.9	2.8	2.7	2.0	2.0	1.8
		Aggregate Prices				
Het Ω_c, het Ω	1.7	0.7	-	1.5	1.7	0.7
Hom Ω_c, het Ω	2.5	0.7	-	1.8	2.0	0.7
Het Ω_c, δ=0	1.9	0.7	-	1.8	1.7	0.7

Table 2: GDP and Aggregate Price Volatility, Heterogeneous Price Stickiness vs. Flexible Prices

This table reports the aggregate monthly GDP volatility from sectoral shocks in a heterogeneous price rigidity economy relative to the volatility in an otherwise identical calibration with flexible prices. Ω_c *represents the vector of GDP shares,* Ω *the matrix of input-output linkages, and* δ *the intermediate input share. We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics except for the flex-price calibration. Monetary policy differs across columns: (1) Strict nominal GDP targeting, (2) standard Taylor rule, (3) strict price-level targeting, (4) augmented-Taylor rule with output growth, (5) augmented-Taylor rule with persistence in nominal interest rate, and (6) standard Taylor rule with output gap.*

	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
		GDP				
Het Ω_c, het Ω	4.2%	3.1%	123.7%	3.7%	5.4%	5.6%
Hom Ω_c, het Ω	2.8%	2.0%	68.6%	2.5%	2.5%	6.9%
Het Ω_c, δ=0	5.9%	5.0%	100.4%	5.3%	5.4%	8.9%
		Aggregate Prices				
Het Ω_c, het Ω	4.2%	2.7%	-	4.3%	6.6%	2.8%
Hom Ω_c, het Ω	2.8%	2.3%	-	3.6%	3.6%	2.3%
Het Ω_c, δ=0	5.9%	5.0%	-	5.6%	6.6%	5.0%

Table 3: Correlation of Sectoral Multipliers, Heterogeneous Price Stickiness vs. Flexible Prices

This table reports correlations of sectoral multipliers between a specification with heterogeneous nominal price rigidity and one with flexible prices for the whole sample of sectors and for a subsample in the top 10% of sectoral multipliers in the flex-price economy. We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics except for the flexible prices calibration. Monetary policy differs across columns: (1) Strict nominal GDP targeting, (2) standard Taylor rule, (3) strict price-level targeting, (4) augmented-Taylor rule with output growth, (5) augmented-Taylor rule with persistence in nominal interest rate, and (6) standard Taylor rule with output gap.

	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
GDP						
Whole Sample	68%	50%	42%	66%	61%	68%
10% Upper Tail	62%	43%	34%	59%	44%	61%
Aggregate Prices						
Whole Sample	68%	83%	34%	71%	67%	83%
10% Upper Tail	62%	79%	47%	65%	51%	79%

Table 4: Estimated Shape Parameter in Sectoral Multipliers' Pareto Distributions

This table reports the estimates of the shape parameter for a Pareto distribution of sectoral multipliers with standard deviations in parentheses. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics except for the flex-price calibration.

	Het Stickiness	**Flex Prices**
Het Ω_c, het Ω	0.78	1.20
	(0.19)	(0.29)
Hom Ω_c, het Ω	1.76	2.17
	(0.43)	(0.53)
Het Ω_c, δ=0	0.75	1.02
	(0.18)	(0.25)

Table 5: Correlation of Sectoral Multipliers, Alternative Monetary Policy Rules

This table reports the correlations of sectoral multipliers for a specification with monetary policy following a standard Taylor rule and the five alternative monetary policy rules we study. In all cases, We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics. Monetary policy differs across columns: (1) Strict nominal GDP targeting, (2) standard Taylor rule, (3) strict price-level targeting, (4) augmented-Taylor rule with output growth, (5) augmented-Taylor rule with persistence in nominal interest rate, and (6) standard Taylor rule with output gap.

	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
GDP	97%	100%	99%	97%	99%	98%
Aggregate Prices	97%	100%	-	98%	96%	100%

Table 6: Data and Model-Generated Moments

This table reports the volatility of monthly series of real consumption expenditures, the consumer price index, and the mean of volatility of sectoral prices. The first column reports computation from HP-filtered U.S. data from Jan 1985 to Dec 2005. The second and third columns report model-generated moments when productivity shocks are assumed indiosyncratic at the sectoral level and aggregate, respectively. We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics.

