American Option Valuation: Implied Calibration of GARCH Pricing-Models

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First version: July 2009
This version: August 2009

Abstract

This article analyzes the issue of American option valuation when the underlying exhibits a GARCH-type volatility process. We propose the usage of Rubinstein's Edgeworth binomial tree (EBT) in contrast to simulation-based methods being considered in previous sudies. The EBT-based valuation approach makes an implied calibration of the pricing model feasible. By empirically analyzing the pricing performance of American index and equity options, we illustrate the superiority of the proposed approach.

JEL classification: G13, C22, C51

Keywords: American Options, GARCH, Implied Calibration, Edgeworth Binomial Tree

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I Introduction

Following the seminal work of Engle (1982) and Bollerslev (1986), GARCH-based financial time series models have been shown to be capable of capturing many stylized facts of financial time series, mainly the leptokurtosis of asset returns and volatility clustering.[1] Moreover, motivated by the weak empirical performance of the Black and Scholes (1973) option pricing formula, producing the well known volatility smile phenomenon,[2] GARCH models have been successfully introduced for option pricing. These approaches primarily build upon the local risk neutral valuation principle of Rubinstein (1976) and Brennan (1979), that was later generalized for a GARCH framework by Duan (1995).

Numerous empirical studies analyzing the pricing performance of European options for various GARCH specifications exist. Bollerslev and Mikkelsen (1999), Heston and Nandi (2000), Christoffersen and Jacobs (2004), Hsieh and Ritchken (2005), Christoffersen et al. (2006), Duan et al. (2006b), Badescu et al. (2008), and Christoffersen et al. (2008) have illustrated that GARCH models are well capable of pricing (European) S&P 500 index options. Studies for other markets include Härdle and Hafner (2000), Lehnert (2003), and Menn and Rachev (2005) for DAX index options, Duan and Zhang (2001) for Hang Seng index options, and Lehar et al. (2002) for FTSE 100 index options.

However, when it comes to the option pricing of underlyings with GARCH volatility, one usually has to rely on simulation-based valuation approaches.[3] If the options under consideration are of the American-type, a simulation-based approach is made more complicated by the fact that an optimal exercise policy has to be determined simultaneously throughout the option's life. As a result, all of the studies mentioned previously have only considered European options, although most exchange traded options are American and, thus, exercisable early.

The first study that considered the pricing of American style options in a GARCH volatility framework we are aware of is Stentoft (2005). Relying on historical maximum-likelihood parameter estimates and the least squares Monte Carlo (LSM)

[1] See, e.g., Bollerslev et al. (1992) for an excellent overview.

[2] See, e.g., Rubinstein (1994).

[3] One exception is the closed-form valuation formula for European options derived by Heston and Nandi (2000).

method developed by Longstaff and Schwartz (2001) to approximate the early exercise premium he considered the pricing of (American) S&P 100 index options and stock options. The major disadvantage of this approach comes from the fact that only historical information is used for parameter estimation, as an implied calibration of the options pricing model is not feasible in reasonable computer time for most real world applications when relying on simulation-based approaches. This is unfortunate, as it well known that using option implied parameter estimates improves the pricing performance significantly.[4].
In this paper, we therefore empirically investigate the potential of implied calibration of GARCH options pricing models when dealing with American options. To the best of our knowledge, no empirical study on this issue currently exists. To accomplish this goal, we apply an option valuation technique based on Edgeworth binomial trees, proposed by Rubinstein (1998) and Duan et al. (2003). This approach, resting on the ideas presented by Rubinstein (1994) and an Edgeworth expansion, allows us to build a recombing tree under general distribution functions. Duan et al. (2003) demonstrated how to adopt this approach in a GARCH framework, but did not test the model empirically.
Thus, the main contribution of our paper is twofold. First, we are the first to empirically study the pricing performance of an implicitly calibrated GARCH options pricing model when dealing with American options. As a benchmark, we consider the historical LSM approach suggested by Stentoft (2005). Second, we study the empirical option valuation performance of Rubinstein's Edgeworth tree in a GARCH volatility framework when pricing American style index and stock options that have not been conducted previously.
In the empirical study, we implement an NGARCH-based option pricing model for S&P 100 index options and options written on General Electric stocks. We estimate the structural parameters and value options in two different ways: (i) by employing the historical maximum likelihood approach only using asset returns and valuing options by the least squares Monte Carlo algorithm. This procedure was suggested by Stentoft (2005) when valuing American options under GARCH volatility and serves as benchmark in our study. (ii) we implicitly estimate the GARCH parameters directly under the risk-neutral measure by valuing the options by employing Rubinstein's Edgeworth binomial tree using

[4]See, e.g., Bakshi et al. (1997), Bates (1996), and also Barone-Adesi et al. (2008).

options prices and asset returns. The two considered approaches thus differ with respect to the pricing algorithm and the estimation methodology. Our results show that the latter approach yields superior pricing performance, which can be mainly attributed to the possibility of an implied calibration. However, as noted above, this estimation approach becomes infeasible when relying on simulation-based methods. We therefore provide the first empirical evidence on the suitability of the Edgeworth tree-based pricing approach.

The rest of the paper is organized as follows. Section II introduces GARCH option pricing in the European case, briefly describes the LSM-based pricing of American options, and introduces the pricing by Edgeworth binomial trees. Section III introduces the data set, whereas Section IV describes the estimation approaches. Section V provides the results of our empirical study. Section VI concludes.

II GARCH Option Pricing

In this section, we introduce the models employed in the empirical study. We first briefly introduce the general framework to price European options in a GARCH volatility context. Thereafter, we apply these ideas to the issue of pricing American options. We then present two different approaches. First, the simulation-based methodology which has been proposed and empirically tested by Stentoft (2005) and Stentoft (2008). Second, we introduce a tree based approach to value American style options allowing for an implicit calibration of the GARCH option pricing model.

A. GARCH Volatility Dynamics and the Pricing of European Options

GARCH volatility models were developed by Engle (1982) and Bollerslev (1986). The pricing of options in a GARCH volatility framework usually builds upon the local risk-neutral valuation relationship suggested by Duan (1995).

Based on the first evidence provided by Black (1976), there exists extensive empirical evidence of negative return innovations having a stronger impact on the conditional volatility than positive shocks of a similar magnitude. To capture this stylized fact, Engle and Ng (1993) proposed an asymmetric, but parsimonious extension of the standard GARCH dynamics. This type of GARCH model, frequently labeled NGARCH in the literature, has been proven to perform well for pricing equity index options, as demonstrated by Christoffersen and Jacobs (2004) and Hsieh and Ritchken (2005). Therefore, we follow their approach and consider an NGARCH-type volatility process. The logarithm of asset returns under the data-generating probability measure $\mathbb{P}$ is assumed to follow the dynamics

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = r - q + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \sqrt{h_t}\epsilon_t \tag{1}$$

$$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 h_{t-1}(\epsilon_{t-1} - \theta)^2 \tag{2}$$

$$\epsilon_t \overset{\mathbb{P}}{\sim} N(0,1), \tag{3}$$

where r and q denote the one-period, continuously compounded risk-free rate and dividend yield, respectively; h_t represents the conditional variance of the asset returns and ϵ_t a standard normal random variable. Conditional on the information set $\Im_{t-1}$ available at time $t-1$ the log-normality implies that

$$E^{\mathbb{P}}\left[\frac{S_t}{S_{t-1}}\middle|\Im_{t-1}\right] = e^{r-q+\lambda\sqrt{h_t}}. \tag{4}$$