	Data	**Idiosyncratic Shocks**	**Aggregate Shocks**	**Idiosyncratic Shocks, Hom Price Rigidity**
Real Consumption Expenditure	0.56%	0.56%	0.56%	0.35%
Consumption Price Index	0.40%	0.63%	0.63%	0.43%
Sectoral Prices	2.01%	1.58%	0.48%	1.28%

Online Appendix:
Sectoral Heterogeneity in Nominal Price Rigidity and the Origin of Aggregate Fluctuations

Ernesto Pasten, Raphael Schoenle, and Michael Weber

Not for Publication

A.1 Steady-State Solution and Log-linear System

A. Steady-State Solution

Without loss of generality, set $a_k = 0$. We show below conditions for the existence of a symmetric steady state across firms in which

$$W_k = W,\ Y_{jk} = Y,\ L_{jk} = L,\ Z_{jk} = Z,\ P_{jk} = P \text{ for all } j, k.$$

Symmetry in prices across all firms implies

$$P^c = P^k = P_k = P$$

such that, from equations (2), (3), (11), and (14) in the main body of the paper,

$$\begin{aligned} C_k &= \omega_{ck} C, \\ C_{jk} &= \frac{1}{n_k} C_k, \\ Z_{jk}\left(k'\right) &= \omega_{kk'} Z, \\ Z_{jk}\left(j', k'\right) &= \frac{1}{n_{k'}} Z_{jk}\left(k'\right). \end{aligned}$$

The vector $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]'$ represents steady-state sectoral shares in value-added C, $\Omega = \{\omega_{kk'}\}_{k,k'=1}^{K}$ is the matrix of input-output linkages across sectors, and $\aleph \equiv [n_1, ..., n_K]'$ is the vector of steady-state sectoral shares in gross output Y.

It also holds that

$$\begin{aligned} C &= \sum_{k=1}^{K} \int_{\Im_k} C_{jk} dj, \\ Z_{jk} &= \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{jk}\left(j', k'\right) dj' = Z. \end{aligned}$$

From Walras' law in equation (21) and symmetry across firms, it follows

$$Y = C + Z. \tag{A.1}$$

Walras' law also implies for all j, k

$$\begin{aligned} Y_{jk} &= C_{jk} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{j'k'}\left(j, k\right) dj', \\ Y &= \frac{\omega_{ck}}{n_k} C + \frac{1}{n_k} \left(\sum_{k'=1}^{K} n_{k'} \omega_{k'k} \right) Z, \end{aligned}$$

so $\aleph$ satisfies

$$\begin{aligned} n_k &= \left(1-\psi\right) \omega_{ck} + \psi \sum_{k'=1}^{K} n_{k'} \omega_{k'k}, \\ \aleph &= \left(1-\psi\right) \left[I - \psi \Omega'\right]^{-1} \Omega_c, \end{aligned}$$

for $\psi \equiv \frac{Z}{Y}$. Note by construction $\aleph' \iota = 1$, which must hold given the total measure of firms is 1.

Steady-state labor supply from equation (8) is

$$\frac{W_k}{P} = g_k L_k^{\varphi} C^{\sigma}.$$

In a symmetric steady state, $L_k = n_k L$, so this steady state exists if $g_k = n_k^{-\varphi}$ such that $W_k = W$ for all k. Thus, steady-state labor supply is given by

$$\frac{W}{P} = L^{\varphi} C^{\sigma}. \tag{A.2}$$

Households' budget constraint, firms' profits, production function, efficiency of production (from equation (17)) and optimal prices in steady state are, respectively,

$$CP = WL + \Pi \tag{A.3}$$

$$\Pi = PY - WL - PZ \tag{A.4}$$

$$Y = L^{1-\delta} Z^{\delta} \tag{A.5}$$

$$\delta WL = (1-\delta) PZ \tag{A.6}$$

$$sP = \frac{\theta}{\theta - 1} \xi W^{1-\delta} P^{\delta} \tag{A.7}$$

for $\xi \equiv \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta}$.

Equation (A.7) solves

$$\frac{W}{P} = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{1-\delta}}. \tag{A.8}$$

This latter result together with equations (A.5), (A.6), and (A.7) solves

$$\frac{\Pi}{P} = \frac{1}{\theta} Y.$$

Plugging the previous result in equation (A.4) and using equation (A.1) yields

$$\begin{aligned} C &= \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right] Y \\ Z &= \delta \left(\frac{\theta - 1}{\theta} \right) Y, \end{aligned} \tag{A.9}$$

such that $\psi \equiv \delta \left(\frac{\theta - 1}{\theta} \right)$.

This result and equation (A.7) gives

$$L = \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\delta}{1-\delta}} Y,$$

where Y from before together with equations (A.2), (A.9) and (A.8) solves

$$Y = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{(1-\delta)(\sigma+\varphi)}} \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{\frac{\delta \varphi}{(1-\delta)(\sigma+\varphi)}} \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\sigma}{\sigma+\varphi}}.$$

B. Log-linear System

B.1 Aggregation

Aggregate and sectoral consumption which we interpret as real sales of final goods, given by equations (2) and (3), are

$$\begin{aligned} c_t &= \sum_{k=1}^{K} \omega_{ck} c_{kt}, \\ c_{kt} &= \frac{1}{n_k} \int_{\Im_k} c_{jkt} dj. \end{aligned} \tag{A.10}$$

Aggregate and sectoral production of intermediate inputs are

$$\begin{aligned} z_t &= \sum_{k=1}^{K} n_k z_{kt}, \\ z_{kt} &= \frac{1}{n_k} \int_{\Im_k} z_{jkt} dj, \end{aligned} \tag{A.11}$$

where equations (11) and (14) imply that $z_{jk} = \sum_{r=1}^{K} \omega_{kr} z_{jk}(r)$ and $z_{jk}(r) = \frac{1}{n_r} \int_{\Im_r} z_{jk}(j', r) \, dj'$.

Sectoral and aggregate prices are (equations (5), (7), and (13)),

$$\begin{aligned} p_{kt} &= \int_{\Im_k} p_{jk} dj \text{ for } k = 1, ..., K \\ p_t^c &= \sum_{k=1}^{K} \omega_{ck} p_{kt}, \\ p_t^k &= \sum_{k'=1}^{K} \omega_{kk'} p_{k't}. \end{aligned}$$

Aggregation of labor is

$$\begin{aligned} l_t &= \sum_{k=1}^{K} l_{kt}, \\ l_{kt} &= \int_{\Im_k} l_{jkt} dj. \end{aligned} \tag{A.12}$$

B.2 Demand

Households' demands for goods in equations (4) and (6) for all $k = 1, ..., K$ become

$$\begin{aligned} c_{kt} - c_t &= \eta \left(p_t^c - p_{kt}\right), \\ c_{jkt} - c_{kt} &= \theta \left(p_{kt} - p_{jkt}\right). \end{aligned} \tag{A.13}$$

In turn, firm jk's demands for goods in equation (12) and (15) for all $k, r = 1, ..., K$,

$$\begin{aligned} z_{jkt}\left(k'\right) - z_{jkt} &= \eta \left(p_t^k - p_{k't}\right), \\ z_{jkt}\left(j', k'\right) - z_{jkt}\left(k'\right) &= \theta \left(p_{k't} - p_{j'k't}\right). \end{aligned} \tag{A.14}$$

Firms' gross output satisfies Walras' law,

$$y_{jkt} = (1 - \psi)\, c_{jkt} + \psi \sum_{k'=1}^{K} \int_{\Im_{k'}} z_{j'k't}\,(j, k)\, dj'. \tag{A.15}$$

Total gross output follows from the aggregation of equations (21),

$$y_t = (1 - \psi)\, c_t + \psi z_t. \tag{A.16}$$

B.3 IS and Labor Supply

The household Euler equation in equation (9) becomes

$$c_t = \mathbb{E}_t\left[c_{t+1}\right] - \sigma^{-1}\left\{i_t - \left(\mathbb{E}_t\left[p_{t+1}^c\right] - p_t\right)\right\}.$$