Hence, we can interpret λ as the constant unit risk premium. The conditional variance follows an NGARCH process. We impose the typical restrictions $\beta_0 > 0$, $\beta_1 \geq 0$, and $\beta_2 \geq 0$ to guarantee a positive unconditional volatility. The parameter θ determines the so-called leverage effect: a positive value of θ induces a negative correlation between the asset returns and the conditional volatility. Duan (1995), generalizing the results of Rubinstein (1976) and Brennan (1979), derived the locally risk-neutral valuation relationship (LRNVR) when dealing with GARCH volatility dynamics. This is satisfied by a risk-neutral measure $\mathbb{Q}$ if

$$E^{\mathbb{Q}}\left[\frac{S_t}{S_{t-1}}\middle|\Im_{t-1}\right] = e^{r-q} \tag{5}$$

and

$$Var^{\mathbb{Q}}\left[\ln\left(\frac{S_t}{S_{t-1}}\right)\bigg|\Im_{t-1}\right] \overset{a.s.}{=} Var^{\mathbb{P}}\left[\ln\left(\frac{S_t}{S_{t-1}}\right)\bigg|\Im_{t-1}\right]. \tag{6}$$

Under the measure $\mathbb{Q}$, we can derive the risk-neutral asset return process as:

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = r - q - \frac{1}{2}h_t + \sqrt{h_t}\varepsilon_t \tag{7}$$

$$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 h_{t-1}(\varepsilon_{t-1} - \theta - \lambda)^2 \tag{8}$$

$$\varepsilon_t \overset{\mathbb{Q}}{\sim} N(0,1), \tag{9}$$

where $\varepsilon_t = \epsilon_t + \lambda$ is a standard normal random variable under the locally risk-neutral measure $\mathbb{Q}$.

When pricing (European) options in a GARCH-style framework, it is common practice to rely upon simulation-based valuation approaches. This is due to the fact that, on the one hand, the final risk-neutral distribution is not known in closed form, but, on the other hand, the discrete nature of the GARCH framework makes simulation based approaches straightforward to implement.

Using Equations (7) and (8) one can generate random draws of future risk-neutral stock prices as:

$$S_T = S_t e^{(T-t)(r-q) - \frac{1}{2}\sum_{s=t+1}^{T} h_s + \sum_{s=t+1}^{T}\sqrt{h_s}\varepsilon_s}, \tag{10}$$

to be able to approximate the risk-neutral distribution at the option's maturity. This enables us to calculate an estimate of the value V_t of a European option at time t as follows:

$$V_t = \frac{1}{M}\sum_{m=1}^{M} e^{-r(T-t)} \max\{0, d \cdot (S_T(m) - X)\}, \tag{11}$$

where M represents the total number of stock paths, $S_T(m)$ the risk-neutral value of the underlying at maturity of path m, X is the exercise price, and d is a dummy variable equaling one if we price a call option and minus one if the contract is a put.

B. Pricing American Options with GARCH Volatility by Simulation

The valuation of American options is not as straightforward as the valuation of European options. Due to the American feature, one does not only need the risk-neutral distribution of the underlying at maturity, but also has to simultaneously determine an optimal early exercise strategy. This boils down to comparing the value of immediate exercise with the continuation value of the option at every possible exercise point. Recently, Stentoft (2005) suggested applying the least squares Monte Carlo methodology developed by Longstaff and Schwartz (2001) to value American options in the case of the underlying features a GARCH volatility process. This approach has the appealing characteristic that it builds upon the simulation approach and is implemented by adding simple OLS regressions. More specifically, starting at the maturity of the option contract and working backwards, one estimates the path-specific continuation value at every possible exercise point by regressing discounted cash-flows on a constant, as well as some basis functions of the current values of the underlying and the volatility levels. Having approximated the continuation value, one can easily decide whether it is optimal to exercise the option or not. For a more detailed presentation of the algorithm and an analysis regarding different parameters, such as the number of simulation paths or different kinds of basis functions, we refer to Longstaff and Schwartz (2001), Moreno and Navas (2003), and Stentoft (2004).

In our empirical implementation of the LSM-algorithm, we use a total of 100,000 asset paths and assume one exercise possibility per day. We consider the first three Laguerre polynomials for the underlying and the volatility, including the respective cross terms. We model the local variance starting at the unconditional variance implied by the maximum likelihood based GARCH parameters 252 trading days before the valuation date and model the evolution of the conditional volatility according to the volatility updating rule. Furthermore, we make use of antithetic random numbers and the empirical martingale method of Duan and Simonato (1998) to enhance the computational efficiency.

C. Pricing American Options with GARCH Volatility by Binomial Trees

When working in a constant volatility environment, it is common practice to apply lattice based approaches for valuing American options. This approach is used because it can easily determine the optimal exercise strategy. However, the inherent path dependence of GARCH processes implies that standard lattice based approaches grow exponentially in the number of discrete time steps.

Building on the ideas of Jarrow and Rudd (1982), Rubinstein (1998) suggested the usage of an Edgeworth expansion to construct a so-called Edgeworth binomial tree (EBT) for the pricing of American options. This method mitigates the drawback of exponentially growing complexity. The density of a standard binomial distribution is transformed into a density with a mean of zero, a variance of one, and the desired skewness and kurtosis. Thus, only the first four moments of the cumulative risk-neutral asset returns are necessary, which can be analytically approximated.

Based on this modified density, Rubinstein (1998) suggested to model the entire stochastic return process using the implied binomial tree approach presented in Rubinstein (1994). In the following, we briefly describe the tree building procedure for the EBT.[5] Consider an underlying with price S_t; the tree is built to price an option with maturity date T; the time period to maturity is denoted $\tau = T - t$; the cumulative return and annualized volatility are given by $R_T = \ln(S_T/S_t)$ and $\sigma = \sqrt{Var[R_T]/\tau}$. Thus, the standardized return for the option's maturity (in years) under the risk-neutral measure is given by $z_\tau = \dfrac{R_T - E^{\mathbb{Q}}[R_T]}{\sigma\sqrt{\tau}}$.

In a first step, we generate an n-step standardized binomial density with $n+1$ possible values y_j and corresponding probabilities $b(y_j)$, according to:

$$y_j = \frac{(2j) - n}{\sqrt{n}}, \tag{12}$$

[5]See Rubinstein (1998) or Duan et al. (2003).

and

$$b(y_j) \quad = \quad \frac{n!}{j!(n-j)!}\left(\frac{1}{2}\right)^n, \tag{13}$$

where j goes from 0 to n. In a second step, we transform this density, given the prespecified skewness and kurtosis, into a density $f(y)$ with desired third and fourth moments using the Edgeworth expansion:

$$\begin{aligned} f(y) \quad = \quad & \Bigg[1 + \frac{1}{6}\kappa_3(y^3 - 3y) + \frac{1}{24}(\kappa_4 - 3)(y^4 - 6y^2 + 3) + \\ & + \frac{1}{72}\kappa_3^2(y^6 - 15y^4 + 45y^2 - 15)\Bigg] b(y), \end{aligned} \tag{14}$$

where $\kappa_3 = E^{\mathbb{Q}}[z_\tau^3]$ denotes the skewness and $\kappa_4 = E^{\mathbb{Q}}[z_\tau^4]$ the kurtosis of the cumulative standardized return for the options's maturity (in years).