The labor supply condition in equation (8) is

$$w_{kt} - p_t^c = \varphi l_{kt} + \sigma c_t. \tag{A.17}$$

B.4 Firms

Production function:

$$y_{jkt} = a_{kt} + (1 - \delta)\, l_{jkt} + \delta z_{jkt} \tag{A.18}$$

Efficiency condition:

$$w_{kt} - p_t^k = z_{jkt} - l_{jkt} \tag{A.19}$$

Marginal costs:

$$mc_{kt} = (1 - \delta)\, w_{kt} + \delta p_t^k - a_{kt} \tag{A.20}$$

Optimal reset price:

$$p_{kt}^* = (1 - \alpha_k \beta)\, mc_{kt} + \alpha_k \beta \mathbb{E}_t \left[p_{kt+1}^* \right]$$

Sectoral prices:

$$p_{kt} = (1 - \alpha_k)\, p_{kt}^* + \alpha_k p_{kt-1}$$

B.5 Taylor Rule:

$$i_t = \phi_\pi \left(p_t^c - p_{t-1}^c \right) + \phi_c c_t$$

A.2 Solution of Key Equations in Section III

A. Solution of Equation (26)

Setting $\sigma = 1$ and $\varphi = 0$ in equation (A.17) yields

$$w_{kt} = c_t + p_t^c = 0, \tag{A.21}$$

where the equality follows from the assumed monetary policy rule, so equation (A.20) becomes

$$mc_{kt} = \delta p_t^k - a_{kt}. \tag{A.22}$$

Here, sectoral prices for all $k = 1, ..., K$ are governed by

$$\begin{aligned} p_{kt} &= (1 - \lambda_k)\, mc_{kt} \\ &= \delta\,(1 - \lambda_k)\, p_t^k - (1 - \lambda_k)\, a_{kt}, \end{aligned}$$

which in matrix form solves

$$p_t = -\left[\mathbb{I} - \delta\,(\mathbb{I} - \Lambda)\,\Omega\right]^{-1}(\mathbb{I} - \Lambda)\, a_t. \tag{A.23}$$

$p_t \equiv [p_{1t}, ..., p_{Kt}]'$ is the vector of sectoral prices, Λ is a diagonal matrix with the vector $[\lambda_1, ..., \lambda_K]'$ on its diagonal, Ω is the matrix of input-output linkages, and $a_t \equiv [a_{1t}, ..., a_{Kt}]'$ is the vector of realizations of sectoral technology shocks.

The monetary policy rule implies $c_t = -p_t^c$, so

$$c_t = \left(\mathbb{I} - \Lambda'\right)\left[\mathbb{I} - \delta\left(\mathbb{I} - \Lambda'\right)\Omega'\right]^{-1}\Omega_c' a_t. \tag{A.24}$$

which may be written in compact form as

$$c_t = \chi' a_t. \tag{A.25}$$

B. Solution of Equation (38)

When inverse-Frisch elasticity $\varphi > 0$, labor supply and demand now jointly determine wages such that

$$w_{kt} = c_t + p_t^c + \varphi l_{kt}^d \tag{A.26}$$

Thus, with monetary policy targeting $c_t + p_t^c = 0$, it no longer holds that sectoral productivity shocks have no effect on wages. Because the labor market is sectorally segmented, wages may differ across sectors. To see the sources of sectoral wage variation, we start from labor demand implied by the sectoral aggregation of the production function and the efficiency condition on the mix between labor and intermediate inputs,

$$l_{kt}^d = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k \right). \tag{A.27}$$

Conditioning on sectors' gross output y_{kt}, this equation shows that a positive productivity shock in sector k directly decreases demand for labor in the shocked sector by a_{kt} and indirectly in all sectors by the effect of the productivity shock on sector-specific aggregate prices of intermediate inputs, p_t^k. This latter effect is due to firms substituting labor for cheaper intermediate inputs, the price of which the steady-state I/O linkages of sectors with sector k determine.

To see the way that productivity shocks affect sectors' gross output y_{kt}, we use the log-linear expression for Walras law

$$y_{kt} = \frac{(1-\psi)\,\omega_{ck}}{n_k} c_{kt} + \frac{\psi}{n_k} \sum_{k'=1}^{K} n_{k'} \omega_{k'k} z_{k't}\,(k)\,, \tag{A.28}$$

such that sectoral gross output depends on households' demand as final goods and all sectors demand as intermediate inputs. The $\{n_k\}_{k=1}^{\infty}$ are the steady-state shares of sectors in aggregate gross output

$$n_k = (1-\psi)\,\omega_{ck} + \psi \sum_{k'=1}^{K} n_{k'} \omega_{k'k} \text{ for all } k = 1, ..., K. \tag{A.29}$$

$\psi \equiv Z/Y$ is the fraction of total gross output used as intermediate input in steady state.

Log-linear demands from households and sectors on goods produced in sector k are given by

$$\begin{aligned} c_{kt} &= c_t - \eta \left(p_{kt} - p_t^c\right), \\ z_{k't}(k) &= z_{k't} - \eta \left(p_{kt} - p_t^{k'}\right) \text{ for } k' = 1, ..., K. \end{aligned}$$

Thus, when sector k has a positive productivity shock, its demand from households and firms increases in the extent the price of sector k decreases relative to the price of goods produced in other sectors. This force pushes wages up in the shocked sector and down in all other sectors as households and firms decrease demand for all sectors with no positive shock. The strength of this effect depends on steady-state GDP shares for households' demand and steady-state input-output linkages.

Summing up, these effects create interdependence in the determination of wages. For $\varphi > 0$, wages solve

$$w_t = \Theta^{-1} \left[\theta_c c_t + \theta_p p_t - \varphi a_t\right], \tag{A.30}$$

where w_t is the vector of sectoral wages, and the parameters are

$$\begin{aligned} \Theta' &\equiv (1 + \delta\varphi)\,\mathbb{I} - (1 + \varphi)\,\psi D^{-1}\Omega' D; \\ \theta_c &\equiv \left[\mathbb{I} - \psi D^{-1}\Omega' D\right]\iota + \varphi\,(1 - \psi)\,D^{-1}\Omega_c; \\ \theta_p &\equiv \left[\mathbb{I} - \psi D^{-1}\Omega' D\right]\iota\Omega_c' - \varphi\eta\left[\mathbb{I} - (1 - \psi)\,D^{-1}\Omega_c\Omega_c'\right] \\ &\quad + \varphi\left[(\eta - 1)\,\psi D^{-1}\Omega' D\Omega - \delta\Omega\right], \end{aligned}$$

where $\mathbb{I}$ is a $K \times K$ identity matrix, D is a $K \times K$ matrix with vector $[n_k]_{k=1}^K$ on its diagonal, Ω_c is a column-vector of GDP shares $\{\omega_{ck}\}_{k=1}^K$, and Ω is the matrix $[\omega_{k'k}]_{k',k=1}^K$ with steady state input-output linkages across sectors.

This expression collapses to $w_{kt} = c_t + p_t^c$ when $\varphi = 0$. In the special case when $\delta = 0$ (i.e., no intermediate inputs), sectoral wages solve

$$w_t = (1 + \varphi)\,\iota c_t + \left[(1 + \varphi\eta)\,\iota\Omega_c' - \varphi\eta\mathbb{I}\right] p_t - \varphi a_t. \tag{A.31}$$

Although the interaction between price rigidity and sector size and I/O linkages is more involved, as these effects jointly create an interdependence of labor demand across sectors, our key insight from Section III remains: heterogeneity in price rigidity affects the propagation of idiosyncratic sectoral productivity shocks by affecting the responsiveness of sectoral prices to these shocks together with GDP shares and input output linkages.

The general solution for χ' when $\delta > 0$ now becomes

$$\chi' = \Omega_c' \left[\mathbb{I} - \delta \left(\mathbb{I} - \Lambda\right) \Omega - \left(1-\delta\right) \left(\mathbb{I} - \Lambda\right) \Theta^{-1} \left(\theta_p - \theta_c \Omega_c'\right)\right]^{-1} \left(\mathbb{I} - \Lambda\right) \left[\mathbb{I} + \left(1-\delta\right) \varphi \Theta^{-1}\right]. \quad \text{(A.32)}$$

Although now functional forms are more involved, sectoral price rigidity affects the aggregate propagation of sectoral productivity shocks through distorting the effect of the distribution of GDP shares and input-ouput linkages.