As the Edgeworth expansion is only an approximation, we have to rescale the probabilities:

$$P_j = \frac{f(y_j)}{\sum\limits_j f(y_j)} \tag{15}$$

to ensure that the probabilities sum to unity. The variable y is then no longer binomially distributed. Standardizing this variable by subtracting mean $M = \sum_j P_j y_j$ and dividing by standard deviation $V = \sqrt{\sum_j P_j(y_j - M)^2}$ yields a random variable x with mean zero and variance one.[6]

Now, we can obtain the value of the underlying in the last time step and node j as

$$S_{T,j} = S_t e^{\mu\tau + \sigma\sqrt{\tau}x_j}, \tag{16}$$

with

$$\mu = r_{ann} - q_{ann} - \frac{1}{\tau}\ln\sum_{j=0}^{n} P_j e^{\sigma\sqrt{\tau}x_j}, \tag{17}$$

[6]As pointed out by Rubinstein (1998), another problem that can arise is the fact that $f(y)$ may not be non-negative everywhere, and thus, not a valid density function. However, he also demonstrates that there exists a large number of possible values for the third and fourth moments where this problem does not arise.

where r_{ann} and q_{ann} denote the continuously compounded risk-free rate and dividend yield on an annual basis, respectively. According to Equation (17), the expected risk-neutral return equals the risk-free rate (reduced for possible dividend payments). Hence, μ ensures risk-neutrality (see Rubinstein (1998)).

Finally, one constructs the entire stochastic process working backwards from the maturity of the option. Following Rubinstein (1994), we calculate the probabilities p_j of a single path to node j as

$$p_j = \frac{P_j}{\left(\frac{n!}{j!(n-j)!}\right)}. \tag{18}$$

Then, invoking the principle of no-arbitrage, one can calculate the probability and asset price pair (p, S) for the preceding node:

$$\begin{aligned} p &= p_j + p_{j+1}, && (19)\\ p_u &= \frac{p_{j+1}}{p}, && (20)\\ S &= [(1-p_u)S_j + p_u S_{j+1}]e^{-(r-q)\frac{\tau}{n}}. && (21) \end{aligned}$$

Following this recursive algorithm, we can construct the entire tree, which precludes arbitrage, as shown by Rubinstein (1994).

Finally, to be able to construct the EBT as described previously, one has to specify the first four moments of the cumulative return distribution. Duan et al. (1999), as well as Duan et al. (2006a), derived analytical expressions for GARCH, NGARCH, GJR-GARCH and EGARCH processes. These moments can be calculated in the style of Equation (10), according to the following formula:

$$E^{\mathbb{Q}}\left[R_T^k|\Im_0\right] = E^{\mathbb{Q}}\left[\left(T(r-q) - \frac{1}{2}\sum_{s=1}^{T} h_s + \sum_{s=1}^{T}\sqrt{h_s}\varepsilon_s\right)^k\right] \tag{22}$$

for $T \in \{1, 2, ...\}$ and $k \in \{1, 2, 3, 4\}$, where T is the number of discrete time steps and $E^{\mathbb{Q}}[\cdot]$ represents the conditional expectation under the risk-neutral measure. We obtain the required moments by expanding the right-hand side of Equation (22) and applying the expectation operator to the resulting terms. As the resulting formulas are algebraically cumbersome, we refrain from presenting them in the paper.

III Return and Option Data

The data used in the empirical study includes options written on the Standard and Poor's 100 index (OEX) as well as on the stocks of General Electric (GE). Standard and Poor's 100 index options are an obvious choice, as they are one of the most liquid contracts available. In addition, they are the natural American counterpart to the options on the Standard and Poor's 500, which is subject of many empirical studies regarding the pricing performance for European option contracts. GE was chosen, as it is one of the US' major companies and options written on its stock belong to the options on dividend paying underlings heaviest traded during our option sample period. All data was obtained from Bloomberg Financial Services.

The return series of the two underlyings was sampled for the period from July 2, 1991 to December 31, 2008, yielding a total of 4412 observations for each series. Table 1 provides an overview of diverse descriptive statistics and selected diagnostic tests. The table shows that GE has a substantially higher annualized volatility compared to OEX, whereas the skewness and kurtosis indicate that the index as well as the individual stock returns are slightly left skewed and exhibit a high excess kurtosis. Not surprisingly, primarily due to the high kurtosis, the null hypothesis that the returns follow a Gaussian distribution is rejected by the Jarque-Bera test of normality at any reasonable significance level.

The Ljung-Box statistic testing the hypothesis of zero autocorrelation up to lag 20 suggests, both for the returns[7] and the squared returns, the existence of significant autocorrelation in the first and second moments of the return processes. Besides, considering the ARCH-LM test statistic, we can reject the null of independent and identically distributed return innovations for both series. This implies the existence of GARCH effects.

Fitting an NGARCH model according to Equation (2) to the two time series and repeating

[7]We assume that the returns are generated as the sum of a constant and an innovation term.

the Ljung-Box test for the squared residuals yields clear evidence that such a model seems appropriate to adequately capture the dependence structure of the return series. The null of independence cannot be rejected with p-values of 0.82 and 0.58 (see line $\hat{Q}^2(20)$).

The empirical study is conducted using three years of option data for options traded on the Chicago Board of Options Exchanges (CBOE). The data was sampled weekly on Wednesdays for the period of January 2006 through December 2008. When a particular Wednesday was a holiday, we used the following trading day. The weekly sampling frequency enables us to analyze a comparably long data sample in a reasonable amount of computer time. Moreover, following Dumas et al. (1998), such a procedure stands in the tradition of multiple other empirical studies on option pricing (e.g., Christoffersen and Jacobs (2004) and Stentoft (2005) amongst others). We chose Wednesdays for our study, as fewer holidays occur on a Wednesday. In our sample period, however, only one holiday fell on a Wednesday. For further possible advantages of this approach, refer to Dumas et al. (1998).

Prior to analyzing our data, we cleaned our sample for all contracts violating the simple no-arbitrage condition:

$$V_t \leq \max\{d \cdot (S_t - X), 0\}, \tag{23}$$

as the continuation value should not be negative. Here, V_t denotes the market price of the option at time t; d is a dummy variable equaling one when we consider a call option and minus one for a put option; S_t denotes the price of the underlying at time t; and X represents the exercise price.

Following Dumas et al. (1998), we excluded all options with less than six and more than 100 trading days to maturity. Moreover, only options on the S&P 100 which prices lie between moneyness bounds of $[-0.05; 0.05]$, with moneyness Mon defined as

$$Mon = \frac{d \cdot (S_t - X)}{X} \tag{24}$$

were used. For options on GE, however, we abstained from such a filter criterion due to the discrete tick size. As the individual stock has a significantly smaller price compared to the OEX, we would only include a comparably small band of different strikes, if using a relative moneyness filter.

In addition, we follow Lehnert (2003) and excluded all options with a trading volume of less than 20 contracts on a specific Wednesday to circumvent any liquidity related pricing biases. As we apply a relative loss function in our model comparison, we exclude all contracts with prices of less than \$0.25. Following the study of Hsieh and Ritchken (2005) we divide each of our three sample years of option data into two halves. The first halves are denoted in-sample (is), while the second halves are denoted out-of-sample (oos) periods.

We further subdivide the options sample with respect to maturity and moneyness to get a more comprehensive picture of the data set and the pricing results in the following sections.

Regarding moneyness, all options with moneyness between $[-0.05; -0.02)$ are denoted as out of the money (OTM), between $[-0.02; 0.02)$ as at the money (ATM), and between $[0.02; 0.05)$ as in the money (ITM). Option contracts with less than 41 trading days to maturity are considered to be short-term, from 41 to 70 trading days to be middle-term, and maturities from 71 to 100 trading days long-term contracts. To keep the presentation manageable, we do not report results for call and put options separately.

Table 2 (OEX) and Table 3 (GE) present selected sample statistics for the option prices for the different moneyness and maturity bins, as well as for the entire data set. We report the number of observations, the average option price, the average trading volume, and the implied volatility, which enhances a comparison across moneyness. The implied volatility is calculated using a standard binomial tree.

One can observe that the number of observations is of equal size for the in- and out-of-sample bins. Most contracts considered are classified as short-term, which is not unexpected, as trading increases when maturity approaches. This can be directly observed from the volume statistics in Panel C.

It is remarkable that the average option price out-of-sample is higher than the in-sample for all moneyness and maturity bins. Looking at yearly data, we find that this phenomenon only occurs in the years 2007 and 2008 and can possibly be explained by a higher volatility in the second halves of these years. Panel D of Table 2 and Table 3 clearly indicate that the binomial tree implied volatility is not constant, but rather seems to follow systematic moneyness and maturity patterns.

IV Estimation Methodology

In this section, we discuss the parameter estimation methodology. First, we describe the estimation of the NGARCH parameters under the physical probability measure $\mathbb{P}$ and how to transform these estimates into risk-neutral parameters under the measure $\mathbb{Q}$. This approach only uses the information contained in the historical evolution of the underlying. Second, we describe the implied estimation approach of the NGARCH parameters, directly under the risk-neutral measure using option data as well as asset returns.

A. Historical Estimation using Asset Returns Only

In a first step, we estimate the parameters under the physical probability measure using the method of maximum likelihood. To be more specific, we maximize the following log-likelihood function (conditioned on the first observation):

$$\begin{aligned} \ln L(\vartheta; R_t) \quad = \quad & -\frac{1}{2}\Bigg[(T-1)\ln(2\pi) + \sum_{t=2}^{T}\ln(h_t) + \\ & + \sum_{t=2}^{T}\frac{(R_t - r + q - \lambda\sqrt{h_t} + \frac{1}{2}h_t)^2}{h_t}\Bigg], \end{aligned} \tag{25}$$

where ϑ denotes the vector of GARCH parameters; R_t represents the time series of continuously compounded returns of the underlying; and h_t follows the GARCH process described by Equation (2). According to the chosen GARCH-in-mean model, we have to estimate five parameters, namely β_0, β_1, β_2, θ, and λ. To reduce this number and improve the reliability of the estimation procedure, we apply the so-called variance targeting methodology of Engle and Mezrich (1996), demanding that the unconditional variance equals the sample variance. Besides the usual non-negativity constraints of the β-parameters, we impose the restriction that the process is covariance stationary. This is equal to require a persistence ν smaller than one, where ν is given by

$$\nu = \beta_1 + \beta_2(1 + \theta^2). \tag{26}$$

We estimate these parameters for every year in our option sample over a period of 15 years, ending with the in-sample period of the specific year, e.g., for 2006 from July 2, 1991 to June 30, 2006. We use a comparably long estimation window, as it is well known in the literature that it is difficult to estimate GARCH parameters precisely from return data with short samples. We use the mean of the bid-ask-price of the one month T-Bill rate at the end of the respective estimation window as the risk-free interest rate, as most option contracts in our sample lie in the first maturity bin. The current dividend yield of the OEX of the specific year is taken as dividend yield in the estimation, both continuously compounded and converted to a daily basis with a factor of 1/252. For GE we account for the actual cash dividends.

B. *Implied Estimation using Option Prices and Asset Returns*

The estimation of GARCH parameters using asset returns only draws on past information. The values of options, however, are determined by the expected future price movements of the underlying. As such, it seems likely that an implied calibration of GARCH parameters enlarges the available information set and ultimately results in a smaller pricing error. A further advantage of this approach is rooted in the fact that one directly obtains risk-neutral parameters, since an additional risk neutralization is obsolete. As pointed out by Barone-Adesi et al. (2008), an estimation under $\mathbb{P}$ and subsequent transformation to risk-neutral parameters under $\mathbb{Q}$, leads to a rather poor pricing performance of European options and is dominated by an implied calibration.

Thus, implied model calibration can now be seen as the standard approach to value European options. However, efficient calculation schemes were lacking to translate this procedure to the issue of pricing American options. Therefore, we explain the methodology used subsequently in some detail.

We estimate the GARCH parameters applying a non-linear least squares optimization approach using one day of option data. For each optimization, we consider the mean absolute percentage error (MAPE) as the objective function:

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\frac{|\tilde{V}_i - V_i|}{V_i}, \tag{27}$$

where $\tilde{V}_i$ and V_i denote the model price and market price of the i-th option, respectively, and N denotes the number of option contracts on a particular Wednesday. We apply a relative loss function to assign out of the money options sufficient weight. However, in contrast to the majority of empirical papers, we do not use the root mean squared error as the MAPE calculates a percent error and is, in our opinion, easier to interpret. The model prices are computed using the Edgeworth binomial tree described in Section II. The local volatility is determined by taking the unconditional volatility implied by the GARCH parameters 252 trading days before the specific pricing day and updated by using the observed return data. To be more specific, we update the volatility from h_{t-1} to h_t by substituting

$$\varepsilon_{t-1} \quad = \quad \left(\frac{R_{t-1} - r + q + \frac{1}{2}h_{t-1}}{\sqrt{h_{t-1}}}\right) \tag{28}$$

into

$$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 h_{t-1}(\varepsilon_{t-1} - \theta - \lambda)^2 \tag{29}$$

resulting in an updating rule consisting of observables only:

$$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 h_{t-1}\left[\left(\frac{R_{t-1} - r + q + \frac{1}{2}h_{t-1}}{\sqrt{h_{t-1}}}\right) - \theta - \lambda\right]^2. \tag{30}$$

Due to estimation under the risk neutral measure, we cannot identify θ and λ separately, as we can only estimate $\theta^* = \theta + \lambda$. This issue, however, is no problem in an option pricing context, as we only require the sum of the two parameters. We use the maximum likelihood parameters as starting values for the first optimization in every year and thereafter use the estimates of the previous week as starting values.

For the implementation of the Edgeworth binomial tree, we apply a two step procedure. First, we calculate the first four moments of the cumulative return over the maturity of the respective option. Second, we implicitly construct the tree, starting at the maturity of the option according to Section II.C and assuming one time step for every day to maturity in the tree construction with one exercise possibility per day. The interest rate and the dividend yield is hold constant over the maturity of the option.

For options on GE, we assume that we know the actual cash dividend as well as the dividend day ex ante. According to the dividend history of GE and regarding the fact that the majority of the options fall into the short to medium maturity bins, this assumption does not seem to be very restrictive and is in accordance with previous literature.[8] Furthermore, we hypothesize that any cash dividend fully spills over to the stock price, ignoring tax aspects, transaction costs, or investor specific preferences. We account for these discrete dividends by first calculating the values of the underlying, according to Subsection III.C, and then subtracting the sum of the compounded dividends. As the sole exception from the described construction of the tree for the index, we, whenever a dividend payment occurs, add this amount back to the before implicit calculated stock price after the exercise decision has been made. This methodology ensures that the implicitly calculated asset price at time t_0 corresponds to the actually observed stock price.