From a different angle, to further explore the effect of positive inverse-Frisch elasticity, consider the special case of no input-output linkages ($\delta = 0$), so

$$\chi' = \Omega_c' \left(\mathbb{I} - \Phi\right), \quad \text{(A.33)}$$

where Φ is a diagonal matrix with entries

$$\frac{1-\lambda_k}{1+\varphi\eta\left(1-\lambda_k\right)} \left[1 - \varphi\left(\eta - 1\right) \sum_{k'=1}^{K} \frac{\omega_{ck'}\left(1-\lambda_{k'}\right)}{1+\varphi\eta\left(1-\lambda_{k'}\right)}\right]^{-1}, \quad \text{(A.34)}$$

for $k = 1, ..., K$ on its diagonal. Note $\Phi = \Lambda$ when $\varphi = 0$. According to equation (28), the inverse-Frisch elasticity φ has two opposite effects on the capacity of price rigidity to generate aggregate volatility from sectoral productivity shocks. On the one hand, if sector k has more flexible prices, its demand responds by more to its own productivity shocks, so wages in the shocked sector respond by more. This effect is captured by the denominator of the term outside the brackets. On the other hand, the response of prices in the shocked sector has an effect on the demand of other sectors. This effect is captured by the term in brackets which is common to all sectors. Thus, in the absence of input-output linkages ($\delta = 0$), more elastic labor supply reduces the quantitative importance of price rigidity to generate fluctuations. However, because both effects operate through sectoral demand, the effect of φ depends on the elasticity of substitution across sectors, η. Quantitatively, empirical estimates suggest η is small (see Atalay (2017) and Feenstra, Luck, Obstfeld, and Russ (2018)).

A.3 Proofs

Most proofs below are modifications of the arguments in Gabaix (2011), Proposition 2, which rely heavily on the Levy's Theorem (as in Theorem 3.7.2 in Durrett (2013) on p. 138).

Theorem 5 (Levy's Theorem) *Suppose $X_1, ..., X_K$ are i.i.d. with a distribution that satisfies*

(i) $\lim_{x\to\infty} \Pr[X_1 > x] / \Pr[|X_1| > x] = \theta \in (0,1)$

(ii) $\Pr[|X_1| > x] = x^{-\zeta} L(x)$ *with* $\zeta < 2$ *and* $L(x)$ *satisfies* $\lim_{x\to\infty} L(tx)/L(x) = 1$.

Let $S_K = \sum_{k=1}^{K} X_k$,

$$a_K = \inf\{x : \Pr[|X_1| > x] \leq 1/K\} \text{ and } b_K = K\mathbb{E}\left[X_1 \mathbf{1}_{|X_1|\leq a_K}\right], \tag{A.35}$$

As $K \to \infty$, $(S_K - b_K)/a_K \xrightarrow{d} u$, *where* u *has a nondegenerated distribution.*

A. Proof of Proposition 1

In the following proofs, we go through three cases: first, when both first and second moments exist, second, when only the first moment exists, and third, when neither first nor second moments exist.

Generally, when there are no intermediate inputs ($\delta = 0$) and price rigidity is homogeneous across sectors ($\lambda_k = \lambda$ for all k),

$$\|\chi\|_2 = \frac{1-\lambda}{K^{1/2}\overline{C}_k}\sqrt{\frac{1}{K}\sum_{k=1}^{K} C_k^2}. \tag{A.36}$$

Given the power-law distribution of C_k, the first and second moments of C_k exist when $\beta_c > 2$, so

$$K^{1/2}\|\chi\|_2 \longrightarrow \frac{\sqrt{\mathbb{E}\left[C_k^2\right]}}{\mathbb{E}\left[C_k\right]}. \tag{A.37}$$

In contrast, when $\beta_c \in (1,2)$, only the first moment exists. In such cases, by the Levy's theorem,

$$K^{-2/\beta_c}\sum_{k=1}^{K} C_k^2 \xrightarrow{d} u_0^2, \tag{A.38}$$

where u_0^2 is a random variable following a Levy's distribution with exponent $\beta_c/2$ since $\Pr\left[C_k^2 > x\right] = x_0^{\beta} x^{-\beta_c/2}$.