We re-calibrate this model every week and only use the options of the particular Wednesday for the optimization. Heston and Nandi (2000) argued that the estimation of the required parameters using only one day of option data might be problematic. Firstly, such a procedure might lead to an over-fitting of the data, which would lead to a good in-sample pricing performance. The out-of-sample performance, however, might significantly deteriorate. Moreover, this approach could unnecessarily restrict the available information set, as it seems possible that the historical evolution of the underlying could

[8]See, e.g., Stentoft (2005).

contain additional valuable information over and above the information contained in option prices. In our opinion, the first objection should not gain any empirical validity as, for example, the average number of contracts for OEX is greater than 31 options per Wednesday. The second argument is mitigated by the use of the volatility updating rule used to determine the starting volatility. For robustness reasons, we tested a model for OEX options which calibrated the parameters over the option data over the last five weeks. The results, however, showed that this approach is inferior compared to weekly updating, and thus, not reported to save space.

Hsieh and Ritchken (2005) proposed to include the starting variance as an additional parameter to the optimization algorithm. Robustness tests, however, indicated that, for our option sample, such a procedure results in a worse pricing performance, especially in the case of out-of-sample valuation.

Finally, we would like to emphasize that the tree based valuation is about 500 times faster then the simulation based approach and thus, much more suitable for real-world applications.

V Empirical Results

In this section, we present the main results of our empirical study. First, we report the estimated parameter values. Second, we present and discuss the pricing performance of the two different approaches, and finally, further analyze the resulting pricing errors in a regression analysis.

A. Parameter Estimates

Table 4 summarizes the estimated parameters using the maximum likelihood approach and provides a statistical overview. All GARCH parameters seem to be quite stable over time, which is, of course, due to the fact that the estimation windows are largely overlapping. The λ-parameter, though, is, especially for OEX, an exception, as it is quite volatile. This finding, however, is in accordance with the previous literature and nothing special for this data sample. As λ only enters the mean equation, it might be possible that this parameter is only poorly estimated. Furthermore, the leverage parameter θ is positive for both underlyings. This finding implies a negative correlation between the return and the conditional volatility and results in a left-skewed distribution when considering multiple periods. The persistence of both processes is quite high for all periods, again in line with the existing literature. The annual volatility implied by the GARCH parameters is roughly 17 % for OEX and 26 % for GE.

Table 5 summarizes the parameter estimates of the implied estimation approach using option and return data. We report the mean and the standard deviation of the weekly estimated parameters. It is noteworthy that β_0 is the parameter with the highest fluctuation, followed by β_1, whereas β_2 and θ^* are estimated relatively stable which is in line with the literature (see, e.g., Heston and Nandi (2000)). The parameter β_2 determines the volatility of volatility, whereas θ^* influences the skewness of the multi-period returns. Comparing these estimates with those obtained using the historical data and the method of maximum likelihood it is evident that β_0 as well as β_2 are partially estimated substantially higher, whereas β_1 takes on consistently smaller values. For θ^*, however, we cannot draw any clear cut conclusions. Regarding the OEX data, the risk-neutral estimates are clearly higher than the sum of the maximum likelihood based parameters θ and λ. Considering the GE estimates, however, the implied calibration approach results in slightly lower estimates for the years 2006 and 2007 and in a substantially higher parameter for the year 2008.

Considering the persistency, it becomes clear that the physically estimated parameters are considerably more persistent. This might be at least partially explained by the extreme amplitudes of the implied volatilities during the period of the option sample. For example, in March 2007, the implied volatility of options on OEX more than doubled to quickly return to its base level again. Engle and Mustafa (1993) showed in an analysis of implied volatilities of options on the S&P 500 around the stock market crash of 1987, that the volatility dynamic changed structurally directly after the Black Monday resulting in a significantly lower persistency of the process. For options on the index the annualized volatility implied by the mean parameters of the non-linear least squares approach is lower for 2006, but higher for the following years, compared to the maximum likelihood based volatilities. In contrast, the volatility of GE is lower until 2007, but higher for 2008. As the volatility of the non-linear least squares approach is based on mean parameters, an entirely true explanation seems impossible. Looking at the implied volatilities we see, however, that at least until the first half of 2007, the options implied volatility is considerably lower than the volatility implied by the maximum likelihood based GARCH parameters. The latter fact is the result of a quasi average consideration over the last 15 years and imply as such, through the inclusion of the very volatile markets around the burst of the dotcom bubble, itself a high volatility. Additionally, it seems possible that the parameters of the two different estimation methodologies are generally different, as the risk-neutral and the physical estimation procedures might emphasize different moments of the cumulative asset returns distribution. The variation of the implied estimated parameters over time indicates that a weekly implied re-calibrating might improve the pricing performance compared to the maximum likelihood based approach.

B. Option Valuation

Each option in the sample is valued using the two estimation and valuation approaches described above. First, as benchmark, we consider the LSM-based method proposed and analyzed by Stentoft (2005). Second, we employ the implicitly calibrated parameters and the Edgeworth binomial tree.

Additionally, we contrast these two models with GARCH volatility with the standard model of constant volatility using a standard binomial tree. Here again, we use the number of days to maturity as time steps for the tree construction. To be a competitive benchmark, we re-estimate the constant volatility for the options weekly using all options of the respective Wednesday. For GE, we proceed in the same fashion as in the EBT to deal with the cash dividend. This procedure ensures that the binomial tree remains recombining, and thus, numerically tractable. An adjustment of the variance seems obsolete as the weekly constant volatility is optimally determined from the data.

Table 6 (OEX) and Table 7 (GE) summarize the model performances overall and for different moneyness and maturity bins. The performance measure employed is the MAPE which was used for estimation. For the historical estimation and LSM valuation approach, in-sample pricing errors are based on the parameters of Subsection *A.* estimated with data until June 30 of the respective year. Out-of-sample errors are based on these same parameters. This means that the parameters are held constant for every specific year.

For the Edgeworth and the standard binomial tree, we calculate in-sample errors with the parameters and volatility of the week used for estimation. Out-of-sample, we use the parameters and volatility of the previous week.

Contrasting the in-sample pricing performance of the implicitly estimated Edgeworth binomial tree with the LSM approach for the two underlyings, we observe that the former yields substantially smaller pricing errors. For the OEX, the MAPE amounts to 22.08 % for the LSM approach, compared to 8.49 % when using the implied Edgeworth tree, which is almost three times smaller. For the options of GE, the overall performance is better for both approaches; again, a much better performance of the EBT approach, which yields an in-sample MAPE of 3.24 % compared to 12.84 % of the LSM approach, is observed. Interestingly, the latter is even outperformed by the simple binomial tree with constant volatility.

It is noteworthy that the result of smaller pricing errors of the EBT approach holds true for every subcategory with the largest differences being for out-of-the-money contracts with only short time to maturity. Even the standard binomial tree with the assumption of constant volatility results for GE in smaller errors for every bin; for options on OEX, however, the LSM-simulation and the binomial tree seem to be overall comparable with the first model, being superior for some categories, whereas the latter one for others. The Edgeworth binomial tree, however, is clearly superior to the standard lattice.

These in-sample results are not too surprising, as the GARCH parameters for the simulation are held constant, whereas for the other two models, the required parameters are optimally determined by the option data. As correctly noted by Bakshi et al. (1997), a true model comparison can only be performed in an out-of-sample context, as in-sample, the superior pricing results might just be the result of an overfitting of the data.

Looking at out-of-sample pricing errors, it is especially evident for the standard binomial lattice that the good in-sample results are merely the result of data-fitting. The pricing performance worsens for nearly every bin. Although this point is also true for the

Edgeworth binomial tree, the latter approach, however, remains clearly superior compared to the LSM-based approach, resulting in smaller pricing errors for every bin. Overall, the implicit Edgeworth tree yields a MAPE of 14.01 % for the OEX options and 10.20 % for General Electric. In contrast, the benchmark approach of Stentoft (2005) produces overall pricing errors of 17.23 % and 12.97 %.

Looking at the performance of the LSM-based approach, it seems surprising that the out-of-sample pricing errors are smaller than the in-sample ones for nearly every bin. The fact, however, that we can hardly talk of in-sample when estimating the GARCH parameter with maximum likelihood using a historical return series of 15 years, can be seen as an indicator for why the pricing performance in-sample need not be higher than that of out-of-sample.

C. Analysis of Pricing Errors

Looking at the pricing performance of the different models, the results clearly suggest that there might still be some systematic factors driving the pricing errors. Therefore, we analyze the errors in more detail by performing a simple regression analysis. We regress the MAPE's of the respective models on a constant (Con), days to maturity (DTM), squared days to maturity (DTM^2), moneyness (Mon), squared moneyness (Mon^2), trading volume (Vol), and a dummy variable (Put), which is set to one if the option is a put and zero otherwise. To be able to compare the results, we re-scale the moneyness to lie between zero and one. The estimated equation read:

$$\begin{aligned} MAPE &= \alpha_0 + \alpha_1 DTM + \alpha_2 DTM^2 + \alpha_3 Mon + \alpha_4 Mon^2 + \\ &\quad + \alpha_5 Vol + \alpha_6 Put + \xi \qquad (31) \\ \xi &\sim N(0, \sigma_t^2). \qquad (32) \end{aligned}$$

Table 8 (OEX) and Table 9 (GE) summarize the results of this regression. Heteroscedasticity-robust t-statistics are reported in parenthesis; in brackets, we provide the partial R^2.

As all regressors except the trading volume are known in advance, potential systematic pricing errors could be considered in the pricing models. Looking at the results for OEX (Table 8), we can conclude that nearly all coefficients are, at least at the 5 % level, significantly different from zero. Days to maturity for the LSM approach and trading volume for the binomial tree, however, are exceptions. Options with longer maturities are priced better for the standard and Edgeworth binomial trees, this effect however weakens with a longer time to maturity. All models price options that are in the money more precise than the out of the money options, whereas this effect also decreases with moneyness. Trading volume seems to have only negligible effects on the pricing performance.

The LSM and Edgeworth binomial tree-based prices for put options are more precise than for calls. This result is reversed for the the standard binomial lattice. Contrasting the models with variable variance with the constant volatility tree, it is clear that these models have a considerably lower R^2. Thus, days to maturity, moneyness, volume and the put-call dummy variable have a less systematic impact on pricing errors. According to the partial R^2, the smaller R^2 can especially be attributed to a less systematic influence of the moneyness effects. Considering the results for GE (Table 9), one can observe that these are very similar. The models with time-varying volatility show a significantly lower R^2 here, too, whereas this effect can again be assigned to the moneyness parameters. Looking at the overall results, we can conclude that the Edgeworth binomial tree does not only result in the smallest mean absolute pricing error, it is also less exposed to systematic pricing errors. However, days to maturity and moneyness still have a significant impact on pricing performance.

VI Conclusion

In this paper, we consider the problem of calibrating option pricing models when the underlying asset exhibit a GARCH type volatility and the option is of the American-type. This is of high relevance, as GARCH option pricing models are popular among practitioners and academics alike. Although most exchange traded option contracts are early exercisable, previous empirical research has mainly considered European options.

In a recent paper, Stentoft (2005) suggested coping with the American feature by using the least squares monte carlo simulation approach. Although this algorithm is extremely popular, we argue in this paper that it might not be the preferable approach when a GARCH option pricing model needs to be calibrated. In contrast, we suggest the usage of Rubinstein's Edgeworth binomial tree, which allows for an implicit model calibration. The empirical results show that this method is indeed superior.

Obvious areas for future research are the consideration of other GARCH specifications when pricing with the Edgeworth binomial tree. Moreover, it would be interesting, how other lattice-based approaches as the Johnson binomial tree suggested by Simonato (2009), performs for pricing American options.

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Table 1: **Sample Statistics for Return Series**

This table reports descriptive statistics for the daily continuously compounded returns for the S&P 100 and General Electric. The sample period is January 2, 1991 to December 31, 2008 for a total of N=4412 observations. For the Jarque-Bera statistic (JB) the brackets right of the statistic show the p-value for the significance of the difference of the third and fourth empirical moments and their theoretical values from the Gaussian distribution. $Q(20)$ *reports the value of the Ljung-Box test statistic for up to 20th order autocorrelation in the returns, whereas* $Q^2(20)$ *and* $\hat{Q}^2(20)$ *shows the statistic for the squared returns and GARCH residuals, respectively.* $LM(5)$ *reports the ARCH-LM test statistic for up to 5th order serial correlation in the returns. Next to these statistics, p-values are reported.*

	OEX		*GE*	
Statistic	Estimate	p–value	Estimate	p–value
N	4412	-	4412	-
μ	3 1.9846E-04	-	.1497E-04	-
σ	0.0018	-	0.0177	-
σ_{ann}	0.1865	-	0.2803	-
Min	-0.0919	-	-0.1368	-
Max	0.1066	-	0.1276	-
$Skewness$	-0.1919	-	-0.1388	-
$Kurtosis$	12.2235	-	10.1708	-
JB	15666.2687	[0.0000]	9466.9751	[0.0000]
$Q(20)$	119.3203	[0.0000]	72.0299	[0.0000]
$Q^2(20)$	5880.1405	[0.0000]	3836.1590	[0.0000]
$\hat{Q}^2(20)$	14.2171	[0.8193]	18.1822	[0.5754]
$LM(5)$	963.8075	[0.0000]	699.4798	[0.0000]

Table 2: **Descriptive Statistics for the S&P 100 (OEX) Option Sample**

This table reports descriptive statistics for options on the S&P 100 index quoted at the closing of every Wednesday for the sample period from January 2, 2006 to December 31, 2008. The implied volatilities are calculated using a standard binomial tree. DTM denotes days to maturity, OTM, ATM, and ITM represent out-, at-, and in-the-money, respectively. is stands for in-sample, oos for out-of-sample.

Panel A. Number of Options Contracts

	$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
	is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	750	691	201	154	33	33	984	878
ATM	1074	980	224	198	38	49	1336	1227
ITM	244	251	25	33	7	4	276	288
All	2068	1922	450	385	78	86	2596	2393

Panel B. Average Price

	$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
	is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	3.57	5.57	7.13	9.56	11.61	14.99	4.57	6.63
ATM	9.85	12.19	15.38	18.08	18.84	23.38	11.03	13.59
ITM	24.37	27.36	27.99	35.73	31.07	35.88	24.87	28.44
All	9.29	11.79	12.40	16.19	16.88	20.74	10.05	12.82

Panel C. Average Volume

	$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
	is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	937.89	770.17	212.36	209.85	164.64	93.33	763.76	646.45
ATM	1232.68	1034.53	165.27	152.86	128.16	182.57	1022.30	858.23
ITM	209.84	223.07	232.32	244.79	76.00	97.75	208.49	223.82
All	1005.09	833.52	190.03	183.54	138.91	144.38	837.78	704.18

Panel D. Implied Volatility

	$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
	is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	15.10%	20.30%	13.53%	16.73%	14.42%	16.45%	14.76%	19.53%
ATM	14.62%	19.38%	14.35%	17.47%	13.58%	16.44%	14.54%	18.95%
ITM	17.62%	23.72%	14.42%	24.67%	14.53%	17.61%	17.25%	23.74%
All	15.15%	20.28%	13.99%	17.79%	14.02%	16.50%	14.91%	19.74%

Table 3: **Descriptive Statistics for the General Electric (GE) Option Sample**

This table reports descriptive statistics for options on General Electric quoted at the closing of every Wednesday for the sample period from January 2, 2006 to December 31, 2008. The implied volatilities are calculated using a standard binomial tree. DTM denotes days to maturity, OTM, ATM, and ITM represent out-, at-, and, in-the-money, respectively. is stands for in-sample, oos for out-of-sample.