Thus,

$$K^{1-1/\beta_c} \|\chi\|_2 \xrightarrow{d} \frac{u_0}{\mathbb{E}[C_k]}. \tag{A.39}$$

When $\beta_c = 1$, the first and second moments of C_k do not exist. For the first moment, by Levy's theorem,

$$\left(\overline{C}_k - \log K\right) \xrightarrow{d} g, \tag{A.40}$$

where g is a random variable following a Levy distribution.

The second moment is equivalent to the result above and hence

$$(\log K) \|\chi\|_2 \xrightarrow{d} u'. \tag{A.41}$$

B. Proof of Proposition 4

When $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, and $\Omega_c = \frac{1}{K}\iota$, we know

$$\begin{aligned}
\|\chi\|_2 &\geq \frac{1-\lambda}{K}\sqrt{\sum_{k=1}^{K}\left[1+\delta' d_k + \delta'^2 q_k\right]^2} \\
&\geq (1-\lambda)\sqrt{\frac{1+2\delta'+2\delta'^2}{K} + \frac{\delta'^2}{K^2}\sum_{k=1}^{K}\left[d_k^2 + 2\delta' d_k q_k + \delta'^2 q_k^2\right]}.
\end{aligned}$$

Following the same argument as in Proposition 2,

$$\begin{aligned}
K^{-2/\beta_d}\sum_{k=1}^{K} d_k^2 &\longrightarrow u_d^2, \\
K^{-2/\beta_q}\sum_{k=1}^{K} q_k^2 &\longrightarrow u_q^2, \\
K^{-1/\beta_z}\sum_{k=1}^{K} d_k q_k &\longrightarrow u_z^2,
\end{aligned}$$

where u_d^2, u_q^2 and u_z^2 are random variables. Thus, if $\beta_z \geq 2\min\{\beta_d, \beta_q\}$,

$$v_c \geq \frac{u_3}{K^{1-1/\min\{\beta_d,\beta_q\}}} v \tag{A.42}$$

where u_3^2 is a random variable.

A.4 Input-Output Linkages

We combine the make and use tables to construct an industry-by-industry matrix that details how much of an industry's inputs other industries produce. We use the make table ($MAKE$) to determine the share of each commodity each industry k produces. We define the market share ($SHARE$) of industry k's production of commodities as

$$SHARE = MAKE \oslash (\mathbb{I} \times MAKE),$$

where $\mathbb{I}$ is a matrix of ones with suitable dimensions and $\oslash$ represents the Hadamard division (element by element).

We multiply the share and use tables (USE) to calculate the dollar amount industry k' sells to industry k. We label this matrix revenue share ($REVSHARE$), which is a supplier industry-by-consumer industry matrix,

$$REVSHARE = SHARE \times USE.$$

We then use the revenue-share matrix to calculate the percentage of industry k inputs purchased from industry k', and label the resulting matrix $SUPPSHARE$

$$SUPPSHARE = [REVSHARE \oslash (\mathbb{I} \times USE)]'. \tag{A.43}$$

The input-share matrix in this equation is an industry-by-industry matrix and therefore consistently maps into our model.[1]

[1]Ozdagli and Weber (2016) follow a similar approach.

A.5 Alternative Calibrations

Table A.1: GDP and Aggregate Price volatility, Heterogeneous vs. Homogeneous Price Stickiness, Alternative Calibrations

This table reports the aggregate monthly GDP volatility from sectoral shocks in a heterogeneous price rigidity economy relative to the volatility in an otherwise identical calibration with homogeneous nominal price rigidity with passive monetary policy. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics. Alternative calibrations differ across columns: (1) Baseline as column (1) in Table 1, (2) baseline with inverted-Frisch elasticity $\varphi = 2$, (3) baseline with $\varphi = 2$ and elasticity of substitutio within sectors $\theta = 11$, (4) baseline with $\varphi = 2$, $\theta = 11$ and elasticity of substitution across sectors $\eta = 1$, (5) baseline with $\varphi = 2$, $\theta = 11$, $\eta = 1$ and persistence of idiosyncratic shocks $\rho_k = 0.5$.