Panel A. Number of Options Contracts

	$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
	is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	20	84	51	91	46	90	117	265
ATM	235	271	118	144	79	126	432	541
ITM	275	288	135	167	98	123	508	578
All	530	643	304	402	223	339	1057	1384

Panel B. Average Price

	$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
	is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	0.42	0.56	0.47	0.60	0.49	0.62	0.47	0.60
ATM	0.93	1.15	1.27	1.44	1.42	1.78	1.11	1.37
ITM	4.73	5.72	5.78	6.55	5.85	6.37	5.23	6.10
ALL	2.88	3.12	3.14	3.38	3.18	3.14	3.02	3.20

Panel C. Average Volume

	$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
	is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	2035.50	3799.01	1641.00	1618.75	2377.93	1454.19	1998.17	2253.96
ATM	3857.15	4340.93	1948.35	1987.94	1873.32	1645.83	2972.98	3086.93
ITM	767.39	1324.74	442.14	473.91	440.57	535.08	617.91	910.87
All	2185.23	2919.18	1227.91	1275.40	1347.77	1191.94	1733.22	2018.65

Panel D. Implied Volatility

	$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
	is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	27.78%	52.06%	24.42%	42.49%	21.52%	23.45%	23.47%	39.06%
ATM	18.56%	25.96%	18.38%	21.44%	17.88%	19.73%	18.37%	23.30%
ITM	23.26%	29.96%	21.68%	27.51%	21.09%	23.27%	22.37%	27.83%
ALL	21.35%	31.16%	20.86%	28.73%	20.15%	22.00%	20.91%	28.21%

Table 4: **MLE Estimates of the GARCH–Parameter**

This table reports GARCH parameters using MLE to the daily log return series of the S&P 100 index and General Electric over a period of fifteen years. r and q represent the continuously compounded one-month T-Bill rate and dividend yield respectively, at the end of the particular period, converted to a daily basis. ν denotes the persistence and σ_{ann} refers to the annualized unconditional volatility, both based on the respective GARCH parameters, whereas the last row contains the value of the Log-Likelihood function.

	2006		*2007*		*2008*	
Parameter	OEX	GE	OEX	GE	OEX	GE
r	1.7922E-04	1.7922E-04	1.6326E-04	1.6326E-04	5.6344E-05	5.6344E-05
q	9.1400E-05	NA	8.3248E-05	NA	1.0224E-04	NA
λ	2.7778E-02	3.4768E-02	3.3798E-02	3.9615E-02	4.5767E-02	4.3504E-02
β_0	1.0991E-06	8.3154E-07	1.3177E-06	7.8980E-07	1.5284E-06	1.2291E-06
β_1	0.8935	0.9423	0.8742	0.9421	0.8650	0.9323
β_2	0.0557	0.0358	0.0640	0.0377	0.0664	0.0466
θ	0.8571	0.7255	0.8826	0.6763	0.9148	0.5981
ν	0.9901	0.9969	0.9880	0.9970	0.9870	0.9956
σ_{ann}	0.1672	0.2610	0.1663	0.2582	0.1723	0.2645
LL	12488.36	10714.70	12542.03	10803.47	12366.44	10678.73

Table 5: **NLS Estimates of the GARCH–Parameter**

This table reports the mean of the risk-neutral GARCH parameters using a non linear least-squares routine minimizing the loss-function of the respective week with a weekly re-calibration of the model. Standard deviations of the estimates are shown in square brackets below. ν refers to the risk-neutral volatility persistence, σ_{ann} to the risk-neutral annualized unconditional volatility, implied by the mean of the estimated GARCH parameters.

	2006		*2007*		*2008*	
Parameter	OEX	GE	OEX	GE	OEX	GE
β_0	9.0686E-06	2.4310E-05	1.9439E-05	4.4206E-05	1.2140E-04	1.3670E-04
	8.4230E-06	1.3391E-05	2.1527E-05	3.2449E-05	2.4929E-04	1.8454E-04
β_1	0.5543	0.4571	0.4827	0.4351	0.4622	0.4390
	0.2059	0.2572	0.1831	0.2139	0.2575	0.2376
β_2	0.1323	0.1945	0.1147	0.1859	0.1292	0.1492
	0.0628	0.0919	0.0372	0.0826	0.0515	0.0586
θ^*	1.2062	0.6868	1.5175	0.6687	1.3146	1.1667
	0.3946	0.3227	0.4441	0.3172	0.6508	0.2651
ν	0.8792	0.7434	0.8615	0.7041	0.8148	0.7914
σ_{ann}	0.1375	0.1545	0.1880	0.1940	0.4064	0.4064

Table 6: **Model Performance for options on the S&P 100 Index**

This table shows the overall pricing performance of the different models for the years 2006 to 2008 using the mean absolute percentage error (MAPE) as loss function. DTM denotes days to maturity, OTM, ATM, and ITM represent out, at and in the money, respectively. Lattice denotes the binomial lattice, LSM the least squares monte carlo algorithm, and EBT the Edgeworth binomial tree. is stands for in-sample, oos for out-of-sample.

		$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
		is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	Lattice	0.4200	0.4933	0.2633	0.3225	0.2709	0.1937	0.3830	0.4521
	LSM	0.4078	0.3154	0.3430	0.2331	0.2399	0.1486	0.3889	0.2947
	EBT	0.1274	0.2374	0.1007	0.1486	0.1111	0.1999	0.1214	0.2204
ATM	Lattice	0.1087	0.1521	0.1362	0.1543	0.1844	0.1537	0.1155	0.1525
	LSM	0.1296	0.1081	0.1236	0.1133	0.1252	0.0769	0.1285	0.1077
	EBT	0.0614	0.1014	0.0691	0.0891	0.0998	0.1193	0.0638	0.1001
ITM	Lattice	0.0727	0.0865	0.1095	0.1130	0.1265	0.1376	0.0774	0.0902
	LSM	0.0657	0.0699	0.0829	0.1120	0.1095	0.0790	0.0684	0.0748
	EBT	0.0550	0.0669	0.0726	0.0568	0.0829	0.0475	0.0573	0.0654
All	Lattice	0.2174	0.2662	0.1915	0.2181	0.2158	0.1683	0.2128	0.2549
	LSM	0.2229	0.1776	0.2194	0.1611	0.1723	0.1045	0.2208	0.1723
	EBT	0.0846	0.1458	0.0834	0.1101	0.1031	0.1469	0.0849	0.1401

Table 7: **Model Performance for options on General Electric**

This table shows the overall pricing performance of the different models for the years 2006 to 2008 using the mean absolute percentage error (MAPE) as loss function. DTM denotes days to maturity, OTM, ATM, and ITM represent out, at and in the money, respectively. Lattice denotes the binomial lattice, LSM the least squares monte carlo algorithm, and EBT the Edgeworth binomial tree. is stands for in-sample, oos for out-of-sample.

		$5 < DTM \leqslant 40$		$40 < DTM \leqslant 70$		$70 < DTM \leqslant 100$		All	
		is	*oos*	*is*	*oos*	*is*	*oos*	*is*	*oos*
OTM	Lattice	0.2764	0.4885	0.2419	0.4740	0.1960	0.3142	0.2298	0.4244
	LSM	0.5243	0.3477	0.4364	0.3452	0.3447	0.2875	0.4153	0.3264
	EBT	0.0585	0.2660	0.0335	0.3141	0.0293	0.2180	0.0361	0.2662
ATM	Lattice	0.0580	0.1140	0.0490	0.0896	0.0506	0.0856	0.0542	0.1009
	LSM	0.1466	0.1298	0.1485	0.1177	0.1834	0.1401	0.1538	0.1290
	EBT	0.0509	0.0908	0.0363	0.0957	0.0245	0.0940	0.0421	0.0928
ITM	Lattice	0.0271	0.0419	0.0281	0.0408	0.0278	0.0411	0.0275	0.0414
	LSM	0.0381	0.0391	0.0426	0.0362	0.0450	0.0488	0.0406	0.0403
	EBT	0.0254	0.0355	0.0211	0.0332	0.0205	0.0371	0.0233	0.0352
All	Lattice	0.0502	0.1306	0.0721	0.1564	0.0706	0.1301	0.0608	0.1380
	LSM	0.1045	0.1176	0.1498	0.1354	0.1559	0.1461	0.1284	0.1297
	EBT	0.0379	0.0889	0.0291	0.1192	0.0237	0.1063	0.0324	0.1020

Table 8: **MAPE Regression results for options on the S&P 100 index**

This table reports the results from regressing the MAPE on a constant, maturity (DTM), moneyness (Mon), volume as well as a Put-Call Dummy. Heteroscedasticity-robust t-statistics are shown in parenthesis; in brackets, we report the partial R^2. is stands for in-sample, oos for out-of-sample.

	Lattice		LSM		EBT	
	is	*oos*	*is*	*oos*	*is*	*oos*
	0.6014	0.7481	0.6136	0.4519	0.2063	0.5762
Con	(30.1920)	(17.6476)	(15.5766)	(17.0630)	(15.5374)	(4.0686)
	[0.2911]	[0.2545]	[0.1171]	[0.1777]	[0.1690]	[0.0575]
	-0.0042	-0.0040	0.0005	-0.0003	-0.0023	-0.0169
DTM	(-6.2870)	(-4.5435)	(0.4872)	(-0.4104)	(-4.7961)	(-1.6601)
	[0.0084]	[0.0041]	[0.0000]	[0.0001]	[0.0127]	[0.0278]
	3.73E-05	2.23E-05	-1.67E-05	-8.83E-06	2.67E-05	2.28E-04
DTM^2	(5.0922)	(2.2731)	(-1.5358)	(-1.0494)	(4.9954)	(1.4463)
	[0.0050]	[0.0009]	[0.0004]	[0.0003]	[0.0125]	[0.0364]
	-1.4510	-1.5727	-1.3096	-1.0015	-0.3551	-0.7520
Mon	(-23.2861)	(-12.8778)	(-11.8085)	(-11.5277)	(-10.3546)	(-6.5442)
	[0.1687]	[0.1107]	[0.0531]	[0.0859]	[0.0498]	[0.0096]
	0.9937	1.0320	0.8524	0.6823	0.2535	0.4671
Mon^2	(16.1873)	(9.6736)	(8.3307)	(8.6905)	(7.2497)	(5.4129)
	[0.0613]	[0.0403]	[0.0174]	[0.0337]	[0.0197]	[0.0031]
	-2.45E-07	2.69E-06	1.77E-05	9.26E-06	3.91E-06	-8.15E-06
Vol	(-0.0877)	(0.6762)	(3.0278)	(2.3859)	(2.1007)	(-0.9117)
	[0.0000]	[0.0001]	[0.0028]	[0.0018]	[0.0017]	[0.0003]
	0.1041	-0.0048	-0.1326	-0.0265	-0.0049	-0.0034
Put	(15.7446)	(-0.4127)	(-9.8472)	(-2.9891)	(-1.2901)	(-0.1692)
	[0.0501]	[0.0001]	[0.0315]	[0.0031]	[0.0006]	[0.0000]
R^2	0.4647	0.2768	0.1825	0.2022	0.1261	0.0625

Table 9: **MAPE Regression results for options on General Electric**

This table reports the results from regressing the MAPE on a constant, maturity (DTM), moneyness (Mon), volume as well as a Put-Call Dummy. Heteroscedasticity-robust t-statistics are shown in parenthesis; in brackets, we report the partial R^2. is stands for in-sample, oos for out-of-sample.

	Lattice		LSM		EBT	
	is	*oos*	*is*	*oos*	*is*	*oos*
	2.7674	1.6097	3.6207	1.1953	0.3486	1.0122
Con	(10.7808)	(12.3936)	(6.8283)	(13.5776)	(4.9834)	(6.3038
	[0.2661]	[0.2891]	[0.0682]	[0.2801]	[0.0179]	[0.0861]
	-0.0004	-0.0005	0.0006	-0.0010	-0.0008	-0.0040
DTM	(-1.2551)	(-0.6608)	(0.6168)	(-1.6116)	(-2.9722)	(-2.2587)
	[0.0007]	[0.0002]	[0.0002]	[0.0012]	[0.0096]	[0.0088]
	3.90E-06	3.39E-06	-3.09E-06	1.08E-05	5.73E-06	4.76E-05
DTM^2	(1.1459)	(0.4950)	(-0.3144)	(1.9533)	(2.4641)	(2.3523)
	[0.0006]	[0.0001]	[0.0001]	[0.0017]	[0.0058]	[0.0143]
	-12.3098	-5.7220	-15.4835	-4.0170	-1.2485	-3.1818
Mon	(-10.1575)	(-9.4067)	(-6.4374)	(-10.4959)	(-3.9918)	(-5.2960)
	[0.2322]	[0.2144]	[0.0550]	[0.1856]	[0.0101]	[0.0499]
	13.5871	4.7714	16.3160	3.1852	1.2060	2.4338
Mon^2	(9.5270)	(6.4532)	(6.0923)	(7.2733)	(3.4480)	(4.0999)
	[0.1970]	[0.1111]	[0.0425]	[0.0870]	[0.0066]	[0.0218]
	-2.10E-06	2.21E-06	2.73E-06	2.31E-06	1.81E-06	1.02E-06
Vol	(-2.4019)	(2.3277)	(1.2911)	(2.5182)	(2.1168)	(1.0454)
	[0.0048]	[0.0018]	[0.0012]	[0.0035]	[0.0152]	[0.0003]
	0.0064	-0.0035	-0.0251	-0.0144	0.0052	-0.0157
Put	(1.3504)	(-0.3117)	(-1.5892)	(-1.7672)	(1.7478)	(-1.0137)
	[0.0010]	[0.0001]	[0.0023]	[0.0016]	[0.0027]	[0.0008]
R^2	0.3996	0.3964	0.1574	0.3858	0.1073	0.1323