	(1)	**(2)**	**(3)**	**(4)**	**(5)**
GDP and Aggregate Prices					
Het Ω_c, het Ω	1.7	1.5	1.6	1.8	1.3
Hom Ω_c, het Ω	2.5	1.8	1.9	2.4	2.1
Het Ω_c, δ=0	1.9	1.3	1.3	1.5	1.2

Table A.2: GDP and Aggregate Price Volatility, Heterogeneous Price Stickiness vs. Flexible Prices, Alternative Calibrations

This table reports the aggregate monthly GDP volatility from sectoral shocks in a heterogeneous price rigidity economy relative to the volatility in an otherwise identical calibration with flexible prices with passive monetary policy. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics except for the flex-price calibration. Alternative calibrations differ across columns: (1) Baseline as column (1) in Table 1, (2) baseline with inverted-Frisch elasticity $\varphi = 2$, (3) baseline with $\varphi = 2$ and elasticity of substitutio within sectors $\theta = 11$, (4) baseline with $\varphi = 2$, $\theta = 11$ and elasticity of substitution across sectors $\eta = 1$, (5) baseline with $\varphi = 2$, $\theta = 11$, $\eta = 1$ and persistence of idiosyncratic shocks $\rho_k = 0.5$.

	(1)	**(2)**	**(3)**	**(4)**	**(5)**
GDP and Aggregate Prices					
Het Ω_c, het Ω	4.2%	6.3%	6.8%	7.8%	19.4%
Hom Ω_c, het Ω	2.8%	2.5%	3.5%	7.6%	21.3%
Het Ω_c, δ=0	5.9%	12.2%	12.5%	12.4%	27.5%

Table A.3: Correlation of Sectoral Multipliers, Heterogeneous Price Stickiness vs. Flexible Prices

This table reports correlations of sectoral multipliers between a specification with heterogeneous nominal price rigidity and one with flexible prices for the whole sample of sectors and for a subsample in the top 10% of sectoral multipliers in the flex-price economy for passive monetary policy. We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics except for the flexible prices calibration. Alternative calibrations differ across columns: (1) Baseline as column (1) in Table 1, (2) baseline with inverted-Frisch elasticity $\varphi = 2$, (3) baseline with $\varphi = 2$ and elasticity of substitutio within sectors $\theta = 11$, (4) baseline with $\varphi = 2$, $\theta = 11$ and elasticity of substitution across sectors $\eta = 1$, (5) baseline with $\varphi = 2$, $\theta = 11$, $\eta = 1$ and persistence of idiosyncratic shocks $\rho_k = 0.5$.

	(1)	**(2)**	**(3)**	**(4)**	**(5)**
GDP and Aggregate Prices					
Whole Sample	68%	86%	88%	81%	89%
10% Upper Tail	62%	84%	85%	72%	85%

Table A.4: Estimated Shape Parameter in Sectoral Multipliers' Pareto Distributions, Alternative Calibraions

This table reports the estimates of the shape parameter for a Pareto distribution of sectoral multipliers with standard deviations in parentheses. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the GDP shares and input-output tables from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index from the Bureau of Labor Statistics except for the flex-price calibration. Monetary policy is assumed passive. Alternative calibrations differ across columns: (1) Baseline as column (1) in Table 1, (2) baseline with inverted-Frisch elasticity $\varphi = 2$, (3) baseline with $\varphi = 2$ and elasticity of substitutio within sectors $\theta = 11$, (4) baseline with $\varphi = 2$, $\theta = 11$ and elasticity of substitution across sectors $\eta = 1$, (5) baseline with $\varphi = 2$, $\theta = 11$, $\eta = 1$ and persistence of idiosyncratic shocks $\rho_k = 0.5$.

	(1)	**(2)**	**(3)**	**(4)**	**(5)**
Het Ω_c, het Ω	0.78	1.11	1.22	1.32	1.17
	(0.19)	(0.27)	(0.30)	(0.32)	(0.28)
Hom Ω_c, het Ω	1.76	2.10	2.12	2.33	1.90
	(0.43)	(0.51)	(0.51)	(0.57)	(0.52)
Het Ω_c, δ=0	0.75	0.83	0.84	0.79	1.02
	(0.18)	(0.20)	(0.20)	(0.19)	(0.25)